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# The Maxwell-Cassano Equations of an Electromagnetic-Nuclear Field Yield the Fermion Masses

**Claude Michael Cassano**

Santa Clara, California, USA

**ABSTRACT**

The Maxwell-Cassano equations yield a fermion architecture table equivalent to that of the fermion Standard Model. Given a pair of constants defined by an affine transformation relating them to two rational fractions, a set of two equations determine all fermion masses.

***Corresponding author**

Claude Michael Cassano, Santa Clara, California, USA, Tel: 1-650-603-1264; E-mail: cloudmichael@hotmail.com

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## Introduction

Using the principles of the analysis of a linear function of a linear variable, constuctive algebras developed using the weighted matrix product leads to the d' Alembertian operator and it's factorization, and a space with all smooth functions satisfying Maxwell's equations [1-7]. This leads to the Helmholtzian operator and factorization, and a space in which all smooth functions satisfy the Maxwell-Cassano equations (which generalizes both Maxwell's equations and the Dirac equation) - a linearization of the Klein-Gordon equations [8-14]. These insights lead to a fermion architecture providing a firm mathematical foundation of the hadrons (mesons and baryons); which, when combined with a pair of constants defined by an affine transformation relating them to two rational fractions, a set of two equations determine all fermion masses, based on the fermion Standard Model architecture table equivalent [15, 16 & 20].

## Analysis Details and Results

Merely a cursory look demonstrates that the Helmholtzian operator and factorization is a generalization of the d'Alembertian operator and it's factorization [17].

$$\mathbf{J} = \begin{pmatrix} J^1 \\ J^2 \\ J^3 \\ J^0 \end{pmatrix} = \begin{pmatrix} (\Box - |m|^2)f^1 \\ (\Box - |m|^2)f^2 \\ (\Box - |m|^2)f^3 \\ (\Box - |m|^2)f^0 \end{pmatrix} =$$

$$= \begin{pmatrix} D_0 & D_3^{\leftrightarrow} & -D_2^{\leftrightarrow} & D_1 \\ -D_3^{\leftrightarrow} & D_0 & D_1^{\leftrightarrow} & D_2 \\ D_2^{\leftrightarrow} & -D_1^{\leftrightarrow} & D_0 & D_3 \\ D_1^{\updownarrow} & D_2^{\updownarrow} & D_3^{\updownarrow} & -D_0^{\updownarrow} \end{pmatrix} \begin{pmatrix} D_0^{\updownarrow} & -D_3^{\leftrightarrow} & D_2^{\leftrightarrow} & D_1 \\ D_3^{\leftrightarrow} & D_0^{\updownarrow} & -D_1^{\leftrightarrow} & D_2 \\ -D_2^{\leftrightarrow} & D_1^{\leftrightarrow} & D_0^{\updownarrow} & D_3 \\ D_1^{\updownarrow} & D_2^{\updownarrow} & D_3^{\updownarrow} & -D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix} =$$

$$= \begin{pmatrix} -D_0 & D_3^{\leftrightarrow} & -D_2^{\leftrightarrow} & -D_1 \\ -D_3^{\leftrightarrow} & -D_0 & D_1^{\leftrightarrow} & -D_2 \\ D_2^{\leftrightarrow} & -D_1^{\leftrightarrow} & -D_0 & -D_3 \\ -D_1^{\updownarrow} & -D_2^{\updownarrow} & -D_3^{\updownarrow} & D_0^{\updownarrow} \end{pmatrix} \begin{pmatrix} -D_0^{\updownarrow} & -D_3^{\leftrightarrow} & D_2^{\leftrightarrow} & -D_1 \\ D_3^{\leftrightarrow} & -D_0^{\updownarrow} & -D_1^{\leftrightarrow} & -D_2 \\ -D_2^{\leftrightarrow} & D_1^{\leftrightarrow} & -D_0^{\updownarrow} & -D_3 \\ -D_1^{\updownarrow} & -D_2^{\updownarrow} & -D_3^{\updownarrow} & D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix} \quad (1.1)$$

$$= \begin{pmatrix} -D_0 & D_3^{\leftrightarrow} & -D_2^{\leftrightarrow} & -D_1 \\ -D_3^{\leftrightarrow} & -D_0 & D_1^{\leftrightarrow} & -D_2 \\ D_2^{\leftrightarrow} & -D_1^{\leftrightarrow} & -D_0 & -D_3 \\ -D_1^{\updownarrow} & -D_2^{\updownarrow} & -D_3^{\updownarrow} & D_0^{\updownarrow} \end{pmatrix} \begin{pmatrix} B_0^1 + E^1 \\ B_0^2 + E^2 \\ B_0^3 + E^3 \\ -\overset{m}{\nabla}{}_0^{*} \cdot \mathbf{f} \end{pmatrix} = \quad (1.2a)$$

$$= \begin{pmatrix} D_0 & D_3^{\leftrightarrow} & -D_2^{\leftrightarrow} & D_1 \\ -D_3^{\leftrightarrow} & D_0 & D_1^{\leftrightarrow} & D_2 \\ D_2^{\leftrightarrow} & -D_1^{\leftrightarrow} & D_0 & D_3 \\ D_1^{\updownarrow} & D_2^{\updownarrow} & D_3^{\updownarrow} & -D_0^{\updownarrow} \end{pmatrix} \begin{pmatrix} B_0^1 - E^1 \\ B_0^2 - E^2 \\ B_0^3 - E^3 \\ \overset{m}{\nabla}_0 \cdot \mathbf{f}^{*} \end{pmatrix} = \quad (1.2b)$$

where:

