Ricci Solitons on $\alpha$-para Kenmotsu Manifolds with Semi Symmetric Metric Connection

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ABSTRACT
In this paper we introduce notion of Ricci solitons in $\alpha$-para Kenmotsu manifold with semi-symmetric metric connection. We have found the relations between curvature tensor, Ricci tensors and scalar curvature of $\alpha$-para Kenmotsu manifold with semi-symmetric metric connection. We have proved that 3-dimensional $\alpha$-para Kenmotsu manifold with semi-symmetric metric connection is an $\eta$-Einstein manifold and the Ricci soliton defined on this manifold is named expanding and steady with respect to the value of $\lambda$ constant. It is proved that Conharmonically flat $\alpha$-para Kenmotsu manifold with semi-symmetric metric connection is $\eta$-Einstein manifold.

Mathematics Subject Classification: 53C15, 53C25, 53C40, 53C50

Keywords: Curvature Tensor, Ricci Solitons, Semi-Symmetric Metric Connection

Introduction
In 1972 Kemmotsu studied a class of contact Riemannian manifolds satisfying some special conditions and this manifolds is known as Kenmotsu manifold [1]. Sharma and Sinha started to study of the Ricci solitons in contact geometry in 1983 [2]. Later Mukut Mani Tripathi, Cornelia Livia Bejan and Mircea Crasmareanu, and others extensively studied Ricci solitons in contact metric manifolds [3, 4]. In 1985, almost paracontact geometry was introduced by Kanesuki and Williams and then it was continued by many authors [5]. Nagaraja ve Premalatha studied exclusively about Ricci solitons on Kenmotsu manifold in 2012. Agashe and Chafle, Liang, Pravonovic and Sengupta, Yildiz and Cetinkaya studied semi-symmetric non-metric connection in different ways [6-11]. A systematic study of almost paracontact metric manifolds was carried out by Zamkovoy [12]. However such structures were also studied by Buchner and Rosca. Rosca and Vencheke [13]. Further almost Para-Hermitian Structure on the tangent of an almost Para-Co hermitian manifold was studied by Bejan [3]. A class of $\alpha$-para kenmotsu manifolds was studied by Srivastva and Srivastva [14]. We can observe that the concircular curvature tensor on Pseudo-Riemannian manifold to be of constant curvature. Hayden introduced Semi-symmetric linear connection on a Riemannian manifold [8]. Let M be an n-dimensional Riemannian manifold of class C-endowed with the Riemannian metric g and $\nabla$ be the Levi-Civita connection on $\mathbb{M}^n$. A linear connection $\nabla$ defined on $\mathbb{M}^n$ is said to be semi symmetric if its torsion tensor T is of the form [15].

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$T(X,Y) = \eta(Y)X - \eta(X)Y$
where $\xi$ is a vector field and $\eta$ is a 1-form defined by
$g(\xi, \xi) = \eta(X)$
for all vector field $X \in \chi(M^n)$, $\gamma(M^n)$ is the set of all differentiable vector fields on $M^n$. A relation between the semi-symmetric metric connection $\nabla$ and the Levi-Civita connection $\nabla$ on $M^n$ has been obtained by Yano which is given as [16]
$\Xi_X Y = \nabla_X Y + \eta(Y)X - g(X, Y)\xi$  (1.1)

Preliminaries
A differentiable manifold $M^n$ of dimension n is said to have an almost paracontact ($\phi, \xi, \eta$)-structure if it admits an (1,1) tensor field $\phi$, a unique vector field $\xi$, and a 1-form $\eta$ such that:
$\phi^2 = I - \eta \otimes \xi,$
$\phi \xi = 0,$
$\eta \circ \phi = 0$  (2.1)
$\eta(\xi) = 1$  (2.2)
for any vector field $X, Y$ on $M^n$. The manifold $M^n$ equipped with an almost paracontact structure ($\phi, \xi, \eta$) is called almost paracontact manifold. In addition, if an almost paracontact manifold admits a pseudo-Riemannian metric satisfying
\begin{align}
g(X,\xi) &= \eta(X) \quad (2.3) \\
g(\phi X, \phi Y) &= -g(X, Y) + \eta(X)\eta(Y) \quad (2.4) \\
g(\phi X, Y) &= -g(X, \phi Y) \quad (2.5)
\end{align}

