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Ricci Solitons on α-para Kenmotsu Manifolds with Semi Symmetric Metric Connection

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ABSTRACT

In this paper we introduce notion of Ricci solitons in α-para Kenmotsu manifold with semi -symmetric metric connection. We have found the relations between curvature tensor, Ricci tensors and scalar curvature of α-para Kenmotsu manifold with semi-symmetic metric connection.We have proved that 3-dimensional α-para Kenmotsu manifold with semi -symmetric metric connection is an η-Einstein manifold and the Ricci soliton defined on this manifold is named expanding and steady with respect to the value of λ constant.It is proved that Conharmonically flat α-para Kenmotsu manifold with semi-symmetric metric connection is η-Einstein manifold.

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Introduction

In 1972 Kemmotsu studied a class of contact Riemannian manifolds satisfying some special conditions and this manifolds is known as Kenmotsu manifold [1].Sharma and Sinha started to study of the Ricci solitons in contact geometry in 1983 [2]. Later Mukut Mani Tripathi,Cornelia Livia Bejan and Mircea Crasmareanu, and others extensively studied Ricci solitons in contact metric manifolds [3, 4]. In 1985, almost paracontact geometry was introduced by kaneyuki and williams and then it was continued by many authors [5]. Nagaraja ve premalatha studied exclusively about Ricci solitons on Kenmotsu manifold in 2012 .Agashe and Chafle ,Liang,pravonovic and Sengupta,Yildiz and Cetinkaya studied semi-symmetric non-metric connection in different ways [6-11].

A systematic study of almost paracontact metric manifolds was carried out by Zamkovoy [12]. However such structures were also studied by Buchner and Rosca. Rossca and Venhecke [13]. Further almost Para-Hermitian Structure on the tangent of an almost Para-Co hermitian manifolds was studied by Bejan [3]. A class of α-para kenmotsu manifolds was studied by srivastva and srivastva [14]. We can observe that the concircular curvature tensor on Pseudo-Riemannian manifold to be of constant curvature. Hayden introduced Semi-symmetric linear connection on a Riemannian manifold [8]. Let M be an n-dimensional Riemannian manifold of class C-endowed with the Riemannian metric g and ▯be the Levi-Civita Connection on M^n. A linear connection ▯ defined on M^n is said to be semi symmetric if its torsion tensor T is of the form [15].

$$T(X,Y) = \eta(Y)X - \eta(X)Y$$

where ξ is a vector field and η is a 1-form defined by

$$g(X,\xi) = \eta(X)$$

for all vector field X ∈ $\chi(M^n)$ where, $\chi(M^n)$ is the set of all differentiable vector fields on M^n. A relation between the semi-symmetric metric connection ▯ and the Levi-Civita connection ▯ on M^n has been obtained by Yano which is given as [16]

$$\bar{▯}_X Y = ▯_X Y + \eta(Y)X - g(X,Y)\xi \qquad (1.1)$$

Preliminaries

A differentiable manifold M^n of dimension n is said to have an almost paracontact (ϕ,ξ,η)-structure if it admits an (1,1) tensor field ϕ,a unique vector field ξ,1-form η such that :

$$\phi^2 = I - \eta \otimes \xi,$$

$$\phi\xi = 0,$$

$$\eta \circ \phi = 0 \qquad (2.1)$$

$$\eta(\xi) = 1 \qquad (2.2)$$

for any vector field X,Y on M^n .The manifold M^n equipped with an almost paracontact structure (ϕ,ξ,η) is called almost paracontact manifold . In addition,if an almost paracontact manifold admits a pseudo-Riemannian metric satisfying

$$g(X,\xi) = \eta(X) \tag{2.3}$$

$$g(\phi X,\phi Y) = -g(X,Y) + \eta(X)\eta(Y) \tag{2.4}$$

$$g(\phi X, Y) = -g(X,\phi Y) \tag{2.5}$$

for any vector field X,Y on M^n,where ϕ is a (1,1) tensor field, ξ is a vector field,η is a 1-form and g is the Riemannian metric.Then M is called an almost contact manifold.For an almost contact manifold M,it follows that [8]

