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Unveiling the Dynamics of Epileptic Seizures Through Nonlinear EEG Analysis

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ABSTRACT

Epileptic seizures are sudden disruptions in brain activity that can have life-threatening consequences due to their unpredictable nature. Analyzing electroencephalogram (EEG) signals using nonlinear methods offers valuable insights into the brain's behavior during different seizure phases: interictal (between seizures), preictal (preceding a seizure), and ictal (during a seizure). This study analyses the nonlinear characteristics of EEG signals collected from 10 epileptic patients. We investigate their dynamical differences as the seizure progresses through the interictal, preictal and ictal stages using nonlinear measures such as Hurst exponent, Lyapunov exponent, Sample entropy and Higuchi Fractal Dimension. Furthermore, the characteristics of the EEG signals are visualized by the Phase space portrait, Power spectral density, Recurrence Plot and quantified by means of Recurrence Quantification Analysis measures such as Determinism (DET), Average diagonal length (L_{avg}) and Recurrence Time Entropy (RTE). The results show that ictal EEG signals have lower values for entropy and fractal dimension than their interictal and preictal EEG signals, indicating the reduced complexity in the brain during the onset of the seizure. Similarly, the Hurst exponent was found to be greater than 0.5 for all three stages, with a rising trend in values as the brain passes from the interictal to the ictal stages. This demonstrates the persistent behavior and greater predictability of EEG signals in epileptic patients. The Lyapunov exponent revealed the existence of chaos in the brain. The long diagonal structures characterizing periodic behavior of the seizure EEG signals were identified from the Recurrence plot. We observed that the seizure EEG signals have high values for DET and $Lavg$ and low values for RTE compared to interictal and preictal EEG signals, indicating that the seizure EEG signals were highly deterministic, recurrent and less complex. The phase portrait and the spectral analysis also justify these observations. As a result, this study suggests that during these three stages, the underlying dynamics of the epileptic brain is not only a chaotic complex dynamical system, but it is also highly deterministic in the sense that prediction of seizure activity is possible in the short term. These findings shed light on the insights into the dynamics of brain activity and may serve as a theoretical foundation for further research on the clinical diagnosis and for developing empirical models that will aid in better prediction of seizures in patients with epilepsy.

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Introduction

Epilepsy is one of the most dominant and devastating neurological disorders that affect people worldwide due to the abnormal and synchronous firing of the neuronal population in the brain. This disorder is a condition characterized by the spontaneous occurrence of multiple seizures. According to recent research, approximately 70 million people worldwide have epilepsy, with nearly 90% of them residing in developed countries[1]. As a result of the sudden and unpredictable onset of seizures, epileptic patients are at risk of suffering various types of severe injuries, including accidents, burns, falls, drownings, and possibly even impairment of the nervous system. These threats not only have a negative impact on their normal quality of life but can even lead to death in some cases. The Electroencephalogram (EEG) is a promising tool capable of recording electrical activity in the brain and providing a way to communicate with the external world to diagnose epilepsy [2]. The possibility of Nonlinear Time Series Analysis(NTSA) opens a new window for a better understanding of EEG signal dynamics, allowing us to gain insight into the pathological and physiological states of the brain.

Over the past few decades, several studies using non-linear methods have been addressed in biomedical applications, particularly in the brain, in order to unravel the hidden dynamics and prediction of various brain diseases [3]. As the human brain represents a complex and chaotic dynamical system, the EEG signals recorded from its cortex are chaotic, complex, nonlinear, and non-stationary [3,4]. With powerful NTSA, it could be possible to detect the changes in the EEG and obtain sufficient information about the state of the brain. The dynamics of the brain do not disclose all of the underlying parameters. It is common to observe the evolution of only one parameter as time-series data. The behavior of such a complex dynamical system can be analyzed by reconstructing the phase space of univariate EEG time series data, which can provide much better information about the state of the system regardless of the underlying governing laws. Several studies have shown that non-linear analysis of the EEG signals has been successfully applied for the detection and prediction of seizures. For instance, an estimation of the correlation dimension reveals a decreased activity during an episode of epileptic seizure compared with the healthy subjects, indicating low dimensional chaos and reduced complexity in the epileptic brain [5,6]. An exponential divergence between nearby trajectories in a dynamical system can be quantified by estimating the Lyapunov exponent. Studies reported a lowering of the Lyapunov exponent values during the onset of seizures, indicating a reduced amount of chaos in the brain

[6,7]. In a new wavelet chaos methodology was introduced to effectively detect seizures and epilepsy using nonlinear measures in the form of correlation dimension and the Largest Lyapunov exponent by extracting five subbands from the EEG signals [8]. One of the basic features of a time series has been its self-affine nature. As the name implies, self-affine time series cannot be distinguishable from a small portion when the appropriate factor scales them. Fractal time series have the property of being self-affine, so the dimension will always be a non-integer value. Fractal Dimension (FD) is a powerful nonlinear tool for determining the complexity and self-affinity of physiological signals. Several methods have been employed to compute the FD in the EEG for the detection of epileptic seizures. FD methods such as Katz FD and Higuchi FD were studied to compare the EEG signals of healthy and epilepsy groups [6]. The results show that both methods reduce FD values in epileptic groups. Similarly, an improved version of the Generalised FD was reported to distinguish between EEG signals of healthy and epileptic subjects [9]. Entropy- based nonlinear studies have been extensively employed help to assess the degree of complexity and the rate of information generation of the underlying dynamical system. Studies have reported significant changes in ApEn, SampEn and Kolmogorov entropy by lowering their values in the epileptic region compared to the nonepileptic region [6, 10-12]. The variations in the interictal and ictal phases of epileptic activity in patients with absence seizure EEG were recognized using Permutation entropy [13]. Various entropy-based measures were also implemented to discriminate between normal and epileptic EEG signals to enhance the accuracy of the model for automatic seizure classification [10].