$$D_i^{+} \equiv (\partial_i + m_i) \quad , \quad D_i^{-} \equiv (\partial_i - m_i) \quad , \quad \partial_i \equiv \frac{\partial}{\partial x^i} \quad , \quad m_i \text{ constants} \tag{2}$$

$$D_i \equiv \begin{pmatrix} D_i^{+} & 0 \\ 0 & D_i^{-} \end{pmatrix} , \; D_i^{\Updownarrow} \equiv \begin{pmatrix} D_i^{-} & 0 \\ 0 & D_i^{+} \end{pmatrix} , \; D_i^{\Leftrightarrow} \equiv \begin{pmatrix} 0 & D_i^{-} \\ D_i^{+} & 0 \end{pmatrix} , \; D_i^{\Leftrightarrow\Updownarrow} \equiv \begin{pmatrix} 0 & D_i^{+} \\ D_i^{-} & 0 \end{pmatrix} \tag{3}$$

Similarly, mass-generalized electric and magnetic potentials for the Helmholtzian operator factorization :

$$\begin{aligned} \mathbf{E} &= \mathbf{w}^{4;1}\left(-D_0^{\Updownarrow}f^1 - D_1 f^0\right) + \mathbf{w}^{4;2}\left(-D_0^{\Updownarrow}f^2 - D_2 f^0\right) + \mathbf{w}^{4;3}\left(-D_0^{\Updownarrow}f^3 - D_3 f^0\right) \\ \mathbf{B} &= \mathbf{w}^{4;1}(D_2 f^3 - D_3 f^2) + \mathbf{w}^{4;2}(-D_1 f^3 + D_3 f^1) + \mathbf{w}^{4;3}(D_1 f^2 - D_2 f^1) \\ \mathbf{E}_{\Updownarrow} &= \mathbf{w}^{4;1}\left(-D_0^{\Leftrightarrow\Updownarrow}f^1 - D_1^{\Leftrightarrow}f^0\right) + \mathbf{w}^{4;2}\left(-D_0^{\Leftrightarrow\Updownarrow}f^2 - D_2^{\Leftrightarrow}f^0\right) + \mathbf{w}^{4;3}\left(-D_0^{\Leftrightarrow\Updownarrow}f^3 - D_3^{\Leftrightarrow}f^0\right) \\ \mathbf{B}_{\Updownarrow} &= \mathbf{w}^{4;1}(D_2^{\Leftrightarrow}f^3 - D_3^{\Leftrightarrow}f^2) + \mathbf{w}^{4;2}(-D_1^{\Leftrightarrow}f^3 + D_3^{\Leftrightarrow}f^1) + \mathbf{w}^{4;3}(D_1^{\Leftrightarrow}f^2 - D_2^{\Leftrightarrow}f^1) \quad ; \end{aligned} \tag{4}$$

where:

| | |
|---|---|
| $\mathbf{f} \equiv \mathbf{w}^{4;1}f^{\mu}$ | $f^{\mu} \equiv \begin{pmatrix} f_{+}^{\mu} \\ f_{-}^{\mu} \end{pmatrix}$ |

These mass-generalized Maxwell's equations may be simpy written:

| | | | |
|---|---|---|---|
| $\mathbf{0} = (\partial_0 - m_0)\vec{\mathbf{B}} + (\vec{\nabla} + \vec{\mathbf{m}}) \times \vec{\mathbf{E}}$ ; | $0 = (\vec{\nabla} + \vec{\mathbf{m}}) \cdot \vec{\mathbf{B}}$ | ; Homgeneous | (4a) |
| $\vec{\mathbf{J}} = (\partial_0 + m_0)\vec{\mathbf{E}} - (\vec{\nabla} - \vec{\mathbf{m}}) \times \vec{\mathbf{B}}$ ; | $\rho = (\vec{\nabla} - \vec{\mathbf{m}}) \cdot \vec{\mathbf{E}}$ | ; Inhomgeneous | |

as the **Maxwell-Cassano equations** of an electromagnetic-nuclear field [9].

In free space, the thus defined **E** and **B** (generalizations of the electric and magnetic field strengths) also satisfy the Klein-Gordon equations, so have a particle-nature. (Also, the potential of the time-independent Klein-Gordon equations is the Yukawa potential[18].)

Identifying a particle-nature member **R** as either an **E** or a **B** , and $\mathbf{R}_+$ as either an $\mathbf{E}_+$ or a $\mathbf{B}_+$ , then a notation consistent with common usage would denote it's particle-nature anti-member $\overline{\mathbf{R}}_+$ as the corresponding $\overline{\mathbf{E}}_+$ or a $\overline{\mathbf{B}}_+$ (and correspondingly for $\overline{\mathbf{R}}_-$ , $\overline{\mathbf{E}}_-$ , & $\overline{\mathbf{B}}_-$ ). And, of course, the particle-nature anti-member components correspond in the same way. Each of these members satisfies the Klein-Gordon equation, but only really do so as three-vectors with three components or triplets. And, each bag of triplets must be triplets or triplets of triplets or triplets of triplets of triplets, and so on (i.e.: $3^n$ of triplets).

