

Quantum-Mechanical Substantiation of the Periodic System of Isotopes. Models of Nuclear Orbitals

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ABSTRACT

The topic of this article lies in the field of problems: substantiating the periodic system of isotopes and the principle of multilevel periodicity using quantum mechanical calculations, combining strong and electromagnetic interactions, and searching for the fundamental cause of periodicity in general. This article is a theoretical section and a continuation of the article: "Periodic system of isotopes", in which the system was checked against 10 types of experimental data, the periodic change of properties at the level of nuclei and the vertical symmetry of subgroups of isotopes were found. Periodic system of isotopes was constructed with the help of a special algorithm, the principle of multilevel periodicity of the atom, from the electrons to the nucleus. As a description of the multilevel periodicity, this paper presents a unified system of quantum numbers, which is used to describe both electron and nucleon shells (binomial probabilistic interpretation). With the binomial interpretation the problem of a particle in a one-dimensional potential well has been solved; quantum-mechanical calculations for the probability functions of the orbitals and periods of both electrons and nucleons have been performed - characteristic equations have been obtained, the projections of electronic orbitals have been reproduced and the binomial interpretation has been shown to correspond to the family of spherical harmonics. For the electron orbitals the calculation and analysis of solutions of the Schrödinger equation for the binomial interpretation of quantum numbers have been performed. The spatial nature of quantum numbers, for this interpretation, in the form of degrees of freedom is shown. Based on the principle of multilevel periodicity, expressions are derived and planar projections of nucleon nucleon orbitals are constructed, and similarity of the forms with electron orbitals is analyzed and revealed. A critical analysis of the modern spherical coordinate system was made, possible errors in the construction of electron orbitals were shown and, taking into account the drawbacks, two alternative spherical coordinate systems were proposed, for which Lamé coefficients were calculated and Laplace equations were derived. As a search for the fundamental cause of multilevel periodicity, a spatial model with changing degrees of freedom $0-n$ is presented, its manifestation in nature (crystal forms) is found; a number of experiments are proposed; the predictions about the applicability of the multilevel periodicity principle in quark theory are made.

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Received: June 09, 2021; Accepted: June 21, 2021; Published: August 07, 2021

Keywords: Periodic System, Isotope, Period, Subgroup, "Nuclear" Orbital, Binomial Probabilistic Interpretation of Quantum Numbers, Alternative Spherical Coordinate Systems, Unification Theories, Multidimensional Spaces.

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Introduction

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5. Multidimensional interference model of the intra-atomic space. Hypotheses of occurrence for the principle of multilevel periodicity.

This paper is a theoretical section and continuation of the article: "The periodic system of isotopes". The periodic system of isotopes is constructed using the principle of multilevel periodicity (PMP). The objectives of the article are the following: interpretation of

the principle of multilevel periodicity in the context of quantum mechanics, explanation of the principle; demonstration of a general approach for constructing wave functions of both electron and nucleon shells in a single format of reasoning and calculations; presentation of some ideas that can be used in nuclear modeling, planning reactions and experiments using the periodic system of isotopes, as well as an explanation of the fundamental reasons for the emergence of multilevel periodicity and the phenomenon of periodicity in general.

Due to the periodic system of isotopes, a change in the properties of nuclei for the periods is shown together with their vertical symmetry in the subgroups in accordance with the following experimental data:

- the distribution of heavy fission fragments (a "double-humped" barrier – symmetry in the subgroups);
- the distribution of quadruple moment anomalies;
- the distribution of the amplitude and sign for the magnetic moment values;
- there is a periodic change in the range of values for the coherent neutron scattering length in the even periods and a small change in the odd periods;

- the mass boundaries of isotope synthesis in stars correspond to the nuclear periodic structure;
- the presence of radioactivity in heavy isotopes (beginning of the period 8);
- the presence of cluster radioactivity (period 8);
- the presence of stability “islands” (^{232}Th , ^{234}U , ^{235}U , ^{238}U are in the center of the dn-orbital for the period 8);
- the distribution of the lifetime for radioactive isotopes;
- the distribution of anomalies for the thermal neutron capture cross-section;
- the distribution of cross-sections for the α -particle ejection reaction (n , α);
- concentration of the main isotopes in polymetallic ores Zn, Pb, etc.