for any vector field \(X,Y\) on \(M^n\), where \(\phi\) is a (1,1) tensor field, \(\xi\) is a vector field, \(\eta\) is a 1-form and \(g\) is the Riemannian metric. Then \(M\) is called an almost contact manifold. For an almost contact manifold \(M^n\), it follows that [8]

\begin{align}
(\nabla_X \phi)Y &= \nabla_X \phi Y - \phi(\nabla_X Y) \quad (2.6) \\
(\nabla_X \eta)Y &= \nabla_X \eta(Y) - \eta(\nabla_X Y) \quad (2.7)
\end{align}

Let \(R\) be the Riemann curvature tensor, \(S\) the Ricci curvature tensor, \(Q\) the Ricci operator. We have

\begin{align}
S(X,Y) &= \sum_{i=1}^n g(R(e_i, X)e_i, Y) \quad (2.8) \\
QX &= -\sum_{i=1}^n R(e_i, X)e_i \quad (2.9)
\end{align}

and

\begin{equation}
S(X,Y) = g(QX,Y) \quad (2.10)
\end{equation}

for any vector field \(X,Y\) on \(M^n\). Then the manifold is called paracontact metric structure and the manifold \(M\) equipped with an almost paracontact metric structure is called an almost paracontact metric manifold. Further in addition, if the structure \((\phi, \xi, \eta, g)\) satisfies

\begin{equation}
d\eta(X,Y) = g(X,\phi Y) \quad (2.11)
\end{equation}

for any vector fields \(X,Y\) on \(M^n\), then the manifold is called paracontact metric structure and the corresponding structure \((\phi, \xi, \eta, g)\), is called a paracontact structure with the associated metric \(g\) [17]. On an almost paracontact metric manifold, the (1,2) tensor field \(N\), defined as

\begin{equation}
N\phi = [\phi, \phi] - 2d\eta \otimes \xi \quad (2.12)
\end{equation}

Where \([\phi, \phi]\) is the nijenhuis tensor of \(\phi\). If \(N\) vanishes identically, then we say that the manifold \(M^n\) is a normal almost paracontact metric manifold. The normality condition implies that the almost paracomplex structure \(J\) defined on \(M^n\times R^n\)

\begin{equation}
J(X, t \frac{d}{dt}) = (\phi X + \lambda(\xi), \eta(X) \frac{d}{dt})
\end{equation}

is integrable. Here \(X\) is tangent to \(M^n\), \(t\) is the coordinate of \(R\) and \(\lambda\) is a differentiable function on \(M^n\times R^n\).

For an almost paracontact metric 3-dimensional manifold \(M^3\), the following three conditions are mutually equivalent:

(i) there exist smooth functions \(\alpha, \beta\) on \(M^3\) such that

\begin{equation}
(\nabla_X \phi)Y = \alpha(g(\phi X, Y)\xi - \eta(Y)\phi X) + \beta(g(X, Y)\xi - \eta(Y)X) \quad (2.13)
\end{equation}

(ii) \(M^3\) is normal,

(iii) there exist smooth functions \(\alpha, \beta\) on \(M^3\) such that

\begin{equation}
\nabla_X \xi = \alpha(X - \eta(X)\xi) + \beta\phi X \quad (2.14)
\end{equation}

where \(\nabla\) is the Levi-Civita connection of pseudo-Riemannian metric \(g\). A normal almost paraconact metric 3-dimensional manifold is called:

(a) Para-Cosymplectic manifold if \(\alpha = \beta = 0\),

(b) quasi-para Sasakian manifold if and only if \(\alpha = 0\) and \(\beta \neq 0\),

(c) \(\alpha\)-para Sasakian manifold if and only if \(\alpha = 0\) and \(\beta\) is a non-zero constant, in particular para-Sasakian manifold if \(\beta = 1\).

\(\alpha\)-para Kenmotsu manifold if \(\alpha\) is a non-zero constant and \(\beta = 0\) in particular para-Kenmotsu manifold if \(\alpha = 1\).