As shown in [20], the fermion Standard Model table may be simply built:

| | | |
|---|---|---|
| $e^- = e(1) = \overline{(E^1,E^2,E^3)}_1$ | $\mu^- = e(2) = \overline{(E^1,E^2,E^3)}_2$ | $\tau^- = e(3) = \overline{(E^1,E^2,E^3)}_3$ |
| $\nu_e = \nu(1) = (B^1,B^2,B^3)_1$ | $\nu_\mu = \nu(2) = (B^1,B^2,B^3)_2$ | $\nu_\tau = \nu(3) = (B^1,B^2,B^3)_3$ |
| $u_R = u_1(1) = (B^1,E^2,E^3)_1$ | $c_R = u_1(2) = (B^1,E^2,E^3)_2$ | $t_R = u_1(3) = (B^1,E^2,E^3)_3$ |
| $u_G = u_2(1) = (E^1,B^2,E^3)_1$ | $c_G = u_2(2) = (E^1,B^2,E^3)_2$ | $t_G = u_2(3) = (E^1,B^2,E^3)_3$ |
| $u_B = u_3(1) = (E^1,E^2,B^3)_1$ | $c_B = u_3(2) = (E^1,E^2,B^3)_2$ | $t_B = u_3(3) = (E^1,E^2,B^3)_3$ |
| $d_R = d_1(1) = \overline{(E^1,B^2,B^3)}_1$ | $s_R = d_1(2) = \overline{(E^1,B^2,B^3)}_2$ | $b_R = d_1(3) = \overline{(E^1,B^2,B^3)}_3$ |
| $d_G = d_2(1) = \overline{(B^1,E^2,B^3)}_1$ | $s_G = d_2(2) = \overline{(B^1,E^2,B^3)}_2$ | $b_G = d_2(3) = \overline{(B^1,E^2,B^3)}_3$ |
| $d_B = d_3(1) = \overline{(B^1,B^2,E^3)}_1$ | $s_B = d_3(2) = \overline{(B^1,B^2,E^3)}_2$ | $b_B = d_3(3) = \overline{(B^1,B^2,E^3)}_3$ |

(5.1)

and equivalent:

| | | |
|---|---|---|
| $e^- = e(1) = \overline{(E^1,E^2,E^3)}_1$ | $\mu^- = e(2) = \overline{(E^1,E^2,E^3)}_2$ | $\tau^- = e(3) = \overline{(E^1,E^2,E^3)}_3$ |
| $\nu_e = \nu(1) = (B^1,B^2,B^3)_1$ | $\nu_\mu = \nu(2) = (B^1,B^2,B^3)_2$ | $\nu_\tau = \nu(3) = (B^1,B^2,B^3)_3$ |
| $u_R = u_1(1) = (B^1,E^2,E^3)_1$ | $c_R = u_1(2) = (B^1,E^2,E^3)_2$ | $t_R = u_1(3) = (B^1,E^2,E^3)_3$ |
| $u_G = u_0(1) = (E^1,B^2,E^3)_1$ | $c_G = u_0(2) = (E^1,B^2,E^3)_2$ | $t_G = u_0(3) = (E^1,B^2,E^3)_3$ |
| $u_B = u_{-1}(1) = (E^1,E^2,B^3)_1$ | $c_B = u_{-1}(2) = (E^1,E^2,B^3)_2$ | $t_B = u_{-1}(3) = (E^1,E^2,B^3)_3$ |
| $d_R = d_1(1) = \overline{(E^1,B^2,B^3)}_1$ | $s_R = d_1(2) = \overline{(E^1,B^2,B^3)}_2$ | $b_R = d_1(3) = \overline{(E^1,B^2,B^3)}_3$ |
| $d_G = d_0(1) = \overline{(B^1,E^2,B^3)}_1$ | $s_G = d_0(2) = \overline{(B^1,E^2,B^3)}_2$ | $b_G = d_0(3) = \overline{(B^1,E^2,B^3)}_3$ |
| $d_B = d_{-1}(1) = \overline{(B^1,B^2,E^3)}_1$ | $s_B = d_{-1}(2) = \overline{(B^1,B^2,E^3)}_2$ | $b_B = d_{-1}(3) = \overline{(B^1,B^2,E^3)}_3$ |

(5.2)

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If the fermion masses may be described by the mass-generalized Maxwell's equations, then denote them as follows:

| | | |
|---|---|---|
| $m(3,1) = m_e : e^- = e(1)$ | $m(3,2) = m_\mu : \mu^- = e(2)$ | $m(3,3) = m_\tau : \tau^- = e(3)$ |
| $m(0,1) = m_{\nu_e} : \nu_e = \nu(1)$ | $m(0,2) = m_{\nu_\mu} : \nu_\mu = \nu(2)$ | $m(0,3) = m_{\nu_\tau} : \nu_\tau = \nu(3)$ |
| $m(2,1) = m_u : u_X = u_X(1)$ | $m(2,2) = m_c : c_X = u_X(2)$ | $m(2,3) = m_t : t_X = u_X(3)$ |
| $m(1,1) = m_d : d_X = d_X(1)$ | $m(1,2) = m_s : s_X = d_X(2)$ | $m(1,3) = m_b : b_X = d_X(3)$ |

(6)

Where for an object's mass: $m(h, i)$ :
$h$ indicates the number of $E$ 's ln the object's $\mathbf{S_R}$ architecture.
$i$ indicates the generation of the object's $\mathbf{S_R}$ architecture.

$$\frac{m(h,1)}{m\left(\left[h+\delta_{-1}^{(-1)^{T_0(h+1)}}\right]\delta_{-1}^{(-1)^{T_0(h+1)}},1\right)} = f(h) \qquad (7)$$

where:

$f(h) \equiv (h^2+1)\varphi(h)$

| $h$ | $h^2+1$ |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 5 |
| 3 | 10 |

$\varphi(h) \equiv \left(\frac{1}{2}\lambda(h^2+1)^{2h}\right)^{T_0(T_0(h+1))}$

| $h$ | $\varphi(h)$ |
|---|---|
| 0 | 1 |
| 1 | 1 |
| 2 | 1 |
| 3 | $5\lambda\cdot(3^2+1)^{3+2}$ |

$$\Rightarrow \varphi(h) = \begin{cases} 1 & , h = 0,1,2 \\ 5\lambda\cdot 10^5 & , h = 3 \end{cases}$$