Today, the Mendeleev’s periodic system is considered to be an empirical rule: it cannot be expressed via mathematical expression. A distinctive advantage of PMP is that this principle suggests a single algorithm for determining the size of the orbitals and periods for electrons and nucleons (fig. 1 c). This allows reproducing the periodic system of elements within the principle. This feature allows describing the electron and nucleon shells in a single format of numbers similar to quantum ones.

The quantum numbers of electrons and the number of moment projections were selected on the basis of the experience for the Mendeleev’s periodic system (fig. 1 a). It was proposed to use quantum numbers in nuclear physics (color, baryon charge, strangeness, isotopic spin) to simulate strong interaction, internal structure of the nucleus and nucleons themselves. It should be noted that the quantum numbers for electrons and nucleons are, in fact, isolated systems separated by different laws of strong and electromagnetic interactions. A unified approach to determining the size of periods and orbitals proposed by PMP requires solving a rather complicated issue of combining electron and nucleon shells in a single quantum system. The principle of multilevel periodicity itself can be subsumed under the category and tasks of the Grand Unified Theory [1].

The main task of the Grand Unified Theories is to describe strong, weak, electromagnetic and gravitational interactions in a unified manner. A distinctive feature of unifying theories, among others, is the search for a suitable geometric formalism, in which there is a tendency towards modeling multidimensional simulation. A quantum field theory in the form of a standard model is the only experimentally confirmed theory for high-energy particles. The quantum field theory is used to study the behavior of physical systems with an infinitely large number of degrees of freedom. In this theory, the algebraic models of Hilbert spaces (n-dimensional analogues of a 3D Euclidean space) are used [2]. In string theory, a quantum string is a hypothetical infinitely thin one-dimensional object with the length of 10-35 m. All the elementary particles are formed by string vibrations. The nature of physical interaction is defined by a string vibration pattern. There are bosonic, fermionic, closed-open, supersymmetric strings (superstrings), etc. A superstring is a hypothetical multidimensional object, which has ambiguous definition and, at the same time, contains supersymmetry — the correspondence of weak and strong interactions [3].

The nucleus consists of many particles. It is difficult to accurately calculate or predict the behavior of nucleons via simulation of their behavior and interaction one at a time. At the same time, for the methods of statistical physics, the number of nucleons (hundreds) is too small, while the nucleons are structured by shells having

different sizes, hierarchies and properties. Such conditions resulted in a sufficiently large number of nuclear models. Today, there are at least 12 models of atomic nucleus. Among them, there are three models describing the existence and parameters of shells inside the nucleus: the nuclear shell, cluster and statistical models. The nuclear shell model [4, 5]; the nuclear statistical model (J. Frenkel 1936; L. Landau, 1937); the nuclear cluster model (J. A. Wheeler, 1937).

The disciplines and tools, using which the nuclear and atomic models are constructed, can be divided into three categories: physical (properties, results of experiments and observations, hypotheses), algebraic (calculations, mathematical models) and geometric (functional and configuration spaces, spatial models, visualization). It should be noted that in the recognized and widespread models of the atomic and nuclear structure, physical and algebraic formalism is of predominant importance, while geometric formalism is of subordinate importance and is often underdeveloped or adjusted to the results of calculations and experiments. The existence of periodicity for the elements and isotopes suggests the existence of a fundamental cause for atom segmentation (division into periods and subgroups). The basic cause of quantization, the discreteness of which is reflected in quantum numbers, is also not quite clear. In natural phenomena, these causes should be found in the form of either a general structural pattern of matter in space and time or in the form of unknown binding interaction, unknown particle or something else. The search for particles and properties of interactions has been carried out rather successfully for a long time, but an empirical nature of the periodic system has not yet been “deflated”. Knowledge on the structure and nature of time is extremely poor. In this regard, the search of a general physical system for describing the shells of electrons and nucleons should be carried out by means of quantum mechanics and in terms of a wave function for comparison with generally accepted results and visualization achievements. Moreover, not only physical, algebraic or geometric formalism should be used, but also a correspondence to this cause should be found in the patterns existing in nature. Visualization is of great practical and academic importance, especially for finding correspondences in nature, as well as for the development of applied use (for more details, see Section 5). These are the criteria and main directions described in this article.