$$(\nabla_X \phi)Y = \nabla_X \phi Y - \phi(\nabla_X Y) \tag{2.6}$$

$$(\nabla_X \eta)Y = \nabla_X \eta(Y) - \eta(\nabla_X Y) \tag{2.7}$$

Let R be Riemann curvature tensor,S Ricci curvature tensor,Q Ricci operator we have

$$S(X,Y) = \sum_{i=1}^{n} g(R(e_i,X)Y,e_i) \tag{2.8}$$

$$QX = -\sum_{i=1}^{n} R(e_i,X)e_i \tag{2.9}$$

and

$$S(X,Y) = g(QX,Y) \tag{2.10}$$

for any vector field X,Y on M^n,then (ϕ,ξ,η,g),is called an almost paracontact metric structure and the manifold M equipped with an almost paracontact metric structure is called an almost paracontact metric manifold.Further in addition, if the structure (ϕ,ξ,η,g) satisfies

$$d\eta(X,Y) = g(X,\phi Y) \tag{2.11}$$

for any vector fields X,Y on M^n .Then the manifold is called paracontact metric manifold and the corresponding structure (ϕ,ξ,η,g) , is called a paracontact structure with the associated metric g [17]. On an almost paracontact metric manifold,the (1,2) tensor field N_ϕ defined as

$$N_\phi = [\phi,\phi] - 2d\eta \otimes \xi \tag{2.12}$$

Where $[\phi,\phi]$ is the nijenhuis tensor of ϕ.If N vanishes identically,then we say that the manifold M^n is a normal almost parametric metric manifold. The normality condition implies that the almost paracomplex structure J defined on Mn×R

$$J(X,\lambda\frac{d}{dt}) = (\phi X + \lambda(\xi), \eta(X)\frac{d}{dt}),$$

is integrable . Here X is tangent to M_n , t is the coordinate of R and λ is a differentiable function on M^n×R.

For an almost paracontact metric 3-dimensional man ifold M^3, the following three conditions are mutually equivalent :

(i) there exist smooth functions α,β on M^3 such that

$$(\nabla_X \phi)Y = \alpha(g(\phi X,Y)\xi - \eta(Y)\phi X) + \beta(g(X,Y)\xi) - \eta(Y)X) \tag{2.13}$$

(ii) M^3 is normal,

(iii) there exist smooth functions α,β on M^3 such that

$$\nabla_X \xi = \alpha(X - \eta(X)\xi) + \beta\phi X \tag{2.14}$$

where ∇ is Levi-Civita connection of pseudo-Riemannian metric g. A normal almost paracontact metric 3-dimensional manifold is called

(a) Para-Cosymplectic manifold if $\alpha = \beta = 0$,

(b) quasi-para Sasakian manifold if and only if $\alpha = 0$ and $\beta \neq 0$,

(c) β-para Sasakian manifold if and only if $\alpha = 0$ and β is a non-zero constant,in particular para-Sasakian manifold if $\beta = -1$

(d) α-para Kenmotsu manifold if α is a non-zero constant and $\beta = 0$ in particular para-Kenmotsu manifold if $\alpha=1$.

On 3-Dimensional α-para Kenmotsu Manifold with Semi-Symmetric Metric Connection

In 3-dimensional α-para Kenmotsu manifold, the Ricci tensor S of Levi-Civita connection ∇ is given by

$$S(X,Y) = g(R(e_1,X)Y,e_1) - g(R(\phi e_1,X)Y,\phi e_1) + g(R(\xi,X)Y,\xi).$$

Let M^3 (ϕ,ξ,η,g) be an α-para Kenmotsu manifold [13],then we have

$$\begin{aligned} R(X,Y)Z &= (\tfrac{r}{2} + 2\alpha^2)[g(Y,Z)X - g(X,Z)Y] \\ &-(\tfrac{r}{2} + 3\alpha^2)[\eta(X)g(Y,Z) - \eta(Y)g(X,Z)]\xi \\ &+(\tfrac{r}{2} + 3\alpha^2)[\eta(X)Y - \eta(Y)X]\eta(Z) \end{aligned} \tag{3.1}$$