\section*{On 3-Dimensional \(\alpha\)-para Kenmotsu Manifold with Semi-Symmetric Metric Connection}

In 3-dimensional \(\alpha\)-para Kenmotsu manifold, the Ricci tensor \(S\) of Levi-Civita connection \(\nabla\) is given by

\begin{equation}
S(X,Y) = g(R(\phi e_1, X)e_1, Y) - g(R(\phi e_2, X)e_2, Y) + g(R(\xi, X)e_1, Y). 
\end{equation}

Let \(M^3(\phi, \xi, \eta, g)\) be an \(\alpha\)-para Kenmotsu manifold [13], then we have

\begin{equation}
R(X, Y)Z = \left(\xi + 2\alpha^2\right)g(Y, Z)X - g(X, Z)Y - \left(\xi + 3\alpha^2\right)\eta(X)g(Y, Z)\xi + \left(\xi + 3\alpha^2\right)\eta(X)g(Y, Z)\eta(X) \quad (3.1)
\end{equation}

Replace \(Z = \xi\) in equation (3.1), we get

\begin{equation}
R(X, Y) = \alpha^2 \eta(X)Y - \eta(Y)X, \quad (3.2)
\end{equation}

\begin{equation}
S(X, Y) = \xi + 2\alpha^2 g(Y, X) - \xi + 3\alpha^2 \eta(X)\eta(Y) \quad (3.3)
\end{equation}

\begin{equation}
S(X, \xi) = -2\alpha^2 \eta(X) \quad (3.4)
\end{equation}

\begin{equation}
(\nabla_X \phi)Y = \alpha(g(\phi X, Y)\xi - \eta(Y)\phi X) + \eta(Y)\phi X \quad (3.5)
\end{equation}

\begin{equation}
\nabla_X \xi = \alpha(X - \eta(X)\xi) \quad (3.6)
\end{equation}

Let \(\nabla\) be a linear connection and \(\nabla\) be a Riemann connection of an \(\alpha\)-para Kenmotsu manifold \(M\). This linear connection defined by

\begin{equation}
\nabla_X Y - \nabla_X Y + \eta(Y)X - g(X, Y)\xi. \quad (3.7)
\end{equation}

For \(\alpha\)-para Kenmotsu manifold with semi-symmetric metric connection, using (2.6), (3.5) and (3.7) we have

\begin{equation}
(\nabla_X \phi)Y = \alpha\eta(\phi X, Y)\xi - \eta(Y)\phi X + \eta(Y)\phi X \quad (3.8)
\end{equation}

from equation (3.7), we have

\begin{equation}
\nabla_X \xi = (1 + \alpha)(X - \eta(X)\xi) \quad (3.9)
\end{equation}

Let \(M^3\) be a 3-dimensional \(\alpha\)-para Kenmotsu manifold. The curvature tensor \(R\) of \(M^3\) with respect to the semi-symmetric metric connection \(\nabla\) is defined by

\begin{equation}
R(X, Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z, \quad (3.10)
\end{equation}

with the help of (3.7) and (3.9), we get
By using equations (3.7), (2.2), (2.3), (3.6), (3.9) (3.10), (3.11), (3.12), and (3.13) we get
\[
\bar{L}(X, Y) Z = \bar{L}(X; Y) Z - (1 + 2 \alpha)[g(Y, Z)X - g(X, Z)Y] + (1 + \alpha)[\eta(X) Y - \eta(Y) X] + (1 + \alpha)[\eta(Y) X - \eta(X) Y] Z \xi + \eta(X) Z \xi + \eta(Y) Y \xi.
\]
(3.17)

Thus we have

**Lemma 3.1** Let \( M \) be a 3-dimensional \( \alpha \)-para Kenmotsu manifold with the semi-symmetric metric connection, \( \mathcal{S} \) Ricci curvature tensor and \( \mathcal{Q} \) Ricci operator. Then
\[
\mathcal{S}(X, \xi) = -2\alpha(1 + \alpha)\eta(X).
\]
(3.18)

and
\[
\bar{\mathcal{Q}}\xi = -2\alpha(1 + \alpha)\xi.
\]
(3.19)

Proof. Contracting with \( Y \) and \( Z \) in (3.17) and summing over \( i = 1, 2, \ldots, n \), from (2.8) expression
\[
\sum g(\bar{L}(e_i, \xi), e_i) = \alpha(1 + \alpha)(\sum \eta(e_i) g(Y, e_i) - \eta(Y) \sum g(e_i, e_i))
\]
the proof of (3.18) is completed. Then also using (2.10) and (2.1), (2.2), (2.3), the proof of (3.19) is completed.