The examples for underdevelopment of geometric formalism and visualization methods in quantum mechanics and physics as a whole are the following:

- When obtaining the wave function graphs for electron orbitals, the complex solutions are reduced to the real form. This can be done using artificial algebraic manipulations with coordinates, such as squaring, reduction to the sum or difference of squares: z^2 , x^2-y^2 , etc. Otherwise, many wave functions simply could not be constructed.

- A classical spherical coordinate system is constructed anisotropically. There is a specified Z axis and XOY plane. A horizontal angle is measured in this plane, and a vertical angle is measured either from the XOY plane or from the Z axis. However, there are other X, Y axes and other planes. While using this coordinate system, it is necessary to remember that 3D space is isotropic and all the directions are equal. This is one of the main properties of space. The priority of one axis or plane can lead to errors and one-sided results when simulating spherically organized systems, where all the directions are independent (Section 3. “Disadvantages of the classical spherical coordinate system and alternative spherical CS”).

- The fourth degree of freedom for a particle, introduced by the German physicist Wolfgang Pauli (later called the spin), made it possible to explain a number of phenomena: magnetism, splitting of spectral lines of a fine structure in a magnetic field. However, today there is no single approach to the geometry and visualization of 4D space, as well as to multidimensional spaces in general. These spaces are set parametrically in most cases.

- The complex numbers determine the additional dimension in the “indefinite” direction of the imaginary axis. No method for representation of this axis together with the three real axes has been developed yet.

- In almost all the sources, 4D Minkowski space (fig. 1, b) is presented only in the form of two light cones of a special relativity theory, where one of the spatial coordinate axes is aligned with the time axis. This is actually a 3D image of one cone for simulated 4D space. In this space, a 3D object being under study is represented and described by parallel transfer, also in 3D space. The temporal dimension is not shown and it does not go into or out of the spatial dimension, but is understood. The theory is used to simulate 4 spatial dimensions only algebraically. It is impossible to get information on the structure of 4D space from this theory (for more details, see Section 5 and) [6].

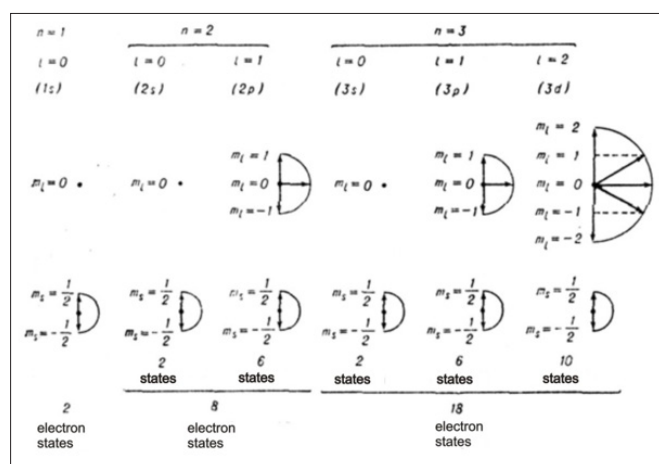


Figure 1 a: Quantum numbers, electron angular momentum and periodic law

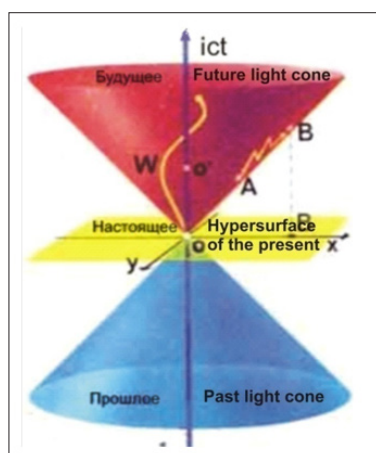


Figure 1 b: Schematic view of 4D Minkowski space

Different aspects of a more general picture for the structure of matter can be better described by different theories. There are several atomic models that can be historically highlighted: “Raisin