The presence or absence of long-term memory in a time series can be evaluated using the Hurst exponent, a commonly used non-linear measure to understand the complexity and predictability of physiological data. A method for predicting the onset of seizures several seconds prior to their occurrence has been proposed by analyzing variations in the Hurst exponent and fractal dimension of the EEG signal [14]. Several studies also proved the Hurst exponent as a non-linear tool for predicting the seizure onset and detecting seizures [15,16]. The ability to discriminate between healthy and epileptic groups was investigated, in which the EEG signals of both groups, especially the epileptic group, exhibit an antipersistent nature ($H<0.5$) [17]. On the contrary, the persistent nature ($H>0.5$) of EEG signals during the seizure has also been reported [15,18,19]. Recent developments in the theory of non-linear dynamics have resulted in novel approaches to visualising and quantifying time series analysis. Over the past years, measures associated with the Recurrence plot (RP) and Recurrence Quantification Analysis (RQA) have been widely used in EEG analysis to provide information on the non- linearities, complexity, and recurrence of brain activity for a better understanding of the dynamical characteristics during an epileptic seizure [20,21]. Moreover, these measures can deal with short datasets with nonlinear, noisy and non-stationary signals. RQA measures have been used to classify the EEG signals between the normal, interictal and ictal groups [20] and to identify pre-seizure states from the invasive EEG recordings of five epileptic patients [22]. The dynamical changes in the RP of the EEG signals were studied on rats, showing different structures during the pre-ictal, inter-ictal and ictal phases [23]. Similarly, three RQA measures, namely Recurrence rate, Determinism and Entropy, were performed on rat EEG signals for predicting epileptic seizures [24]. Studies provide evidence that the electrical activity of the brain in the epileptogenic areas resembles a deterministic process, whereas a stochastic process was observed in non-epileptogenic areas [25]. These studies all emphasize the significance of the NTSA in the EEG signal, which reflects the dynamics of brain activity for detecting and improving the accuracy of predicting seizures in epileptic patients. However, we observed a scarcity of adequate literature describing the nonlinear analysis of the epileptogenic domain, particularly EEG signals from interictal, preictal, and ictal stages in epileptic patients. Although numerous studies have been conducted on seizure EEG signals using nonlinear methods, the focus of these studies has been limited to classification problems and comparisons between healthy and epileptic groups (interictal and ictal stages). It excludes the evaluation of preictal EEG recordings for recognising preictal stages, which aid in predicting seizure onset. Ignoring the preictal stage may result in an insufficient understanding of brain activity suitability for seizure prediction.

In this study, we focus on investigating the nonlinear characteristics of EEG signals from the interictal (time interval between two seizures), preictal (before the seizure), and ictal (during the seizure) stages of a seizure and quantifying them in the form of predictability and complexity of the epileptic brain. We used 150 scalp EEG recordings collected from 10 epileptic patients, and then the three seizure stages were subjected to NTSA. The predictability and complexity of EEG signals are quantified by employing the Hurst exponent, Sample entropy, Lyapunov exponent and Fractal dimension. Furthermore, the dynamical changes at various stages were revealed by the RP and further quantified by three RQA measures, namely Determinism (DET), Average diagonal length(L_{avg}) and Recurrence time entropy (RTE). The DET and L_{avg} helps to identify the deterministic dynamics, recurrent structures and predictability of EEG signals from epileptic brains. RTE is a complexity-based measure that captures the transitions between chaotic and periodic dynamics (and vice-versa) of EEG signals during the various seizure stages. The dynamical information of EEG signals from the epileptic brain could be extracted using NTSA by reconstructing the phase portrait analysis. An estimate of divergence rates between two trajectories in this space can be performed using the maximal Lyapunov exponent, which determines the predictability of a dynamical system. The Hurst exponent (H) determines the predictability and degree of long-term memory in time series data based on values ranging between 0 and 1. A time series with H between 0 and 0.5 suggests a highly complex and antipersistent behavior. When H is close to 0.5, it is said to be Brownian motion, and thus it contains no information about the future, whereas when H is between 0.5 and 1, it shows the likelihood of forecast in persistent time series. The Sample entropy helps quantify the uncertainty and thus predictability of a time series. Fractal analysis of EEG signals reveals its self-similarity and complexity. FD is estimated using various methods, including box-counting, Kantz FD, Sevcik FD, epsilon bracket, Higuchi FD, power spectrum, and Petrosian C and D methods. We used Higuchi FD to characterise different structural properties of EEG signals in this paper because of its simplicity, high precision in results, and ability to estimate the dimension directly from the time domain. Adapting the Fourier transform technique to the EEG signal can provide a detailed description of the frequency components by converting the time domain signal into the frequency domain. The power spectrum is a straightforward method for distinguishing EEG signals between periodic and chaotic motion. Therefore, the power spectrum of a periodic motion has a discrete sharp delta peak, whereas the power spectrum of a chaotic motion has the continuous broadband nature of a noisy spectrum. The following structure is organized throughout the remainder of this paper. Section 2 describes the EEG data source and preprocessing procedure, as well as an overview of the nonlinear time series analysis. The structure of RP and the three RQA measures for analysing dynamical differences at various stages of seizures are introduced. Section 3 contains the results and discussion, along with the limitations of this work. Finally, Section 4 concludes the paper by encapsulating the relevant points of this study.

Materials and Methods

EEG Data and Preprocessing

The EEG data for the present study were obtained from the publicly available dataset with the Neurology and Sleep Centre, Hauz Khas, New Delhi, India [26]. This dataset contains EEG time series segments recorded from ten epileptic patients using the Grass Telefactor Comet AS40 Amplification System, digitized at a sampling rate of 200 Hz and bandpass filtered between 0.5Hz and 70Hz. The recording is done by placing gold-plated electrodes on the surface of the scalp according to the International 10-20 system. The EEG segments were carefully selected and cut out after being visually inspected by the clinical experts for various artifacts (eye blinks or muscle movements) from multiple channels of continuous EEG recording. The dataset is divided into three different folder sets based on the seizure stages, namely interictal, preictal and ictal stages. Each folder contains 50 EEG segments saved in MAT file format, with the duration of each segment lasting 5.12s and having 1024 sample points. Thus, a total of 150 (50 × 3) EEG samples are used for the analysis.

The electrical activity recorded from the cerebral cortex of the brain is often a challenging task to interpret, especially when the signal is contaminated with various artifacts like muscle movement, power lines and eye movements. In order to avoid spurious results, it is essential to denoise the EEG signals as much as possible to gain valuable information about the behavior of each region of seizure. Typically, the human brain generates signals ranging from 0.1Hz to 100 Hz. In this study, frequencies up to 30 Hz are used to analyze the EEG signal, thus avoiding the power line (50Hz/60Hz) and high-frequency components that are regarded as noise. Therefore, as a step for preprocessing, the EEG segments are filtered using an IIR zero-phase 10th order low pass Butterworth filter at a cutoff frequency of 30 Hz.

Nonlinear Time Series Analysis

Nonlinear time series analysis (NTSA) consists of several methods for analysing and extracting dynamic information about the underlying system from the time-series data. This analysis provides insight into the behavior of brain dynamics in the present state and determines the future state, taking into account the complexity and predictability of the EEG time series. The following is a brief description of nonlinear methods used to examine the degree of complexity, predictability, and determinism associated with the EEG signals during various seizure stages in epileptic brains.

Phase Space Reconstruction

The analysis of any time series using non-linear dynamics theory begins with the reconstruction of phase space. This method allows the reconstruction of the whole dynamics of a complex non-linear system from a univariate time series to extract essential information from the underlying system [27]. Hence, we need to reconstruct the phase space of brain dynamics from one-dimensional EEG by using time delay and embedding dimension. Phase space reconstruction is usually achieved using the Takens time delay embedding theorem [28]. According to Takens theorem, the scalar time series of N variables q_1 , q_2 ,q_3 ,…., q_N from a dynamical system is embedded into an m-dimensional phase space vector $X(t)$ can be formulated as

$$\vec{X}(t) = [\, x(t), x(t+\tau), \ldots, x(t+(d-1)\tau) \,] \quad (1)$$

where $t = 1,2,\ldots, N-(d-1)\tau$. Here τ denotes the time delay, d denotes the embedding dimension, and N represents the number of phase points in the reconstructed phase space. For a faithful representation of an attractor which is topologically similar to an attractor of an actual dynamical system, it is essential to specify an appropriate value of embedding dimension and time delay. In this work, the optimal time delay and embedding dimension are estimated by choosing the first local minimum of the Mutual Information function (MI) and the Caos method, respectively, for the phase-space reconstruction [29,30].