Replace Z = ξ in equation (3.1), we get

$$R(X,Y)\xi = \alpha^2\eta(X)Y - \eta(Y)X, \tag{3.2}$$

$$S(X,Y) = (\tfrac{r}{2} + 2\alpha^2)g(X,Y) - (\tfrac{r}{2} + 3\alpha^2)\eta(X)\eta(Y) \tag{3.3}$$

$$S(X,\xi) = -2\alpha^2\eta(X) \tag{3.4}$$

$$(\nabla_X \phi)Y = \alpha(g(\phi X,Y)\xi - \eta(Y)\phi X) \tag{3.5}$$

$$\nabla_X \xi = \alpha(X - \eta(X)\xi) \tag{3.6}$$

Let $\bar{\nabla}$ be a linear connection and ∇ be a Riemann connection of an α-para Kenmotsu manifold M. This $\bar{\nabla}$ linear connection defined by

$$\bar{\nabla}_X Y = \nabla_X Y + \eta(Y)X - g(X,Y)\xi. \tag{3.7}$$

For α-para Kenmotsu manifold with semi-symmetric metric connection ,using (2.6),(3.5) and (3.7) we have

$$(\bar{\nabla}_X \phi)Y = \alpha[g(\phi X,Y)\xi - \eta(Y)\phi X] + \eta(Y)\phi X \tag{3.8}$$

from equation (3.7),we have

$$\bar{\nabla}_X \xi = (1+\alpha)(X - \eta(X)\xi) \tag{3.9}$$

Let M^3 be a 3-dimensional α-para Kenmotsu manifold.The curvature tensor $\bar{R}$ of $M3$ with respect to the semi-symmetric metric connection $\bar{\nabla}$ is defined by

$$\bar{R}(X,Y)Z = \bar{\nabla}_X\bar{\nabla}_Y Z - \bar{\nabla}_Y\bar{\nabla}_X Z - \bar{\nabla}_{[X,Y]} Z, \tag{3.10}$$

with the help of (3.7) and (3.9), we get

$$\bar\nabla_X\bar\nabla_Y Z = \nabla_X\nabla_Y Z + X\eta(Z)Y + \eta(Z)\nabla_X Y - Xg(Y,Z)\xi$$
$$-\alpha g(Y,Z)X + \alpha g(Y,Z)\eta(X)\xi$$
$$+\eta(\nabla_Y Z)\eta(Z)\eta(Y)X - g(Y,Z)X$$
$$-g(X,\nabla_Y Z)\xi - \eta(Z)g(X,Y)\xi + g(Y,Z)\eta(X)\xi \quad (3.11)$$

$$\bar\nabla_Y\bar\nabla_X Z = \nabla_Y\nabla_X Z + Y\eta(Z)X + \eta(Z)\nabla_Y X - Yg(X,Z)\xi$$
$$-\alpha g(X,Z)Y + \alpha g(X,Z)\eta(Y)\xi$$
$$+\eta(\nabla_X Z)\eta(Z)\eta(X)Y - g(X,Z)Y$$
$$-g(Y,\nabla_X Z)\xi - \eta(Z)g(Y,X)\xi + g(X,Z)\eta(Y)\xi \quad (3.12)$$

and

$$-\bar\nabla_{[X,Y]} Z = -\nabla_{[X,Y]} - \eta(Z)\nabla_X Y + \eta(Z)\nabla_Y X$$
$$-g(\nabla_X Y,Z)\xi + g(\nabla_Y X,Z)\xi. \quad (3.13)$$

By using equations (3.7),(2.2),(2.3),(3.6),(3.9)(3.10),(3.11),(3.12) and(3.13) ,we get

$$\bar R(X,Y)Z = R(X,Y)Z - (1+2\alpha)[g(Y,Z)X - g(X,Z)Y]$$
$$+(1+\alpha)[\eta(Y)X - \eta(X)Y]\eta(Z)$$
$$+(1+\alpha)[\eta(X)g(Y,Z) - \eta(Y)g(X,Z)]\xi \quad (3.14)$$

Replace Z = ξ in equation (3.14),using (2.3) and (3.2),we have

$$\bar R(X,Y)\xi = \alpha(1+\alpha)(\eta(X)Y - \eta(Y)X). \quad (3.15)$$

Replace Y = ξ in equation (3.15) and using equation (2.3),we get

$$\bar R(X,\xi)\xi = \alpha(1+\alpha)(\eta(X)\xi - X). \quad (3.16)$$

In (3.15) taking the inner product with Z,we have

$$g(\bar R(X,Y)\xi,Z) = \alpha(1+\alpha)(\eta(X)g(Y,Z) - \eta(Y)g(X,Z)). \quad (3.17)$$