So, the $f(h)$ are: :

$$f(0) \equiv (0^2+1)\varphi(0) = 1$$
$$f(1) \equiv (1^2+1)\varphi(1) = 2$$
$$f(2) \equiv (2^2+1)\varphi(2) = 5$$
$$f(3) \equiv (3^2+1)\varphi(3) = 10\cdot 5\lambda\cdot 10^5 = 5\lambda\cdot 10^6$$

Continuing, the following table may be built:

| | |
|---|---|
| $f(0) = 1$ | $m_{\nu_e} = m(0,1) = m(0,1)f(0) = m(0,1)$ |
| $f(1) = 2$ | $m_d = m(1,1) = m(2,1)f(1) = 2m(2,1)$ |
| $f(2) = 5$ | $m_u = m(2,1) = m(3,1)f(2) = 5m(3,1)$ |
| $f(3) = 5\lambda\cdot 10^6$ | $m_e = m(3,1) = m(0,1)f(3) = 5\lambda\cdot 10^6\cdot m(0,1)$ |

(8)

Thus

$$\begin{aligned} m(n,1) &= \prod_{n-h}^{3}\left[(h^2+1)\left(\tfrac{1}{2}\lambda(h^2+1)^{2h}\right)^{T_0(T_0(h+1))}\right]m(0,1) \\ &= \left(\tfrac{1}{2}\lambda\right)^{T_0(T_0(n+1))}\prod_{n-h}^{3}\left[(h^2+1)\left((h^2+1)^{2h}\right)^{T_0(T_0(h+1))}\right]m(0,1) \end{aligned} \qquad (9)$$

The fermion measured to the greatest accuracy is the electron. It's current measured value is: $m_e \approx .0.51099892811 MeV/c^2$
However, consider:

$$\frac{1}{10}\left[\frac{15}{8}+\frac{1}{4000}\left(\frac{486}{25}\right)\right]e = 0.51099892780470207761443900058 97 \qquad (10a)$$

Since this is right in the middle of the margn of error of a quantity measured to eight significant figures, it is not even remotely out of line to make the assignment:

$$m_e = m(3,1) = \frac{1}{10}\left[\frac{15}{8}+\frac{1}{4000}\left(\frac{486}{25}\right)\right]e \qquad (10b)$$

It may seem odd that a physical constant with units may be calculable, but like converting from centimeters to inches or liters to quarts there is only a proportionality constant involved, so the dimensionless quantity involved would be some multiple of this (which, here may be $\frac{e}{10}$ ).

So, taking the mass of the electron as the basis, from the above analysis (in $MeV/c^2$):

| |
|---|
| $m_e = m(3,1) = 0.5109989278047020776144390005897$ |
| $m_u = m(2,1) = m(3,1)f(2) = 5m(3,1) = 2.5549946390235103880721950029485$ |
| $m_d = m(1,1) = m(2,1)f(1) = 2m(2,1) = 5.1099892780470207761443900058 97$ |

(11)

(since the λ is to this point undetermined, it is not included in this table)

And ∃k : all fermion masses satisfy the following table:

| | |
|---|---|
| $\frac{m(0,2)}{m(0,1)} = \lambda_2$ | $\frac{m(0,3)}{m(0,1)} = \lambda_3$ |
| $\frac{m(1,2)}{m(2,1)} = \left(\frac{23}{25}\right)\cdot(k)$ | $\frac{m(1,3)}{m(2,1)} = \left(\frac{23}{25}\right)^{\frac{1}{2}}\cdot(k)^2$ |
| $\frac{m(2,2)}{m(1,1)} = 1\cdot(6k)$ | $\frac{m(2,3)}{m(1,1)} = 1\cdot\left[\left(\frac{3}{1004}\right)(6k)^2\right]^2$ |
| $\frac{m(3,2)}{m(3,1)} = 1\cdot(5k)$ | $\frac{m(3,3)}{m(3,1)} = 1\cdot\left[\left(\frac{2}{1450}\right)(5k)^2\right]^2$ |

(12)

So, from tables above:

$$\frac{m(0,3)}{m(0,1)} = \lambda_3 \;;\; \frac{m(0,2)}{m(0,1)} = \lambda_2 \Rightarrow \begin{cases} m(0,3) = \lambda_3 m(0,1) \\ m(0,2) = \lambda_2 m(0,1) \end{cases}$$

$$\Rightarrow \begin{cases} m(0,3) = \lambda_3\cdot\left(\frac{2}{\lambda}\cdot 10^{-7}\right)m(3,1) \\ m(0,2) = \lambda_2\cdot\left(\frac{2}{\lambda}\cdot 10^{-7}\right)m(3,1) \\ m(0,1) = \left(\frac{2}{\lambda}\cdot 10^{-7}\right)m(3,1) \end{cases}$$

$$\Rightarrow \begin{cases} m(0,1) = \left(\frac{2}{\lambda}\cdot 10^{-7}\right)m(3,1) \\ m(0,2) = \lambda_2\cdot m(0,1) \\ m(0,3) = \lambda_3\cdot m(0,1) \end{cases}$$