**Lemma 3.2** Let \( M \) be a 3-dimensional \( \alpha \)-para Kenmotsu manifold with the semi-symmetric metric connection, \( r \) scalar curvature tensor, \( \mathcal{S}(X, Y) \) Ricci curvature tensor and \( \mathcal{Q} X \) Ricci operator. Then it follows that
\[
\bar{S}(X, \xi) = (-1 + \frac{\alpha}{2} - 3\alpha + 3\alpha^2)g(X, Y) + (1 - \frac{\alpha}{2} + 3\alpha^2)\eta(X)\eta(Y).
\]
(3.20)

And
\[
\bar{\mathcal{Q}}X = (-1 + \frac{\alpha}{2} - 3\alpha + 3\alpha^2)X + (1 - \frac{\alpha}{2} + 3\alpha^2)\eta(X)\xi.
\]
(3.21)

Proof. Taking inner product of equation (3.14) with \( U \) and using equation (2.3) we have
\[
g(R(\xi, Y) Z, U) = g(\mathcal{S}(X, Y) Z, U) - (1 + 2\alpha)[g(Y, Z)g(X, U) - g(X, Z)g(Y, U)] + (1 + \alpha)[\eta(Y) g(X, U) - \eta(X) g(Y, U)].
\]
(3.22)

Let \( \{e_i, \varphi e_i, \xi\} \) be a local orthonormal \( \varphi \)-basis of vector fields on \( \alpha \)-para Kenmotsu manifold \( M \). Then, we get
\[
\mathcal{S}(X, Y) = (-1 + \frac{\alpha}{2} - 3\alpha + 3\alpha^2)g(X, Y) + (1 - \frac{\alpha}{2} + 3\alpha^2)\eta(X)\eta(Y).
\]
(3.23)

from equation (3.23), we have
\[
\mathcal{F} = -2 + r - 8\alpha
\]
(3.24)

where \( \mathcal{F} \) is the scalar curvature with semi-symmetric metric connection. Using (3.23) and (2.10), it's verified that
\[
g(\bar{R}(\xi, Y) Z, U) = g((-1 + \frac{\alpha}{2} - 3\alpha + 3\alpha^2)X + (1 - \frac{\alpha}{2} + 3\alpha^2)\eta(X)\xi, Y).
\]
(3.25)

from equation (3.25), we get
\[
\bar{Q}X = (-1 + \frac{\alpha}{2} - 3\alpha + 3\alpha^2)X + (1 - \frac{\alpha}{2} + 3\alpha^2)\eta(X)\xi
\]
(3.26)

the proof of (3.21) is completed.
Putting (4.4) in (4.3), we obtain
\[ b = (2a(1 + \alpha) - \lambda)\eta \]  
(4.5)

By applying \( d \) in (4.5), we get
\[ 0 = (2a(1 + \alpha) - \lambda)d\eta \]  
(4.6)

Since \( d\eta \neq 0 \) from, we have
\[ 2a(1 + \alpha) - \lambda = 0 \]  
(4.7)

By using (4.5) and (4.7), we obtain that \( b \) is constant. Hence from (4.2) it is verified
\( S(X,Y) = -b((1 + \alpha) + \lambda)g(X,Y) + b(1 + \alpha)\eta(X)\eta(Y) \)  
(4.8)

which implies that \( M \) is an \( \eta \)-Einstein manifold. This leads to the following

**Theorem 4.1** If in a 3-dimensional \( \alpha \)-para Kenmotsu manifold with the semi symmetric metric connection, the metric \( g \) is a Ricci soliton and \( V \) is a pointwise collinear with \( \xi \), then \( V \) is a constant multiple of \( \xi \) and \( g \) is an \( \eta \)-Einstein manifold of the form (4.8) and Ricci soliton is steady and expanding according as \( \lambda = 2a(1 + \alpha) \) is zero and positive, respectively.