Lyapunov Exponent

The Lyapunov exponent (λ) or the Lyapunov characteristic exponent of a dynamical system is a measure that characterizes the average rate of separation of two nearby trajectories in the phase space. Let $\delta\ 0$ be the initial separation distance between the two trajectories $Z(t)$ and $Z_0(t)$ at time $t = 0$, then $\delta\ t$ at some time, $t = t$ is given by the relation,

$$\delta(t) = e^{\lambda t}\delta(0) \quad (2)$$

where λ is known as the Lyapunov exponent [31]. The rate of separation between the nearby trajectories varies exponentially over time in the phase space, and so does the magnitude of the λ. Therefore, if the value of λ is positive, this means that the nearby trajectories diverge over time, indicating that the system is chaotic, and if the value of λ is negative, the nearby trajectories converge over time, which points to the presence of a periodic regime in the system.

Complexity Analysis

Hurst Exponent (H)

The Hurst exponent is a non-linear parameter for determining the predictability and degree of long-term memory in physiological time series data. It is also an indicator of roughness in a fractal time series based on the value, which ranges between 0 and 1. Moreover, the Hurst exponent gives better insight into analyzing the dynamics of the system and classifies the time series into different types such as persistent, antipersistent and random. When analyzing the dynamics of the cerebral process, if the value of H lies between 0 and 0.5, it indicates that the EEG time series exhibits an antipersistent behavior. In other words, it means that a rise in the trend of a process in the next period would be the opposite of what was in the previous period, i.e. a fall in the trend, indicating a strong long-range negative correlation. Further, if the value of H lies between 0.5 and 1, it indicates that the time series exhibits persistent behavior, which means that a rise in the process trend in the next period would be the same as the previous period, which indicates a strong long-range positive correlation providing more significant information about the past and the present, thereby a better prediction. Finally, if H=0.5, the processes are uncorrelated, and the present has no bearing on the future. As a result, the time series can be considered random or Brownian motion. The calculation of the Hurst exponent using rescaled range analysis (R/S) method is expressed as:

$$H = \frac{\log\frac{R}{S} - \log c}{\log N} \quad (3)$$

Where $\frac{R}{S}$ is the rescaled range, c is a constant and N is the number of observations [32]. The ability of H to classify the time series based on predictability suggests that it might provide a valuable tool for identifying variations in brain activity during different regions of seizures. Additionally, the Hurst exponent can be used to calculate the fractal dimension (D) using the relation:

$$D = 2 - H \tag{4}$$

Higuchi Fractal Dimension

Fractal dimension (FD) is a commonly used measure in non-linear dynamics to characterize the complexity and self-similarity of biological signals. The fractal object has the generic property that exhibits self-similarity across different time scales (scale-invariant) and has a non-integer dimension. Typically, the greater the degree of self-similarity in the signal pattern, the higher the fractal dimension value and vice-versa. The EEG signal has the property of fractal nature as the dynamics of EEG exhibit statistical similarities across different time scales [32]. Unlike other measures, the Higuchi Fractal Dimension (HFD) is evaluated directly from the time domain without reconstructing the attractor in phase space, making it relatively simple to implement in the EEG signal. The dimension of HFD for a one-dimensional EEG signal is always between 1 and 2, which expresses the complexity of a two-dimensional curve representing a signal. A high value of HFD corresponds to a high degree of complexity or self- similarity in the signal.

The procedure for calculating HFD is as follows:

Consider an original time series having N number of sample points $X_1, X_2, X_3, \ldots, X_N$. From this time series, k new time series are constructed, which are represented by X^k as:

$$Xm^k = [X\ (m)\ , X\ (m+k)\ , X\ (m+2k)\ , \ldots X(m + int(N-m)/k)k]$$

where m = 1,2,3…k represents the initial time and k represents the time interval between sample points. Then, the length $L_m\ (k)$ of each curve X^k is calculated as:

$$L_m\,(k) = \frac{1}{k}\left\{ \left[\sum_{i=1}^{int(\frac{N-m}{k})} \left| X\,(m+ik) - X(m+(i-1)k) \right| \right] \frac{N-1}{int(\frac{N-m}{k})} \right\}$$

This calculation of $Lm\ \ k$ is repeated for each k ranging from 1 $to\ kmax$. The length $L(k)$ of the curve is computed by taking the mean of the k values of $Lm\ (k)$ for m = 1,2,3…k as indicated in:

$$L\,(k) = \frac{\sum_{m=1}^{k} L_m(\ )}{k} \tag{5}$$

The FD of a curve is determined from the slope of the least-squares linear best-fitting procedure by plotting log(L(k)) against log(1/k) [33]. The only parameter required for the HFD algorithm is the optimal value of k_{max}. For this, we first calculated the average of the HFD values for different values of $kmax$ in all EEG segments (each segment having 1024 sample points) from each stage. Then we investigate the association between the HFD values in each stage (Interictal, Preictal, and Ictal) with different values of $kmax$ by computing HFD for k_{max}= 2 to100. We observed that an association between the three stages remained consistent where the average HFD plateau reaches saturation once $kmax$ > 25. In the present study, we have chosen k_{max} = 40, as this value clearly distinguishes the maximum separation between the three stages. The average HFD values for the interictal, preictal and ictal stages for different $kmax$ values are depicted in Figure 1. There is also a monotonous increase in HFD values with an increase in k_{max}.

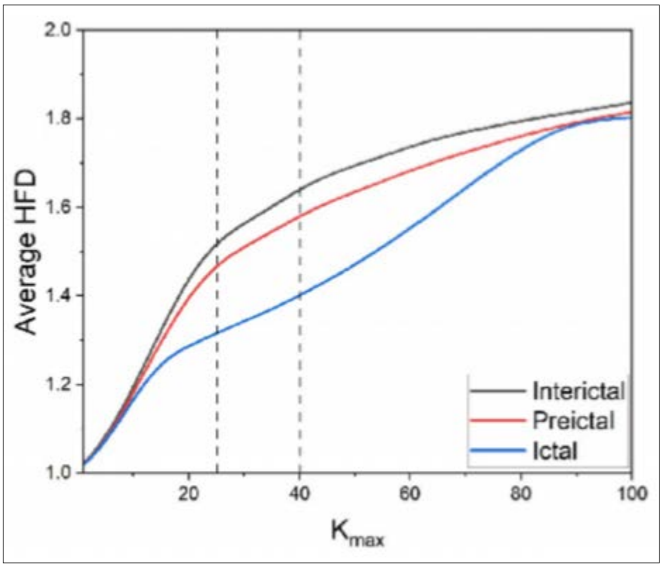

Figure 1: The average HFD of the corresponding three stages (Interictal, Preictal and Ictal) for different values of k_{max} ranging from 2 to 100. A dashed line at k_{max} = 25 represents the starting point at which the HFD plateau reaches saturation. The dashed line at k_{max} = 40 is the value chosen for this study.