From this table 12 the constant may be established: :

$$\begin{aligned} k &= \frac{m(1,2)}{m(2,1)}\left(\frac{25}{23}\right) = \frac{1}{6}\left[\frac{m(2,2)}{m(1,1)}\right] = \frac{1}{5}\left[\frac{m(3,2)}{m(3,1)}\right] \\ &= \sqrt{\frac{m(1,3)}{m(2,1)}\sqrt{\frac{25}{23}}} = \frac{1}{6}\sqrt{\frac{1004}{3}\sqrt{\frac{m(2,3)}{m(1,1)}}} = \frac{1}{5}\sqrt{\frac{1450}{2}\sqrt{\frac{m(3,3)}{m(3,1)}}} \end{aligned} \qquad (14)$$

And, the constant k also satisfies the following relation:

$$k = 4\pi^2 + \frac{15}{8} + \frac{1}{4000}\sum_{k=0}^{\infty}\left(-\frac{1}{20}\right)^k = 4\pi^2 + \frac{15}{8} + \frac{1}{4000}\left(\frac{20}{21}\right) \qquad (15a)$$

41.353655699595529713433202094743 (15b)

So, using the already above determined value:

$m(3,1) = m_e = 0.51099892780470207761443900058 97 MeV/c^2$

$m(3,2) = 5km\ (3,1) = 105.65836861649061337988846727846 MeV/c^2$

It's current measured value is: $m\mu$ = m(3,2) ≈ 105.658371535$MeV/c^2$

$$\frac{m(1,2)}{m(2,1)} = \left(\frac{23}{25}\right)\cdot(k)$$
$$\Rightarrow m_s = m(1,2) = m(2,1)\left(\frac{23}{25}\right)k = 97.205699127171364309497389896187$$

$$\frac{m(2,2)}{m(1,1)} = 1\cdot(6k)$$
$$\Rightarrow m_c = m(2,2) = 6m(1,1)k = 1267.9004233978873605586616073416$$

$$\frac{m(3,2)}{m(3,1)} = 1\cdot(5k)$$
$$\Rightarrow m_\mu = m(3,2) = 5m(3,1)k = 105.65836861649061337988846727846$$

$$\frac{m(1,3)}{m(2,1)} = \left(\frac{23}{25}\right)^{\frac{1}{2}}\cdot(k)^2$$
$$\Rightarrow m_b = m(1,3) = m(2,1)\left(\frac{23}{25}\right)^{\frac{1}{2}}k^2 = 4190.9426907545271186849743851983$$

$$\frac{m(2,3)}{m(1,1)} = 1\cdot\left[\left(\frac{3}{1004}\right)(6k)^2\right]^2$$
$$\Rightarrow m_t = m(2,3) = \left[\left(\frac{3}{1004}\right)(6k)^2\right]^2 m(1,1) = 172924.17191486611744398343538627$$

$$\frac{m(3,3)}{m(3,1)} = 1\cdot\left[\left(\frac{2}{1450}\right)(5k)^2\right]^2$$
$$\Rightarrow m_\tau = m(3,3) = \left[\left(\frac{2}{1450}\right)(5k)^2\right]^2 m(3,1) = 1776.9680674108457768918379570944$$

There is an affine transformation that relates the above two constants to two rational fractions:

$$\frac{m(3,1)}{e/10}\Big/ 1Mev/c^2 = \left(\frac{15}{8}\right) + \frac{486}{25}\left(\frac{1}{4000}\right) \qquad (16a)$$
$$k = \left(\frac{15}{8}\right) + \frac{20}{21}\left(\frac{1}{4000}\right) + 4\pi^2 \qquad (16b)$$

The table 12 may be expressed in the following four lines (17a- d) :

$$m(0,n) = \lambda_{0n} m(0,1) \quad , (\lambda_{01} = 1, \lambda_{02} = \lambda_2, \lambda_{03} = \lambda_3) \quad , (n = 1,2,3) \qquad (17a)$$
$$m(1,n) = \left(\frac{23}{25}\right)^{\frac{1}{n-1}}([n^2 + 2(n-1)(-1)^n]k)^{n-1} m(2,1) \quad , (n = 2,3) \qquad (17b)$$
$$m(2,n) = \left[\left(\frac{3}{1004}\right)^{n-2}([n^2 + 2(n-1)(-1)^n]k)^{n-1}\right]^{n-1} m(1,1) \quad , (n = 2,3) \qquad (17c)$$
$$m(3,n) = \left[\left(\frac{2}{1450}\right)^{n-2}([n^2 + 2(n-1)(-1)^n]k)^{n-1}\right]^{n-1} m(3,1) \quad , (n = 2,3) \qquad (17d)$$

The bottom three 17b d may be further reduced to:

$$m(j,n) = \left[V^{(n-1-T_0(j))^{2T_0(j)-1}}([n^2 + 2(n-1)(-1)^n]k)^{n-1}\right]^{(n-1)^{T_0(j)}} m\left(j + (-1)^{j+1}\cdot\tfrac{1}{2}\left[1-(-1)^{T_0(j+1)}\right], 1\right) \quad , (j = 1,2,3\ ; n = 2,3) \qquad (17e)$$

$$\text{where: } V = \frac{5^{T_0(3-n)+1} - 2^{T_0(3-n)}}{2^{n+(-1)^n T_0(n-1)}\cdot 5^{n-1}\cdot(5\cdot(5+T_0(n-1)) - T_0(n))} \qquad (17e.1)$$

And, all four 17a d may be reduced to:

$$m(j,n) = \lambda_{n1}^{T_0(T_0(3-j))}\left[V^{(n-1-T_0(j))^{2T_0(j)-1}}\left([n^2+2(n-1)(-1)^n]k\right)^{n-1}\right]^{(n-1)^{T_0(j)}} m\left(j+(-1)^{j+1}\cdot\tfrac{1}{2}\left[1-(-1)^{T_0(j+1)}\right],1\right) \quad , \left(j=0,1,2,3\ ;\ n=2,3\right) \tag{17f}$$

$$\text{where: } V = \frac{5^{T_0(3-n)+1}-2^{T_0(3-n)}}{2^{n+(-1)^n T_0(n-1)}\cdot 5^{n-1}\cdot(5\cdot(5+T_0(n-1))-T_0(n))} \tag{17f.1}$$