**Conharmonically Flat \( \alpha \)-para Kenmotsu Manifolds with the Semi-Symmetric Metric Connection**

We have studied conharmonically flat \( \alpha \)-para Kenmotsu manifolds with respect to the semi-symmetric metric connection. In a \( \alpha \)-para Kenmotsu manifold the conharmonic curvature tensor with respect to the semi-symmetric metric connection is given by
\[ \overline{R}(X,Y)Z = \overline{R}(X,Y)Z - [\overline{S}(Y,Z)X - \overline{S}(X,Z)Y + g(Y,Z)\overline{Q}X - g(X,Z)\overline{Q}Y]. \]  
(5.1)

If \( \overline{\mathcal{R}} = 0 \), then the manifold \( M \) is called conharmonically flat manifold with respect to the semi-symmetric metric connection. Let \( M \) be a conharmonically flat manifold with respect to the semi-symmetric metric connection. From (5.1), we have
\[ R(X,Y)Z = S(Y,Z)X - S(X,Z)Y + g(Y,Z)\overline{Q}X - g(X,Z)\overline{Q}Y \]  
(5.2)

(3.20) and (3.21) in (5.1) we get
\[ R(X,Y)Z = (1 + 2a)\eta(Y,Z)X - g(X,Z)Y \]  
(6.1)

now putting \( X = \xi \) in (5.3), we obtain
\[ R(\xi, Y)Z = S(Y,Z)\xi - S(\xi, Z)Y \]  
(5.4)

using (3.1) and (3.4) in (5.4), we get
\[ S(Y,Z)\xi - S(\xi, Z)Y + (-1 - 4a - 2a^2)g(Y,Z)\xi + (1 + 4a + 2a^2)\eta(Y)\eta(Z)Y = 0 \]  
(5.5)

Taking inner product with \( \xi \) in (5.5), we get
\[ S(Y,Z) = (1 + 4a + 2a^2)g(Y,Z) - (1 + 4a + 4a^2)\eta(Y)\eta(Z) \]  
(5.6)

Thus \( M \) is an \( \eta \)-Einstein manifold with respect to the Levi-Civita connection. This leads to the following

**Theorem 5.1** If \( M \) is a conharmonically flat \( \alpha \)-para Kenmotsu manifolds with respect to the semi-symmetric metric connection. Then the manifold \( M \) is an \( \eta \)-Einstein.

**Example**

(A 3-dimensional \( \alpha \)-para Kenmotsu manifold with the semi-symmetric metric connection.) We consider the 3-dimensional manifold \( M = (x,y,z) \in \mathbb{R}^3, z \neq 0 \), where \((x,y,z)\) are the standard coordinates in \( \mathbb{R}^3 \). The vector fields \( e_1, e_2, e_3 \) are linearly independent at each point of \( M \). Let \( g \) be the Riemannian metric defined by
\[ g(e_1, e_1) = g(e_2, e_2) = g(e_3, e_3) = 0. \]

Let \( \eta \) be the 1-form defined by \( \eta(Z) = g(Z, e_3) \) for any \( Z \in \chi(M) \). Let \( \phi \) be the (1,1) tensor field defined by \( \phi(\epsilon_1) = -\epsilon_2, \phi(\epsilon_2) = \epsilon_1, \phi(\epsilon_3) = 0 \).

Then using linearity of \( \phi \) and \( g \) we have
\[ \eta(\epsilon_1) = 1, \phi^2(Z) = -Z + \eta(Z)e_3 \]

\[ g(\phi(Z), \phi(W)) = g(Z,W) - \eta(Z)\eta(W) \]

for any \( Z, W \in \chi(M) \). Now, by direct computations we obtain
\[ [\epsilon_1, \epsilon_2] = 0, [\epsilon_2, \epsilon_3] = -\frac{2}{z}\epsilon_2, [\epsilon_1, \epsilon_3] = -\frac{2}{z}\epsilon_1 \]