Thus we have

Lemma 3.1 Let M be a 3-dimensional α-para Kenmotsu manifold with the semi-symmetric metric connection,$\bar S$ Ricci curvature tensor and $\bar Q$ Ricci operator .Then

$$\bar S(X,\xi) = -2\alpha(1+\alpha)\eta(X) \quad (3.18)$$

and

$$\bar Q\xi = -2\alpha(1+\alpha)\xi \quad (3.19)$$

Proof. Contracting with Y and Z in (3.17) and summing over i=1,2,...,n,from (2.8) expression

$$\sum g(\bar R(e_i,Y)\xi,e_i) = \alpha(1+\alpha)[\sum \eta(e_i)g(Y,e_i) - \eta(Y)\sum g(e_i,e_i)]$$

the proof of (3.18) is completed.Then also usnig (2.10) and (2.1),(2.2),(2.3) ,the proof of (3.19) is completed.

Lemma 3.2 Let M be a 3-dimensional α-para Kenmotsu manifold with the semi-symmetric metric connection,r scalar curvature tensor,$\bar S$ (X,Y) Ricci curvature tensor and $\bar Q$ X Ricci operator. Then it follows that

$$\bar S(X,Y) = (-1+\tfrac{r}{2} - 3\alpha + \alpha^2)g(X,Y) + (1-\tfrac{r}{2} + \alpha - 3\alpha^2)\eta(X)\eta(Y) \quad (3.20)$$

And

$$\bar QX = (-1+\tfrac{r}{2} - 3\alpha + \alpha^2)X + (1-\tfrac{r}{2} + \alpha - 3\alpha^2)\eta(X)\xi \quad (3.21)$$

Proof. Taking inner product of equation (3.14) with U and using equation (2.3) we have

$$g(\bar R(X,Y)Z,U) = g(R(X,Y)Z,U)$$
$$-(1+2\alpha)[g(Y,Z)g(X,U) - g(X,Z)g(Y,U)]$$
$$+(1+\alpha)[\eta(Y)g(X,U) - \eta(X)g(Y,U)]\eta(Z)$$
$$+(1+\alpha)[\eta(X)g(Y,Z) - \eta(Y)g(X,Z)]\eta(U) \quad (3.22)$$

Let $\{e_1,\phi e_1,\xi\}$ be a local orthonormal ϕ-basis of vector fields on α-para Kenmotsu manifold M^3.Then, we get

$$\bar S(X,Y) = (-1+\tfrac{r}{2} - 3\alpha + \alpha^2)g(X,Y) + (1-\tfrac{r}{2} + \alpha - 3\alpha^2)\eta(X)\eta(Y) \quad (3.23)$$

from equation (3.23) ,we have

$$\bar r = -2 + r - 8\alpha \quad (3.24)$$

where $\bar r$ is the scalar curvature with semi-symmetric metric connection.
using (3.23) and (2.10),it's verified that

$$g(\bar QX,Y) = g((-1+\tfrac{r}{2} - 3\alpha + \alpha^2)X + (1-\tfrac{r}{2} + \alpha - 3\alpha^2)\eta(X)\xi),Y) \quad (3.25)$$

from equation (3.25),we get

$$\bar QX = (-1+\tfrac{r}{2} - 3\alpha + \alpha^2)X + (1-\tfrac{r}{2} + \alpha - 3\alpha^2)\eta(X)\xi \quad (3.26)$$

the proof of (3.21) is completed.