Sample Entropy

Sample Entropy (SampEn) is a commonly used method in non-linear dynamics for measuring the uncertainty and complexity of biological signals. Conceptually, SampEn has defined as the negative logarithm of the conditional probability that two similar successions over 'm' points will remain similar over the following '$m+1$' points within a tolerance 'r' while ignoring self- matches using the equation given by [33]:

$$SampEn(m,r,L) = -\log\left(\frac{A^{m+1}(r)}{B^{m}(r)}\right) \tag{6}$$

where, L is the total number of sample points in a time series, m is the embedding dimension, $A^{m+1}(r)$ represents the likelihood that two points will match when they are m + 1, and $B^m(r)$ is the probability that two sequences will match when two points are m apart. Thus, a low value of SampEn corresponds to more regularity, predictability, self-similarity and less complexity in time series data. The ability of SampEn to classify biological signals into deterministic and stochastic signals on the basis of irregularity and persistence with a relatively short number of data points would be beneficial in representing the overall complexity and predictability of EEG signals during an epileptic seizure. Here, m and r are the two parameters that should be assigned for computing the SampEn. However, there is no specific solution for the appropriate selection of these values. Typically, the value of m is usually 2 or 3, and the value of r is usually between 0.1 and 0.25 times the standard deviation of the original time series. Based on previous research, we have chosen m=2 and r=0.2 (20%) for the analysis since these parametric values indicate good statistical validity in estimating the SampEn [34].

Recurrence Plot

In the real-world, many dynamical systems exhibit a fundamental property of revisiting the previously visited states (or recurrence). Recurrence Plot (RP) is a promising graphical tool in non-linear dynamics to visualize complex dynamics of systems on a 2D representation in phase space which shows the recurrences of states. Transforming the phase space from high to low dimension reveals the underlying dynamics and hidden periodicities that provide better insight into the behavior of a dynamical system. The recurrence of the region at time i and at time j is constructed by a $N \times N$ symmetric square matrix R with 0 and 1, which corresponds to black and white dots computed as:

$$R_{ij} = \Theta(\epsilon - \| x_i - x_j \|) \qquad i,j = 1,2,3,....N \tag{7}$$

Where $\Theta(\bullet)$ is the Heaviside function, ϵ is a predefined threshold radius of a sphere centred on x_i, $\|\bullet\|$ is the L_2 norm (Euclidean distance between two points), and N is the total number of considered points [37]. A recurrence is said to have occurred if the point x_j falls within the neighbourhood of radius ϵ of the sphere centred at x_i, then $R_{ij} = 1$ and a black dot would be displayed in the RP, otherwise $R_{ij} = 0$, which displays a white dot. A crucial parameter in the analysis of RP involves choosing an appropriate threshold value for ϵ for obtaining robust results. In this work, we determined the threshold value ϵ as the 4th percentile of the pairwise distance distribution of all points in the phase space in order to ensure the value of the global Recurrence Rate (RR) is always 0.04 [35].

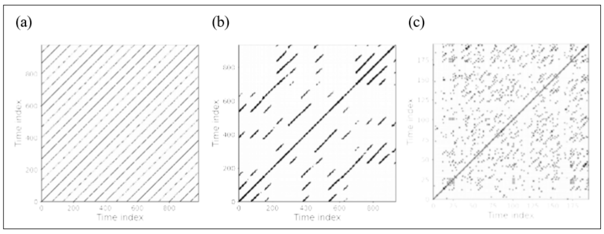

Figure 2: Example for recurrence plot of three types of time series (a) Periodic signal generated from the 3Hz Sine wave ($\tau = 4$,$d = 5$, $\epsilon = 0.18$, $RR = 0.04$). (b) A chaotic signal from the y component of Lorentz attractor ($\tau = 16$, $d = 5$,$\epsilon = 6.78$,$RR = 0.04$) and (c) White noise with mean=0 and variance =2 ($\tau = 2$, $d = 3$,$\epsilon = 1.67$,$RR = 0.04$)

The typology and texture of the RP vary from one dynamical system to another. For example, the RP of periodic systems is characterized by longer diagonal lines parallel to the main diagonal compared to chaotic systems, which are typically exhibited by short diagonal lines along with isolated dots. The diagonal lines in the RP represent the trajectories passing through the same sphere of radius ϵ in the phase space multiple times, which indicates the deterministic or periodic behavior of the dynamical system. The RP for an uncorrelated random signal or stochastic process does not have any parallel diagonal lines; instead, it consists entirely of many black dots that are isolated from one another. These dynamical characteristics of the RP can provide a comprehensive picture of complexities in the brain dynamics at a glance as the seizure progresses through different stages. Figure 2 depicts the simulated examples of the RP of a periodic signal, chaotic signal and uncorrelated noisy signal, where both axes represent time.

Recurrence Quantification Analysis

Recurrence Quantification Analysis (RQA) is a non-linear method for quantifying the visual representation of the RP by means of structures based on diagonal and vertical lines by determining the number and duration of recurrences in order to obtain meaningful information about a complex dynamical system. The main advantage of RQA is its ability to analyze time- series data without making any assumptions about the signals underlying stationarity and non- stationarity. Furthermore, RQA can handle noisy as well as non-stationary time series with relatively short datasets, thereby allowing it to be suitable for quantifying the dynamics of brain activity, particularly for the detection of seizures. In order to investigate the complexity and determinism of the EEG signals from different stages of the seizure of this dataset, we will highlight three RQA quantifiers, namely the Determinism (DET), Average diagonal length (<L>) and the Recurrence time entropy (RTE) for the current study. In the following, the definitions of these RQA measures are briefly outlined. The first measure of RQA is the DET, defined as the fraction of recurrence points that have diagonal lines to all recurrence points in the RP [36]:

$$DET = \frac{\sum_{l=l_{min}}^{N} lP(l)}{\sum_{l=1}^{N} lP(l)} \tag{8}$$

where $P(l)$ is the histogram of the diagonally oriented line of length l in the RP and $lmin$ is the predefined minimum threshold length of a diagonal line (fixed as $lmin = 2$ in this study). Thus, a correlated and periodic process is characterized by long diagonal lines in the RP, indicating a high DET value, whereas an uncorrelated and chaotic process can usually be recognized by short diagonal lines representing a low DET value. As a result, the RQA variable DET typically specifies the predictability or determinism of the system. Another measure in RQA is the L_{avg}, defined as the average length of diagonal structures present in the RP and is interpreted as the mean prediction time of the system given by [36]:

$$L_{avg} = \frac{\sum_{l=l_{min}}^{N} lP(l)}{\sum_{l=l_{min}}^{N} P(l)} \tag{9}$$

The last RQA measure is the RTE, defined as a measure of complexity based on the white vertical (non-recurrent) lines of lengths tk in the RP, which represents the recurrence time, is given by

$$RTE = -\frac{1}{\log(T_{max})} \sum_{k=1}^{T_{max}} p(t_k) \ln p(t_k) \tag{10}$$

Where $p(tk)$ is the probability of a recurrence time tk and T_{max} is the largest recurrence time [35]. This measure is particularly well suited to capture the transitions between chaotic and periodic dynamics (and vice versa) due to its association with the Kolmogorov-Sinai entropy. Therefore, a periodic or regular process generally has a low RTE value, whereas a chaotic or irregular process has a high RTE value.