The neutrino masses are not well established, so there is no neutrino mass corroboration within ranges yet.
However, if the above 17f are valid for: $\lambda_{n1}=1$, $\forall n$; then :

$$m(0,n) = \left[\left(\frac{5^{T_0(3-n)+1}-2^{T_0(3-n)}}{2^{n+(-1)^n T_0(n-1)}\cdot 5^{n-1}\cdot(5\cdot(5+T_0(n-1))-T_0(n))}\right)^{\frac{1}{n-1}}\left([n^2+2(n-1)(-1)^n]k\right)^{n-1}\right]m(0,1) \tag{18}$$

yielding:

$$\begin{aligned}
m(0,1) &= \left(\frac{2}{\lambda}\cdot 10^{-7}\right)m(3,1)\\
&= (2\cdot 10^{-7})\cdot 0.51099892780470207761443900058 97\,MeV/c^2 \quad , \quad (\lambda=1)\\
&=1.02199785560940415522887800117 94\cdot 10^{-7}\,MeV/c^2
\end{aligned} \tag{19a}$$

$$\begin{aligned}
m(0,2) &= \left[\left(\frac{5^{T_0(3-2)+1}-2^{T_0(3-2)}}{2^{2+(-1)^2 T_0(2-1)}\cdot 5^{2-1}\cdot(5\cdot(5+T_0(2-1))-T_0(2))}\right)^{\frac{1}{2-1}}\left(\left[2^2+2(2-1)(-1)^2\right]k\right)^{2-1}\right]m(0,1)\\
&= \left[\left(\frac{5^1-2^0}{2^{2+0}\cdot 5^1\cdot(5\cdot(5+0)-1)}\right)^{1}([4+2]k)^1\right]m(0,1)\\
&= \left[\left(\frac{1}{5\cdot 24}\right)\cdot 6k\right]m(0,1)\\
&=2.06768278497977648567166010473 72\cdot m(0,1)\\
&=2.11316737232981226759776934556 93\cdot 10^{-7}\,MeV/c^2
\end{aligned} \tag{19b}$$

$$\begin{aligned}
m(0,3) &= \left[\left(\frac{5^{T_0(3-3)+1}-2^{T_0(3-3)}}{2^{3+(-1)^n T_0(3-1)}\cdot 5^{3-1}\cdot(5\cdot(5+T_0(3-1))-T_0(3))}\right)^{\frac{1}{2}}\left(\left[3^2+2(3-1)(-1)^3\right]k\right)^{3-1}\right]m(0,1)\\
&= \left[\left(\frac{5^1-2^0}{2^{3+1}\cdot 5^2\cdot(5\cdot(5+1)-1)}\right)^{\frac{1}{2}}([9-4]k)^2\right]m(0,1)\\
&= \left[\left(\frac{1}{2}\sqrt{\frac{1}{29}}\right)\cdot 5k^2\right]m(0,1)\\
&=793.90552609227528632086363624 44\cdot m(0,1)\\
&=811.36974522276120114923446081 431\cdot 10^{-7}\,MeV/c^2
\end{aligned} \tag{19c}$$

[if these are inaccurate, then: $\lambda$, $\lambda_2$, $\lambda_3$ are not 1

AND there may be simple $\lambda$, $\lambda_2$, $\lambda_3$ OR $\lambda_{11}$, $\lambda_{21}$, $\lambda_{31}$ satisfying these appropriately]

And all the fermion masses may be tabulated as follows, along side reported mass values.
(as of this publication) [21]-[35]
( in $Mev/c^2$)

| **Calculated** | **Measured** |
|---|---|
| $m_d = m(1,1) = 5.1099892780470207761443900058 97$ | $m_d = m(1,1) \approx 5.0(0.5)$ |
| $m_u = m(2,1) = 2.5549946390235103880721950029 485$ | $m_u = m(2,1) \approx 2.4(0.6)$ |
| $m_e = m(3,1) = 0.51099892780470207761443900058 97$ | $m_e = m(3,1) \approx 0.510998928(11)$ |
| $m_{\nu_e} = m(0,1) = 0.10219978556094041552288780011794 \times 10^{-6}$ | $m_{\nu_e} = m(0,1) < 10^{-6} \times 2.2$ |
| $m_s = m(1,2) = 97.205699127171364309497389896187$ | $m_s = m(1,2) \approx 95(5)$ |
| $m_c = m(2,2) = 1267.9004233978873605586616073416$ | $m_c = m(2,2) \approx 1275(25)$ |
| $m_\mu = m(3,2) = 105.65836861649061337988846727846$ | $m_\mu = m(3,2) \approx 105.6583715(35)$ |
| $m_{\nu_{\mu e}} = m(0,2) = 0.21131673723298122675977693455693 \times 10^{-6}$ | $m_{\nu_{\mu e}} = m(0,2) < 10^{-6} \times 0.17$ |
| $m_b = m(1,3) = 4190.9426907545271186849743851983$ | $m_b = m(1,3) \approx 4180(30)$ |
| $m_t = m(2,3) = 172924.17191486611744398343538627$ | $m_t = m(2,3) \approx 172970(620)$ |
| $m_\tau = m(3,3) = 1776.9680674108457768918379570944$ | $m_\tau = m(3,3) \approx 1,776.82(16)$ |
| $m_{\nu_\tau} = m(0,3) = 81.1.3697452227612011492344608 1431 \times 10^{-6}$ | $m_{\nu_\tau} = m(0,3) < 10^{-6} \times 18.2$ |

(20)

Recalling the Helmholtzian operator matrix product from [8, 9]:

There are variations between the references on some of the masses, but the tightest ranges have been used, and the value centered in the error range.