Ricci Solitons in α-para Kenmotsu Manifold with Semi-Symmetric Metric Connection

Let M be a 3-dimensional α-para Kenmotsu manifold with the semi-symmetric metric connection and V be pointwise collinear with ξ (i.e.V =bξ , where b is a function).Then

$$(L_V g + 2S + 2\lambda g)(X,Y) = 0$$

implies

$$0 = bg(\bar\nabla_X \xi,Y) + (Xb)\eta(Y) + bg(X,\bar\nabla_Y \xi)$$
$$+(Yb)\eta(X) + 2\bar S(X,Y) + 2\lambda g(X,Y) \quad (4.1)$$

using (3.9) in (4.1) , we get

$$0 = 2b(1+\alpha)g(X,Y) - 2b(1+\alpha)\eta(X)\eta(Y) + (Xb)\eta(Y)$$
$$+(Yb)\eta(X) + 2\bar S(X,Y) + 2\lambda g(X,Y) \quad (4.2)$$

With the substitution of Y with ξ in (4.2) , it follows that

$$(Xb) + (\xi b)\eta(X) + 2\lambda\eta(X) - 4\alpha(1+\alpha)\eta(X) = 0 \quad (4.3)$$

Again replacing X by ξ in (4.3) shows that

$$\xi b = -\lambda + 2\alpha(\alpha+1) \quad (4.4)$$

Putting (4.4) in (4.3), we obtain

$$b = (2\alpha(1+\alpha) - \lambda)\eta \quad (4.5)$$

By applying d in (4.5), we get

$$0 = (2\alpha(1+\alpha) - \lambda)d\eta \quad (4.6)$$

Since $d\eta \neq 0$ from, we have

$$2\alpha(1+\alpha) - \lambda = 0 \quad (4.7)$$

By using (4.5) and (4.7), we obtain that b is constant. Hence from (4.2) it is verified

$$\bar{S}(X,Y) = -b((1+\alpha)+\lambda)g(X,Y) + b(1+\alpha)\eta(X)\eta(Y) \quad (4.8)$$

which implies that M is an η-Einstein manifold. This leads to the following

Theorem 4.1 If in a 3-dimensional α-para Kenmotsu manifold with the semi symmetric metric connection, the metric g is a Ricci soliton and V is a pointwise collinear with ξ ,then V is a constant multiple of ξ and g is an η-Einstein manifold of the form (4.8) and Ricci soliton is steady and expanding according as $\lambda = 2\alpha(1+\alpha)$ is zero and positive , respectively .

Conharmonically Flat *α*-para Kenmotsu Manifolds with the Semi-Symmetric Metric Connection

We have studied conharmonically flat *α*-para Kenmotsu manifolds with respect to the semi-symmetric metric connection. In a *α*-para Kenmotsu manifold the conharmonic curvature tensor with respect to the semi-symmetric metric connection is given by

$$\begin{aligned}\bar{K}(X,Y)Z &= \bar{R}(X,Y)Z \\ &-[\bar{S}(Y,Z)X - \bar{S}(X,Z)Y \\ &+g(Y,Z)\bar{Q}X - g(X,Z)\bar{Q}Y]. \quad (5.1)\end{aligned}$$

If $\bar{K}=0$,then the manifold M is called conharmonically flat manifold with respect to the semi- symmetric metric connection. Let M be a conharmonically flat manifold with respect to the semi-symmetric metric connection. from (5.1),we have

$$\bar{R}(X,Y)Z = \bar{S}(Y,Z)X - \bar{S}(X,Z)Y + g(Y,Z)\bar{Q}X - g(X,Z)\bar{Q}Y \quad (5.2)$$

,(3.20)and (3.21) in (5.1) ,we get

$$\begin{aligned}&R(X,Y)Z - (1+2\alpha)[g(Y,Z)X - g(X,Z)Y] \\ &+(1+\alpha)[\eta(Y)X - \eta(X)Y]\eta(Z) \\ &+(1+\alpha)[\eta(X)g(Y,Z) - \eta(Y)g(X,Z)]\xi \\ &= S(Y,Z)X - S(X,Z)Y \\ &+(\tfrac{r}{2} - 6\alpha - 2)[g(Y,Z)X - g(X,Z)Y] \\ &+(1+\alpha)[\eta(Y)X - \eta(X)Y]\eta(Z) \\ &+(1 - \tfrac{r}{2} + \alpha - 3\alpha^2)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)]\xi \quad (5.3)\end{aligned}$$