bun” by J.J. Thomson (1903); Nuclear model by E. Rutherford (1911); Wave model by L. De Broglie (1924); Quantum mechanical model by E. Schrödinger (1926); Orbital model by G. White (1931); Ring-shaped model (1963) by K. Snelson et al. The orbital and quantum-mechanical models are of the greatest practical importance in chemistry. These models most closely reproduce the periodic law of elements. In this regard, it is necessary to demonstrate how, using the principle of multilevel periodicity, by means of quantum mechanics, it is possible to construct the forms of nuclear orbitals, and within the framework of one system of quantum numbers, in a new way, to reproduce the forms of electron orbitals, proposing a theoretical unification of systems of electron shells and nuclei. And also, most importantly, to offer an explanation for the fundamental reason for the periodicity of chemical elements and isotopes.

Quantum-Mechanical Substantiation of the Principle for the Periodic System of Isotopes

A physical sense for the principle of multilevel periodicity should be considered in more detail. In this aspect, “one of the most excellent properties of quantum mechanics should be demonstrated – how much can be deduced from so little” [7]. The space for the filled shell of the orbital or period can be represented as a configuration space for N regions equal to the number of particles contained in this shell. In accordance with the principle of multilevel periodicity, N is equal to:

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$$N = 2 * (C_{n1k1} + C_{n2k2}) \quad (1)$$

where C_{n1k1} and C_{n2k2} corresponding to the shell are the binomial coefficients of the Pascal’s triangle (fig. 1 c):

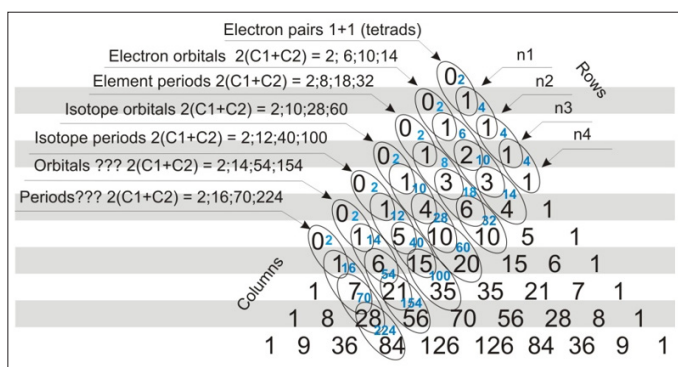


Figure 1 c: The Pascal's triangle. The principle of multilevel periodicity.

Binomial coefficients are calculated by several explicit formulas, for non-negative integers d and f there is the expression (2). In literature, as a rule, denotations n and k are used (2, on the left) for the formulas of binomial coefficients. To avoid confusion with the main quantum number n and the wave number k , the coefficients in (2) are denoted as $n = d$ and $k = f$ (2, on the right).

$$C_n^k = \binom{n}{k} = \frac{n!}{k!(n-k)!} \quad C_d^f = \binom{d}{f} = \frac{d!}{f!(d-f)!} \quad (2)$$

where $d=0, 1, 2, 3, 4 \dots$ and $f=0 \dots d$ are the numerical coefficients. The number of particles contained in the shell of the period and orbital for electrons and nucleons is as follows:

$$N = 2*(C_{d1f1} + C_{d2f2}) \quad (3)$$

Since the principle of multilevel periodicity covers both electronic and nucleon shells, the experimental data should be analyzed for both types of particles. The changes in the coefficients d and f should be compared to each other depending on the shell type, and then the coefficients should be compared to the changes in the quantum numbers: the principal quantum number n and the orbital quantum number l .