All the calculated masses above are accurate well within their margin of error (except possible hypothetical neutrino values).

As seen above, there is an affine transformation that relates the above two constants to two rational fractions:

$$\boxed{\begin{aligned} \frac{m(3,1)}{e/10}\Big/ 1Mev/c^2 &= \left(\frac{15}{8}\right) + \frac{486}{25}\left(\frac{1}{4000}\right) \\ k &= \left(\frac{15}{8}\right) + \frac{20}{21}\left(\frac{1}{4000}\right) + 4\pi^2 \end{aligned}} \tag{21}$$

Thus, from 17 & 19 , the fermion architecture is as follows:

(22)

| | | |
|---|---|---|
| $e^- = e(1) = \overline{(E^1,E^2,E^3)}_1$ | $\mu^- = e(2) = \overline{(E^1,E^2,E^3)}_2$ | $\tau^- = e(3) = \overline{(E^1,E^2,E^3)}_3$ |
| $: m_e = m(3,1) = 0.51099892780470207761443900058 97$ | $: m_\mu = m(3,2) \approx 105.6583715(35)$ | $: m_\tau = m(3,3) \approx 1,776.82(16)$ |
| $\nu_e = \nu(1) = (B^1,B^2,B^3)_1$ | $\nu_\mu = \nu(2) = (B^1,B^2,B^3)_2$ | $\nu_\tau = \nu(3) = (B^1,B^2,B^3)_3$ |
| $: m_{\nu_e} = m(0,1) \approx 0.102199785 \times 10^{-6}$ | $: m_{\nu_\mu} = m(0,2) \approx 0.211316737 \times 10^{-6}$ | $: m_{\nu_\tau} = m(0,3) \approx 81.1.369745 \times 10^{-6}$ |
| $u_R = u_1(1) = (B^1,E^2,E^3)_1$ | $c_R = u_1(2) = (B^1,E^2,E^3)_2$ | $t_R = u_1(3) = (B^1,E^2,E^3)_3$ |
| $: m_u = m(2,1) = 2.5549946390235103880721950029485$ | $: m_c = m(2,2) \approx 1275(25)$ | $: m_t = m(2,3) \approx 172970(620)$ |
| $u_G = u_2(1) = (E^1,B^2,E^3)_1$ | $c_G = u_2(2) = (E^1,B^2,E^3)_2$ | $t_G = u_2(3) = (E^1,B^2,E^3)_3$ |
| $: m_u = m(2,1) = 2.5549946390235103880721950029485$ | $: m_c = m(2,2) \approx 1275(25)$ | $: m_t = m(2,3) \approx 172970(620)$ |
| $u_B = u_3(1) = (E^1,E^2,B^3)_1$ | $c_B = u_3(2) = (E^1,E^2,B^3)_2$ | $t_B = u_3(3) = (E^1,E^2,B^3)_3$ |
| $: m_u = m(2,1) = 2.5549946390235103880721950029485$ | $: m_c = m(2,2) \approx 1275(25)$ | $: m_t = m(2,3) \approx 172970(620)$ |
| $d_R = d_1(1) = \overline{(E^1,B^2,B^3)}_1$ | $s_R = d_1(2) = \overline{(E^1,B^2,B^3)}_2$ | $b_R = d_1(3) = \overline{(E^1,B^2,B^3)}_3$ |
| $: m_d = m(1,1) \approx 5.0(0.5)$ | $: m_s = m(1,2) \approx 95(5)$ | $: m_b = m(1,3) \approx 4180(30)$ |
| $d_G = d_2(1) = \overline{(B^1,E^2,B^3)}_1$ | $s_G = d_2(2) = \overline{(B^1,E^2,B^3)}_2$ | $b_G = d_2(3) = \overline{(B^1,E^2,B^3)}_3$ |
| $: m_d = m(1,1) \approx 5.0(0.5)$ | $: m_s = m(1,2) \approx 95(5)$ | $: m_b = m(1,3) \approx 4180(30)$ |
| $d_B = d_3(1) = \overline{(B^1,B^2,E^3)}_1$ | $s_B = d_3(2) = \overline{(B^1,B^2,E^3)}_2$ | $b_B = d_3(3) = \overline{(B^1,B^2,E^3)}_3$ |
| $: m_d = m(1,1) \approx 5.0(0.5)$ | $: m_s = m(1,2) \approx 95(5)$ | $: m_b = m(1,3) \approx 4180(30)$ |

Fermion architecture & mass table:

$$\boxed{\begin{aligned} \frac{m(3,1)}{e/10}\Big/ 1Mev/c^2 &= \left(\frac{15}{8}\right) + \frac{486}{25}\left(\frac{1}{4000}\right) \\ k &= \left(\frac{15}{8}\right) + \frac{20}{21}\left(\frac{1}{4000}\right) + 4\pi^2 \end{aligned}}$$