Now putting X=ξ in (5.3), we obtain

$$\begin{aligned}R(\xi,Y)Z &= S(Y,Z)\xi - S(\xi,Z)Y \\ &+(\tfrac{r}{2} - 1 - 4\alpha)[g(Y,Z)\xi - \eta(Z)Y] \\ &-(\tfrac{r}{2} + 3\alpha^2)[g(Y,Z) - \eta(Z)\eta(Y)]\xi, \quad (5.4)\end{aligned}$$

using (3.1) and (3.4) in (5.4), we get

$$S(Y,Z)\xi - S(\xi,Z)Y + (-1-4\alpha-2\alpha^2)g(Y,Z)\xi + (1+4\alpha+2\alpha^2)\eta(Z)Y = 0 \quad (5.5)$$

Taking inner product with ξ in (5.5) , we get

$$S(Y,Z) = (1+4\alpha+2\alpha^2)g(Y,Z) - (1+4\alpha+4\alpha^2)\eta(Y)\eta(Z)) \quad (5.6)$$

Thus M is an η-Einstein manifold with respect to the Levi-Civita connection. This leads to the following

Theorem 5.1 If M is a conharmonically flat *α*-para Kenmotsu manifolds with respect to the semi-symmetric metric connection. Then the manifold M is an *η*-Einstein.

Example

(A 3-dimensional α-para Kenmotsu manifold with the semi-symmetric metric connection.) We consider the 3-dimensional manifold $M = (x,y,z) \in R^3, z\neq 0$, where (x,y,z) are the standard coordinates in R^3. The vector fields

$$e_1 = z^2\frac{\partial}{\partial x}, e_2 = z^2\frac{\partial}{\partial y}, e_3 = \frac{\partial}{\partial z}$$

are linearly independent at each point of *M*. Let g be the Riemannian metric defined by

$$g(e_1,e_3) = g(e_2,e_3) = g(e_1,e_2) = 0,$$

$$g(e_1,e_1) = g(e_2,e_2) = g(e_3,e_3) = 1.$$

Let η be the 1-form defined by $\eta(Z) = g(Z, e_3)$ for any $Z \in \chi(M)$. Let ϕ be the (1,1) tensor field

defined by $\phi(e_1) = -e_2$, $\phi(e_2) = e_1$, $\phi(e_3) = 0$.

Then using linearity of ϕ and g we have

$$\eta(e_3) = 1, \phi^2(Z) = -Z + \eta(Z)e_3$$

$$g(\phi Z, \phi W) = g(Z,W) - \eta(Z)\eta(W)$$

for any Z , W ∈ χ(M) .Now , by direct computations we obtain

$$[e_1,e_2] = 0,\ [e_2,e_3] = -\tfrac{2}{z}e_2,\ \ [e_1,e_3] = -\tfrac{2}{z}e_1$$

by using these above equations we get [1]

$$\nabla_{e_i} e_i = \tfrac{2}{z}e_3 \ \ and\ \ \nabla_{e_i} e_3 = -\tfrac{2}{z}e_1 \quad (6.1)$$

$$\nabla_{e_2} e_1 = \nabla_{e_1} e_2 = \nabla_{e_3} e_1 = \nabla_{e_3} e_2 = \nabla_{e_3} e_3 = 0 \quad (6.2)$$

Now we consider at this example for semi-symmetric metric connection . from (3.8) , (6.1) and (6.2)

$$\bar{\nabla}_{e_i} e_i = (\tfrac{2}{z} - 1)e_3 \ \ and\ \ \bar{\nabla}_{e_i} e_3 = (-\tfrac{2}{z} + 1)e_1 \quad (6.3)$$

$$\bar{\nabla}_{e_i}\nabla_{e_j} = \bar{\nabla}_{e_3} e_j = 0 \ and\ \bar{\nabla}_{e_3} = 0 \quad (6.4)$$

where $i \neq j = 1,2$. it's known that

$$\begin{aligned}&\bar{R}(e_i,e_3)e_3 = (\tfrac{6}{z^2} + \tfrac{2}{z})e_i,\ \ \bar{R}(e_i,e_j)e_3 = 0 \\ &\bar{R}(e_i,e_j)e_j = (\tfrac{4}{z} - \tfrac{4}{z^2} - 1)e_i,\ \bar{R}(e_i,e_3)e_j = 0 \quad (6.6) \\ &\bar{R}(e_3,e_i)e_i = (\tfrac{2}{z} - \tfrac{6}{z^2})e_3\end{aligned}$$