Spectral Analysis

EEG signals are the voltage variations of brain activity in the time domain. The Fourier transform can be used to reveal spectral information that is hidden in the time domain signals. With a simple and effective technique based on the Fast Fourier Transform (FFT), it is possible to convert time-domain signals to frequency domain signals and compute the real-valued power spectral density

(PSD) function, which gives the power distribution of the signal over the frequency range. The power spectrum is a simple way of distinguishing the periodic, quasiperiodic and chaotic behavior of dynamical systems [37]. Therefore, the power spectrum of a periodic motion with frequency *f* has a discrete sharp delta peak at *f* as well as peaks at its various harmonics. On the other hand, a chaotic motion is characterized by continuous broadband nature in the power spectrum.

Statistical Analysis
The statistical tests are imperative in the fact that the variations in these quantitative measures are typically relative in nature. In this study, a paired t-test was employed to compare the means of the non-linear measures and RQA measures between the interictal vs preictal, preictal vs ictal and interictal vs ictal stages of seizure at a 95% confidence level in order to determine whether there exists any significant difference.

Results and Discussion
This study aims to explore the nonlinear characteristics of EEG signals from the interictal, preictal, and ictal stages of a seizure using nonlinear time series analysis and quantify them in terms of predictability and complexity. We analysed 150 EEG samples collected from ten epileptic patients during the interictal, preictal, and ictal stages of the seizure. After preprocessing, the nonlinear characteristics of the EEG signals were examined and compared using nonlinear parameters such as Sample entropy, Lyapunov exponent, Higuchi fractal dimension, Hurst exponent, RP, and RQA. The analysis for this work was implemented by utilizing two programming languages: Python and R studio. The nonlinear parameters such as SampEn and HFD are assessed using the "antropy" package in python, Hurst exponent using the nolds package, Recurrence plots and RQA measures with the open-source 'Recurrence plot and quantification' package and the estimation of maximum Lyapunov exponent using the 'nonlinearTseries' package in R studio [38-40].

Nonlinear time series analysis is an effective method for comprehending the underlying dynamical system. As the EEG is a time-varying signal that reflects various physiological conditions in the brain, it can be subjected to nonlinear time series analysis. The EEG waveforms from the interictal preictal and ictal stages are shown in Figure 3(a-c). When we examine the EEG signals in these three stages, we can see that their amplitudes and shapes differ significantly. During the interictal stage, the EEG signal seems unpredictable with a small magnitude and an irregular shape (Figure 3a). As the brain transitions towards an ictal stage, the EEG signal takes on a more regular shape, with much higher amplitudes (Figure 3c). The spectral features of the EEG signal analysed through the PSD reveal frequency components, thereby characterizing the signal as periodic and chaotic motion. The PSD of the three seizure stages is illustrated in Figure 3(d-f). From the PSD of the ictal EEG signal shown in Figure 3(f), it is clearly visible that the frequency is almost narrowed to a single delta peak at 2.5Hz and its various harmonics at 5Hz, 7.5Hz and 10Hz due to the enhanced regularity, which is typical behavior of a periodic system. However, the PSD of interictal and preictal EEG signals depicted in Figure 3(d-e) consists of a continuous broadband noisy spectrum with sharp delta peaks indicating the presence of unstable periodic orbits embedded in the attractor. The dynamical evolution of the epileptic brain can be adequately visualized by reconstructing an attractor in the phase space with an appropriate time lag and embedding dimension. Therefore, we determined the proper time lag by selecting the first local minimum of the Average Mutual Information (AMI) function and the embedding dimension using the algorithm proposed by Cao with the 'nonlinearTseries' package in R. The phase-space representation of the EEG signals during the interictal, preictal and ictal stages are shown in Figure 3 (g-i).

(a) (b) (c)

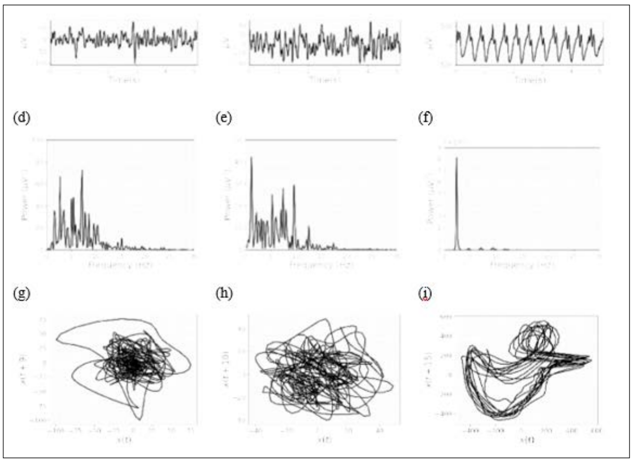

Figure 3: Time series of (a) interictal, (b) preictal and (c) ictal EEG signal (first row), Power spectrum of (d) interictal, (e) preictal and (f) ictal EEG signal (second row) and Phase portrait of (g) interictal, (h) preictal and (i) ictal EEG signal (third row).

The dynamic properties of EEG signals from these three stages demonstrate significant changes in phase space trajectories as time progresses from the interictal to the ictal stage. When compared to the interictal and preictal stages, the morphology of geometric structure in the phase portrait appears more periodic at the time of seizure onset (Figure 3i), supporting the observation of an ictal EEG signal in the power spectrum. Furthermore, the trajectories of an interictal and preictal stage in the phase portrait appear to be a more complex and irregular motion that moves in and out (self-organize) within the attractor.

Complexity Analysis
An evaluation of the sample entropy, Hurst exponent, fractal dimension and Lyapunov exponent of the EEG signal provides quantitative information on the complexity, predictability and randomness. Sample entropy of EEG signals reveals information regarding the complexity and uncertainty of the epileptic brain. The estimated mean values of Sample entropy for the three stages are represented in a box plot, as shown in Figure 4a. Results show that the value of SampEn is substantially lower throughout the seizure period (ictal stage) than during the preictal and interictal periods. This means that when a seizure begins, large groups of neurons attempt to synchronise with one another, resulting in a tremendous electrical discharge that lowers the complexity of the brain. Therefore, as the brain transitions from an interictal to an ictal stage, the decreasing tendency in the SampEn values implies increasing regularity and decreased stochastic nature in the EEG signal, suggesting the deterministic and predictable nature of the brain. In order to ensure that the statistical differences between the different seizure stages are significant, a paired t-test was performed. Results suggest that the difference between the interictal and preictal (p-value = 9.94×10^{-5}), preictal and ictal (p-value $=1.92 \times 10^{-7}$) and interictal and ictal (p-value $=4.74 \times 10^{-26}$) stages are statistically significant. Such small p-values can also ensure

that the observed difference between the sample means reflects the actual differences rather than a chance of random sampling of data. This finding is in accordance with the theoretical results performed between healthy and epileptic groups whose seizures EEG signals had significantly lower entropy values compared to the healthy EEG signals [6,10].