(23)

| | | |
|---|---|---|
| $e^- = e(1) = \overline{(E^1,E^2,E^3)}_1$ | $\mu^- = e(2) = \overline{(E^1,E^2,E^3)}_2$ | $\tau^- = e(3) = \overline{(E^1,E^2,E^3)}_3$ |
| $: m_e = m(3,1)$ | $: m_\mu = m(3,2) = m(3,1) \cdot 1 \cdot (5k)$ | $: m_\tau = m(3,3) = m(3,1) \cdot 1 \cdot \left[\left(\frac{2}{1450}\right)(5k)^2\right]^2$ |
| $\nu_e = \nu(1) = (B^1,B^2,B^3)_1$ | $\nu_\mu = \nu(2) = (B^1,B^2,B^3)_2$ | $\nu_\tau = \nu(3) = (B^1,B^2,B^3)_3$ |
| $: m_{\nu_e} = m(0,1) = \left(\frac{2}{\lambda} \cdot 10^{-7}\right)m(3,1)$ | $: m_{\nu_\mu} = m(0,2) = \left[\left(\frac{1}{5 \cdot 24}\right) \cdot 6k\right]m(0,1)$ | $: m_{\nu_\tau} = m(0,3) = \left[\left(\frac{1}{2}\sqrt{\frac{1}{29}}\right) \cdot 5k^2\right]m(0,1)$ |
| $u_R = u_1(1) = (B^1,E^2,E^3)_1$ | $c_R = u_1(2) = (B^1,E^2,E^3)_2$ | $t_R = u_1(3) = (B^1,E^2,E^3)_3$ |
| $: m_u = m(2,1) = m(3,1) \cdot 5$ | $: m_c = m(2,2) = m(3,1) \cdot 10 \cdot 1 \cdot (6k)$ | $: m_t = m(2,3) = m(3,1) \cdot 10 \cdot 1 \cdot \left[\left(\frac{3}{1004}\right)(6k)^2\right]^2$ |
| $u_G = u_2(1) = (E^1,B^2,E^3)_1$ | $c_G = u_2(2) = (E^1,B^2,E^3)_2$ | $t_G = u_2(3) = (E^1,B^2,E^3)_3$ |
| $: m_u = m(2,1) = m(3,1) \cdot 5$ | $: m_c = m(2,2) = m(3,1) \cdot 10 \cdot 1 \cdot (6k)$ | $: m_t = m(2,3) = m(3,1) \cdot 10 \cdot 1 \cdot \left[\left(\frac{3}{1004}\right)(6k)^2\right]^2$ |
| $u_B = u_3(1) = (E^1,E^2,B^3)_1$ | $c_B = u_3(2) = (E^1,E^2,B^3)_2$ | $t_B = u_3(3) = (E^1,E^2,B^3)_3$ |
| $: m_u = m(2,1) = m(3,1) \cdot 5$ | $: m_c = m(2,2) = m(3,1) \cdot 10 \cdot 1 \cdot (6k)$ | $: m_t = m(2,3) = m(3,1) \cdot 10 \cdot 1 \cdot \left[\left(\frac{3}{1004}\right)(6k)^2\right]^2$ |
| $d_R = d_1(1) = \overline{(E^1,B^2,B^3)}_1$ | $s_R = d_1(2) = \overline{(E^1,B^2,B^3)}_2$ | $b_R = d_1(3) = \overline{(E^1,B^2,B^3)}_3$ |
| $: m_d = m(1,1) = m(3,1) \cdot 10$ | $: m_s = m(1,2) = m(3,1) \cdot 5 \cdot \left(\frac{23}{25}\right) \cdot (k)$ | $: m_b = m(1,3) = m(3,1) \cdot 5 \cdot \left(\frac{23}{25}\right)^{\frac{1}{2}} \cdot (k)^2$ |
| $d_G = d_2(1) = \overline{(B^1,E^2,B^3)}_1$ | $s_G = d_2(2) = \overline{(B^1,E^2,B^3)}_2$ | $b_G = d_2(3) = \overline{(B^1,E^2,B^3)}_3$ |
| $: m_d = m(1,1) = m(3,1) \cdot 10$ | $: m_s = m(1,2) = m(3,1) \cdot 5 \cdot \left(\frac{23}{25}\right) \cdot (k)$ | $: m_b = m(1,3) = m(3,1) \cdot 5 \cdot \left(\frac{23}{25}\right)^{\frac{1}{2}} \cdot (k)^2$ |
| $d_B = d_3(1) = \overline{(B^1,B^2,E^3)}_1$ | $s_B = d_3(2) = \overline{(B^1,B^2,E^3)}_2$ | $b_B = d_3(3) = \overline{(B^1,B^2,E^3)}_3$ |
| $: m_d = m(1,1) = m(3,1) \cdot 10$ | $: m_s = m(1,2) = m(3,1) \cdot 5 \cdot \left(\frac{23}{25}\right) \cdot (k)$ | $: m_b = m(1,3) = m(3,1) \cdot 5 \cdot \left(\frac{23}{25}\right)^{\frac{1}{2}} \cdot (k)^2$ |

Further application of the Maxwell-Cassano equations and the above resulting architecture yields further results for particle characteristics (including masses and charges), but that goes beyond the scope of this article.

## Conclusion

As shown in, the Higgless-fermion Standard Model is not fundamental, but is derivable from the Maxwell-Cassano equations factorization of the foour-vector-doublet Klein-Gordon equation [20]. Therefore, the current Standard Model adapted therefrom is not not a fundamental.Herein, above; the fermion masses are derivable from these Maxwell-Cassano equations. Because the Maxwell-Cassano equations are not massless (as the Yang-Mills/Glashow-Weinberg-Salam are), a Higgs mechanism and boson are superfluous (and strong and weak force field potentials may be realized - though these, also, go beyond the scope of this article).Thus, these insights provide the Helmholtzian operator and factorization, and resulting Maxwell-Cassano equations as a more perfect firm mathematical foundation of the funamental particles and forces [19].

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