by using these above equations we get [1]
\[ \mathbb{E}_{\epsilon_1} \epsilon_1 = \frac{2}{z} \epsilon_3 \text{ and } \mathbb{E}_{\epsilon_1} \epsilon_3 = -\frac{2}{z} \epsilon_1 \]  
(6.1)

\[ \mathbb{E}_{\epsilon_2} \epsilon_1 = \mathbb{E}_{\epsilon_1} \epsilon_2 = \mathbb{E}_{\epsilon_2} \epsilon_3 = \mathbb{E}_{\epsilon_1} \epsilon_3 = \mathbb{E}_{\epsilon_2} \epsilon_1 = \mathbb{E}_{\epsilon_2} \epsilon_3 = 0 \]  
(6.2)

Now we consider at this example for semi-symmetric metric connection. From (3.8), (6.1) and (6.2)
\[ \mathbb{E}_{\epsilon_1} \epsilon_1 = \frac{2}{z} \epsilon_3 \text{ and } \mathbb{E}_{\epsilon_1} \epsilon_3 = (\frac{2}{z} - 1) \epsilon_1 \]  
(6.3)

\[ \mathbb{E}_{\epsilon_2} \epsilon_1 = \mathbb{E}_{\epsilon_1} \epsilon_2 = \mathbb{E}_{\epsilon_2} \epsilon_3 = 0 \text{ and } \mathbb{E}_{\epsilon_2} \epsilon_1 = 0 \]  
(6.4)

where \( i 
eq j = 1,2,3 \). It’s known that
\[ \overline{R}(\epsilon_1, \epsilon_2) \epsilon_3 = \frac{6}{z^2} + \frac{2}{z^2} \epsilon_1, \overline{R}(\epsilon_1, \epsilon_2) \epsilon_3 = 0 \]  
(6.6)

\[ \overline{R}(\epsilon_1, \epsilon_1) \epsilon_1 = \frac{4}{z^2} + \frac{4}{z^2} \epsilon_2, \overline{R}(\epsilon_1, \epsilon_2) \epsilon_1 = 0 \]  
(6.7)

\[ \overline{R}(\epsilon_1, \epsilon_1) \epsilon_3 = \epsilon_1, \overline{R}(\epsilon_1, \epsilon_2) \epsilon_3 = 0 \]  
(6.8)

\[ \overline{R}(\epsilon_1, \epsilon_2) \epsilon_3 = \epsilon_1, \overline{R}(\epsilon_1, \epsilon_2) \epsilon_3 = 0 \]  
(6.9)

\[ \overline{R}(\epsilon_1, \epsilon_1) \epsilon_1 = \frac{2}{z} - \frac{6}{z^2} \epsilon_3 \]

\[ \overline{R}(\epsilon_1, \epsilon_1) \epsilon_3 = \frac{2}{z} - \frac{6}{z^2} \epsilon_3 \]

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\[ \overline{R}(\epsilon_1, \epsilon_1) \epsilon_3 = \frac{2}{z} - \frac{6}{z^2} \epsilon_3 \]
where $i \neq j = 1, 2$. From (2.8) and (6.6) it’s verified that

$$S(e_1, e_1) = \left(\frac{2}{x^2} + \frac{2}{z} - 1\right)$$

$$S(e_2, e_2) = \left(\frac{10}{x^2} + \frac{6}{z} - 1\right)$$

$$S(e_3, e_2) = \left(\frac{12}{x^2} + \frac{4}{z}\right)$$

(6.7)

Conclusion

If in a 3-dimensional $\alpha$-para Kenmotsu manifold with the semi-symmetric metric connection, the metric $g$ is a Ricci soliton and in this study, we gave some curvature conditions for 3-dimensional $\alpha$-para Kenmotsu manifolds with semi-symmetric metric connection In 3-dimensional $\alpha$-para Kenmotsu manifolds with semi-symmetric metric connection is also an $\eta$-Einstein manifold and Ricci soliton defined steady or expanding on this manifold is named with respect to values of $\alpha$ and $\lambda$ constant. We also proved that conharmonically flat $\alpha$-para Kenmotsu manifolds with semi-symmetric metric connection is an $\eta$-Einstein manifold.

References


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