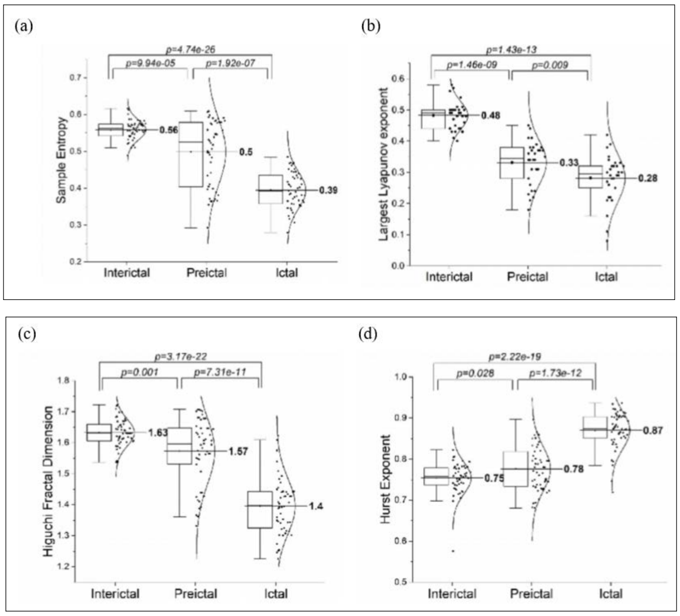

Figure 4: The estimates of (a) Sample entropy, (b) Lyapunov exponent, (c) Higuchi Fractal Dimension and (d) Hurst exponent for the interictal, preictal and ictal stages of a seizure.

The largest Lyapunov exponent (λ) is a measure that determines the predictability and long- term behavior of a dynamical system. A positive value of the λ indicates the presence of chaotic behavior in a system. The box plot of the λ for the interictal, preictal and ictal stages of a seizure is depicted in Figure 4b. It can be seen that the mean values of λ for all three stages turned out to be positive, which is a sign of chaos in the epileptic brain. Furthermore, the ictal stage has a smaller value of λ than the preictal and interictal stages. This smaller value of λ in the ictal stage shows that the amount of chaos in the brain is reduced during the seizure episode. Furthermore, this shows that EEG signals become more predictable and less random as seizure approaches. The high value of λ for the interictal stage is due to the irregularity in the EEG signal, which decreases when a seizure occurs, as observed by the phase portrait analysis and power spectrum. Results from the paired t-test suggested that the differences between the λ values for the interictal and preictal (p-value = 1.46 × 10-9), preictal and ictal (p-value = 0.009) and interictal and ictal (p-value =1.43 × 10-13) stages were statistically significant. Previous studies have reported similar results with the presence of chaos in the EEG of both healthy brains and epileptic brains, though the λ tends to be lower for epileptic ones [6,7]. Another study performed on intracranial EEG signals from ten rats with a genetic model of absence epilepsy showed the chaotic processes with positive values of λ during the interictal EEG and spike-wave discharges (SWDs) [41]. Several studies have been reported on mathematical models of seizure activity and SWDs. The theoretical models, such as the Neural mass model and Neural network model, were compared with the experimental analysis of SWDs using λ from the time series, and the results showed similar signal characteristics, with the value of λ being positive in both models [42].

Fractal analysis of EEG signals reveals information regarding the irregularity, dimensionality and complexity of various physiological conditions of the brain. In this study, the fractal dimension was evaluated using the Higuchi algorithm for quantifying the complexity of the different seizure stages. It can be realized from Figure 4c that the mean value of HFD shows a decreasing trend in the EEG dynamics as the brain transitions from the interictal to the ictal stage. The lowering of HFD values implies a reduced complexity and irregularity in the epileptic brain. This means that the seizure EEG signals are less complex and more predictable compared to the interictal and preictal EEG signals. Results from the paired t-test suggested that the differences between the HFD values for the interictal and preictal (p-value = 0.001), preictal and ictal (p-value $=7.31 \times 10^{-11}$) and interictal and ictal (p-value $=3.17 \times 10^{-22}$) stages were statistically significant. Previous studies have shown that patients with epilepsy had lower fractal dimension measures when compared to the healthy groups, thereby confirming the validity of our result [43]. The predictability and randomness of the time series can be measured by the Hurst exponent parameter. The average of the Hurst exponent values are computed for the different seizure events. It was observed that the values of the Hurst exponent show an increasing trend as the dynamics of the brain progress from the interictal to the ictal stage, as shown in Figure 4d. This rising trend of Hurst exponent values implies reduced randomness and greater predictability of EEG signals in the epileptic brain. Moreover, it should be noted that the Hurst exponent values obtained for all the stages of this dataset are between 0.5 and 1, suggesting that the EEG signals of these stages in the epileptic brain exhibit persistent behavior and long-term memory process (high correlation between the points) of time series. The paired t-test suggested a significant difference between the EEG signals of the Hurst exponent in the interictal and preictal (p-value = 0.028), preictal and ictal (p-value $=1.73 \times 10^{-12}$) and interictal and ictal (p-value $=2.22 \times 10^{-19}$) stages of a seizure. Similar results with a Hurst exponent value greater than 0.5 have been found in other studies [15,18,19]. In contrast, some studies have reported the opposite results, which showed an antipersistent behavior during seizures where the Hurst exponent value was less than between 0 and 0.5 [17]. It is pretty understandable that epileptic activity is caused by the synchronized bursting of neuronal populations in the epileptogenic region, which results in spike discharge in the EEG waveform. More synchronization in the EEG results in a high Hurst exponent value. Our finding agrees with this process by registering high values of the Hurst exponent during seizure activity in the EEG. Furthermore, the fractal dimension (FD) can be derived from the Hurst parameter (H) by the equation (4). From this relation, the average FD values of the corresponding H value can be calculated for the interictal, preictal and ictal stages as 1.24, 1.22 and 1.13, respectively. Interestingly, we find a decreasing trend of FD values from this relation during the evolution of seizures. This finding is in consonance with our earlier observation of FD values evaluated using the Higuchi algorithm. Consequently, these findings emphasize the fact that epileptic seizures tend to be associated with lower complexity, which is evident from low values of fractal dimension and sample entropy, as well as higher predictability and less randomness, which is evident from high values of the Hurst exponent. In order to ensure the complexity, predictability and whether or not the epileptic brain during the seizure stages follows deterministic nature, we further analyzed the dynamic characteristics of recurrence plots and performed recurrence quantification analyses.