where $i \neq j = 1,2$. From (2.8) and (6.6)
it's verified that

$$S(e_1, e_1) = (\frac{-2}{z^2} + \frac{2}{z} - 1)$$

$$S(e_2, e_2) = (\frac{-10}{z^2} + \frac{6}{z} - 1) \quad (6.7)$$

$$S(e_3, e_3) = (\frac{-12}{z^2} + \frac{4}{z})$$

Conclusion

If in a 3-dimensional α-para Kenmotsu manifold with the semi-symmetric metric connection , the metric g is a Ricci soliton and In this study , we gave some curvature conditions for 3-dimensional α-para Kenmotsu manifolds with semi-symmetric metric connection.In 3-dimensional α-para Kenmotsu manifolds with semi-symmetric metric connection is also an η-Einstein manifold and Ricci soliton defined steady or expanding on this manifold is named with respect to values of α and λ constant.We also proved that conharmonically flat α-para Kenmotsu manifolds with semi-symmetric metric connection is an η-Einstein manifold.

References

1. Welyczko J (2009) On Legendre curves in 3-dimensionalnormal almost paracontact metric manifolds, Result Math 54: 377-387.
2. Sharma R, Sinha BB (1983) On para-A-Einstein manifolds, Publications De L'institu Mathematique 34: 211-215.
3. Bejan CL, Crasmareanu M (2011) Ricci solitons in manifolds with quasi-constant curvature, publications Mathematicae, Debrecen 78: 235-243.
4. Tripathi M M (2008) Ricci solitons in contact metric manifolds. http://arxiv.org/abs/0801.4222.
5. Kaneyuki S, Williams FL (1985) Almost paracontact and parahodge structures on manifolds,Nagoya Math J 99: 173-187.
6. Nagaraja HG, Premalatha CR (2012) Ricci solitons in Kenmotsu manifolds, Journal of Mathematical Analysis 3: 18-24.
7. Agashe S, Chafle R (1992) A semi-symmetric non-metric connection on a Riemannian manifold,Indian Journal Pure Applications 23: 399-409.
8. Liang KC (1972) A nonoscillation theorem for the superlinear case of second order differential equation, SIAM Journal on Applied Mathematics 23: 456-459.
9. Bagewadi CS, Ingalahalli G (2012) Ricci solitons in α-Sasakian manifolds, ISRN Geometry 13.
10. Sengupta JDeUC, Binh TQ (2000) On a type of semi-symmetric non-metric connection on a Riemannian manifold. Indian Journal of Pure Applied Mathematics 31: 1659-1670.
11. Yildiz A, Cetinkaya A (2013) Kenmotsu manifolds with the semi-symmetric non-metric connection, preprint.
12. Zamkovoy S (2009) Canonical connections on paracontact manifolds, Ann. Glob. Anal. Geom 36: 37-60.
13. Rossca R, Vanhecke L (1976) S'ur une vari'et'e presque paracok "ahl'erienne munie d'une connexion self-orthogonal involutive,Ann.Sti.Univ."Al.I.Cuza"Ia si 22: 49-58.
14. Srivastava K, Srivastava SK (2014) On a class of α-Para Kenmotsu manifolds, Mediterranean Journal of Mathematics 13: 391-399.
15. Kenmotsu K (1972) A class of almost contact Riemannian manifolds, The Tohoku Mathematical Journal 24: 93-103.
16. Yano K (1970) On semi-symmetric metric connection, Rev. Roum. Math. Pure Appl 15: 1570-1586.
17. Mishra RS, Jami at al-Kuwayt Qism al-Riyaḍiyat 1984) Structures on differentiable manifold and their applications. Chandrama Prakasana 1-396.
18. Blaga AM (2014) η-Ricci solitons on para-Kenmotsu manifolds arXiv:1402. 0223v1.
19. De UC, Shaikh AA (2007) Differential geometry of manifolds, Alpha Science International 298.
20. G Zhen, JLCabrerizo, LM Fernandez, M Fernandez (1997) on ξ-conformally flat contact metric manifolds. Indian J.Pure Appl.Math 28: 725-734.
21. Prvanovic M (1975) On pseudo metric semi-symmetric connection,Publication De L Institut Mathematic Nouvelle Series 18: 157-164.