Recurrence Plot and Recurrence Quantification Analysis

The visual representation through the structural patterns in the RP reveals hints about the underlying dynamical behavior of an epileptic brain during the different stages of the seizure. The RP of an interictal stage seems to exhibit very short diagonal lines with isolated black dots and rectangle-like structures formed by horizontal and vertical lines (Figure 5a). The rectangle- shaped structure represents the brain as being in an intermittent state or a transient process between stochasticity and determinism. The pattern in the RP during the preictal stage alters by the emergence of several short diagonal lines which runs parallel to the main diagonal, signifying that the system is considerably more deterministic, predictable and less chaotic than in the interictal stage (Figure 5b).

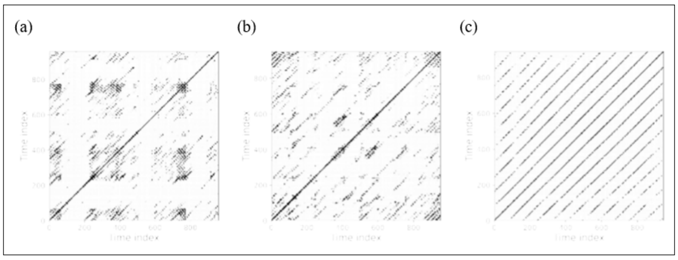

Figure 5: Recurrence plot of (a) interictal, (b) preictal and (c) ictal EEG signal. Here, the threshold value of ϵ for the RPs are chosen as the 4^{th} percentile of distance distribution of all points in the phase space by ensuring the global Recurrence rate=0.04.

Compared to the interictal and preictal stages, the RP of an ictal stage is identified by the enhancement of several diagonal lines having different lengths that closely resemble the RP structure of the periodic signal as shown in Figure. (2a). The presence of long diagonal line patterns in the RP is a signature of determinism, periodicity and predictability of a dynamical system. Further evidence can be seen in the time series of an ictal EEG signal (Figure. 5c) that exhibits certain repetitive patterns over time, and the phase portrait and the PSD clearly demonstrate its periodicity as well due to the synchronous firing of a cluster of neurons in the epileptic focus. In order to further explore the complexity and determinism of the interictal, preictal and ictal from the small-scale structures in the RP, three RQA measures are performed, namely DET, L_{avg} and RTE.

The diagonal structures of the RP are quantified by defining the RQA variable DET, which vividly defines the recurring behavior. The more extended diagonal structure is characterized by a higher value of DET (closer to 1) and higher predictability of the system. Figure. (6a) depicts the box plot of the DETs for the various seizure stages of ten epileptic patients. It can be seen that the DETs in the preictal and ictal stages were generally higher compared with the interictal stage. Moreover, the DETs for the interictal, preictal and ictal stages are greater than 0.94. Higher DET values imply that the dynamical characteristics of the brain during the various phases of seizure are governed by deterministic mechanisms associated with high predictability of the brain. This evidence of the deterministic nature can also be seen in the RP of EEG signals transitioning from interictal to ictal stages by increasing the emergence of long diagonal lines, clearly indicating recurrent behavior (Figure 5). The paired t-test reveals a significant difference in the EEG signals of the DET for the stages between interictal and preictal (p-value = 0.024) and interictal and ictal (p-value = 0.008). However, the DET does not appear to significantly differ between the preictal and ictal (p-value =0.849) stages. This finding is supported by previous research where they observed an increase in the determinism of the EEG data during the seizure state [44]. Similar to DET, the RQA variable L_{avg} measures the average result of the length of diagonal line structure in the RP, considered as the mean prediction time of the system. The high value of L_{avg} points out that the dynamics of the system tend to become more regular and deterministic. From Figure. (6b), we observe that the L_{avg} shows an increasing trend in the evolution of seizures from interictal to ictal stages. This result suggests that during the onset of a seizure, the dynamics of the brain become more regular, with higher deterministic recurrent properties and an increased mean prediction time than during the preictal and interictal stages.

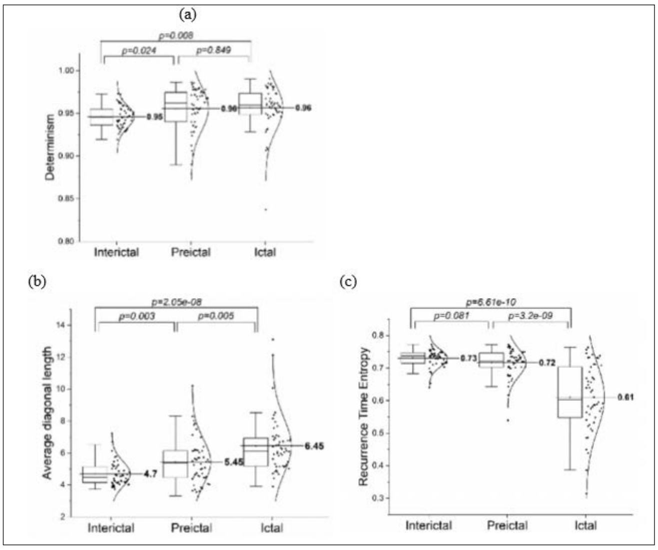

Figure 6: Box plot of (a) Determinism (b) Average diagonal length and (c) Recurrence time entropy for the interictal, preictal and ictal stages of a seizure.

The statistical test suggested that there was a significant difference between the EEG signals of the L_{avg} in the interictal and preictal (p-value = 0.003), preictal and ictal (p-value =0.005) and interictal and ictal (p-value =2.05 × 10-8) stages of a seizure. This observation is on the same path as former studies recognizing the presence of more extended diagonally oriented structures in the preictal and ictal stages compared to the interictal stage and had higher L_{avg} for the EEG signal of an ictal stage than the interictal stage [20,25].

Unlike DET and $Lavg$, the RQA variable Recurrence time entropy (RTE) measures the length of white vertical lines between each pair of recurrence points in the RP, which can be used to detect periodicity and determine the degree of complexity of the signal, with values ranging from 0 to 1. A perfectly periodic process has an RTE value close to 0, whereas a chaotic or stochastic process has an RTE value close to 1. The box plot of the RTE measures for the three seizure phases is shown in Figure (6c). It can be seen that the RTE shows a decreasing trend from the interictal stage to the ictal stage. In other words, the RTE for the ictal stage was significantly lower than those in the interictal and preictal stages. Low RTE indicates that the signal is less complex and more periodic. This finding suggests that the onset of a seizure is associated with lower complexity, implying a decrease in active neuronal processes in the epileptic brain. The paired t-test suggested a significant difference in the EEG signals of the RTE between the preictal and ictal (p-value = 3.2×10^{-9}) and interictal and ictal (p-value = 6.61×10^{-10}), whereas no statistically significant difference was observed for the interictal and preictal (p-value = 0.024) stages. Thus, the RQA measures DET, L_{avg}, and RTE were able to detect significant differences in the dynamics of EEG signals to differentiate interictal, preictal, and ictal stages of epileptic patients. The increasing values of DET and L_{avg} in the epileptogenic region indicate that their activities were more deterministic and recurrent, whereas decreasing values of RTE indicate a lower degree of complexity in this region. In fact, epileptogenic regions play a significant role in developing seizures. Their presence would likely change the characteristics of EEG dynamics in the epileptic brain and increase the frequency of seizures in patients.

In addition to the experimental EEG signal analysis, we have quantified the DET, L_{avg} and RTE measures from the three simulated signals (periodic, chaotic, and random) in order to compare their determinism and complexity with the results of RQA measures observed from the EEG signals of an epileptic brain. This was achieved by generating the Periodic signal using a 3Hz Sine wave, a chaotic signal using the y-component of the Lorentz attractor, and a Gaussian white noise signal with a mean of zero and variance of 2. Table 1 compares the RQA measures for the periodic, chaotic and noisy signals

Table 1: Comparison of RQA Measures for the Periodic, Chaotic and Noisy Signals.

RQA measures	Periodic signal (Sinusoidal)	Chaotic signal (Lorentz)	Noisy signal (Gaussian)
DET	1	0.99	0.05
L_{avg}	39.89	17.208	2.125
RTE	0.146	0.638	0.7105

It is clear that the DET for a perfectly periodic signal of a dynamical system is 1, as expected. As a result, the periodic system is completely deterministic in the sense that the future can be determined with certainty. However, the DET of the signal from a nonlinear chaotic Lorentz attractor has also been closer to 1, indicating the deterministic nature of the dynamical system, and it is known that a specific set of equations governs the underlying dynamics of the system. Even though it has the property of determinism, the presence of chaos makes long-term predictions impossible. On the other hand, the DET is very low for an uncorrelated noisy signal, making it impossible to predict even for short terms. The L_{avg} of a periodic signal is significantly higher than that of a chaotic signal due to the presence of more diagonal lines in the RP, indicating the predictability of the signal. Similarly, the RTE of a periodic signal is also significantly lower than that of noisy and chaotic signals. In this study, the RQA variable DET evaluated from the EEG signals during the three seizure stages of epileptic brains was found to be greater than 0.94. As previously stated, the DET value closer to 1 emphasises the deterministic behaviour of the brain. Furthermore, the calculation of the Lyapunov exponent reveals the presence of a chaotic regime during these stages, with the exponent value decreasing during the ictal stage. The EEG represents the overall dynamics of the brain as a result of numerous processes superimposed on each other. The existence of chaotic properties would imply that the epileptic brain is extremely sensitive to slight alterations in the initial inputs of the brain process, making it difficult for long-term prediction. The deterministic nature suggests that in the epileptic brain, short-term prediction of seizures is possible. When the brain is purely chaotic, even minor changes to the initial state of brain functions can have a significant impact on its behaviour. On the other hand, if the brain is purely deterministic, the onset of seizures can be predicted with certainty, and neither of these things occurs in general.

Despite all the above facts that this study has some limitations. Primarily, the nonlinear analysis of EEG from epileptic patients was considered using a database that was not sufficiently large. Second, each EEG segment from this database is selected based on the continuous signals of different channels by the clinical experts that are almost free from various artifacts (eye blinks, muscle movements, line noise and electrode movements). In contrast, artifact-free EEG signals cannot be achieved in diagnosing and predicting epileptic activity in a real-time scenario. Hence, this study does not draw definite conclusions concerning the complexity, degree of determinism and particularly the synchronization phenomena in the other electrode locations of the brain region as the seizure progresses over time. However, our results provide indications that the EEG signal during the seizure periods of an epileptic brain exhibits a higher degree of determinism and less complexity in addition to the chaotic behavior. Considering all of the issues raised above, our future goal will focus on evaluating these nonlinear measures on real- time continuous EEG signals of epileptic patients using several large databases. It will help generalize these findings into a mathematical model for predicting the behavior of the epileptic brain.

Conclusion

Brain seizures are severe neurological disorders that can impair the mental and physical functioning of epileptic patients. The application of nonlinear dynamics applied to EEG signals provides insight into the nature of the underlying dynamics of seizures, allowing for a better understanding of complex brain activity. This study used nonlinear time series analysis methods to investigate the dynamical characteristics of EEG signals recorded during the interictal, preictal and ictal stages of seizure from 10 epileptic patients. The nonlinear measures such as Lyapunov exponent, Sample entropy, Fractal dimension, Hurst exponent, Recurrence plot and RQA measures like DET, L_{avg} and RTE were performed to reveal significant differences in the nonlinear characteristics and recurrent structures of EEG signals during the different seizure stages. Results suggest that the nonlinear measures such as SampEn, HFD, and the RQA measure RTE significantly decreased as the brain transitioned from the interictal to the ictal stages, implying a reduction in complexity owing to a decrease in active neuronal processes in the epileptic brain. On the other hand, the Hurst exponent shows an increasing trend from the interictal to the ictal stages, indicating predictability and persistent behavior of the brain. When a seizure begins, large groups of neurons attempt to synchronize with one another, resulting in a massive electrical discharge that causes more regularity and high amplitude in the EEG signal. This regularity nature was visualized through the analysis of time series, power spectrum, and phase portrait. However, the regularity of the seizure EEG signal does not imply the absence of chaos in the brain. The presence of a chaotic regime in the brain was confirmed during the three seizure stages by the Lyapunov exponent. It can be concluded that even though the regularity of the EEG signal increases as the brain progress from interictal to ictal stages, the brain remains in a chaotic regime throughout all seizure stages. The RQA measure DET has significantly high values with greater than 0.94 in all seizure stages and L_{avg} shows an increasing trend from the interictal to the ictal stage, indicating the more deterministic nature and less complexity of the epileptic brain. Similarly, the ictal stage was identified by the long diagonal line structure in the RP, which emphasize the regularity and deterministic behavior of the brain, which shows the consistency of our observations. These valuable findings highlight the fact that the underlying dynamics of the epileptic brain is not only a chaotic complex dynamical system during these phases, but it is also deterministic in the sense that short-term prediction of seizure activity is possible for a certain period. Moreover, the aforementioned non-linear measures could effectively detect the changes in the EEG signals and differentiate between the interictal, preictal and ictal stages of a seizure in epileptic patients. Thus, the methods using Nonlinear time series analysis are found to be promising tools that reveal the underlying dynamical differences of EEG in diagnosing and predicting the onset of epileptic seizures.

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Highlights
- We investigate the nonlinear characteristics of EEG signals from the interictal, preictal, and ictal stages of a seizure using nonlinear methods and quantify them in the form of predictability and complexity.
- The Hurst exponent values exceed 0.5 in all stages, signifying the predictability of the EEG signals in the epileptic brain.
- The RQA measure DET exhibits high levels of determinism in all seizure stages.
- The epileptic brain is found to be both chaotic and highly deterministic.

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