Binomial coefficients should be analyzed in terms of two disciplines: geometry and combinatorics. Geometric approach is important for understanding spatial quantization and anisotropy of the wave vector k . combinatorial approach is important for understanding the probabilistic picture within this interpretation of quantum numbers. The interpretation will be called binomial. From the point of view of geometry, the sum $C_{d1f1} + C_{d2f2}$ (3) quantitatively shows the specific volume for the configuration space of the filled shell for $N/2$ pairs of particles, where the volume $V_{shell}/2$ ($C_{d1f1} + C_{d2f2}$) corresponds to each particle at the point moment of time. From the point of view of combinatorics, the binomial coefficients C_{d1f1} and C_{d2f2} (as well as multinomial ones) are the number of d combinations by f or the number of ordered partitions for the d -element set X_1-X_d into Cdf unique subsets consisting of f elements. It is important to pay attention to the term "unique". The Pauli fundamental principle also has the uniqueness property, whereby electrons (fermions as a whole) in the atom can have only a different set of quantum numbers, which is unique for each particle.

In formula (2), the numbers d and f are changed similar to the quantum numbers n and l ($d = 0, 1, 2, 3, 4 \dots$ and $f = 0 \dots d$; the principal quantum number $n = 1, 2, 3, 4$ and the orbital quantum number $l = 0 \dots n$) with the only difference that there is a state

$d = 0$ for d , and when the energy levels are numbered by n , the count is from 1; "0" is present for l (the state $d = 0$ is considered below, following the calculations).

In the classical format, one period of chemical elements corresponds to each n , one orbital corresponds to each l . In the binomial format, a subshell, which is included in the composition of two adjacent periods and orbitals of both electrons and nucleons, corresponds to each d and each f (Table 1). There are important differences in the correspondence of numbers to the shells: in the binomial interpretation, two pairs of numbers d_1, f_1 and d_2, f_2 correspond to each shell, as the number of particles for each shell consists of twice the sum (particle pairs) of two binomial coefficients (3).

Table 1: Calculation of binomial coefficients from d and f (8)

| d\f | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|-----|---|---|----|----|----|----|---|---|
| 0 | 1 | | | | | | | |
| 1 | 1 | 1 | | | | | | |
| 2 | 1 | 2 | 1 | | | | | |
| 3 | 1 | 3 | 3 | 1 | | | | |
| 4 | 1 | 4 | 6 | 4 | 1 | | | |
| 5 | 1 | 5 | 10 | 10 | 5 | 1 | | |
| 6 | 1 | 6 | 15 | 20 | 15 | 6 | 1 | |
| 7 | 1 | 7 | 21 | 35 | 35 | 21 | 7 | 1 |

One pair of quantum numbers n and l corresponds to each orbital in the classical format. It follows that from the point of view of the binomial interpretation, each shell, orbital or period, consists of two subshells. At a first glance, this complication may look like regression. However, using this format, being the binomial analog of quantum numbers d and f , the forms of the orbitals for both the electrons of the atom and the nucleons of the nucleus can be constructed and their descriptions can be combined within one theoretical model.

It should be assumed that the coefficients d and f have geometric sense for the degrees of freedom. This assumption is made on the basis of the following two features:

1. While observing the increase in the number of particles N from shell to shell (Table 1), it can be shown that in the configuration space, d and f can be presented by directions. If it is assumed that the particles are fixed in the shell at the point moment of time, a straight line can be drawn between the pair particles (for example, electrons with opposite spin) through the center of an atom; under the influence of Coulomb repulsive forces, the charged particles tend to be evenly distributed in the configuration space of the shell (period or orbital). Therefore, these lines will be distributed equidistantly;
2. This assumption is also true on the basis of the probability function for the binomial distribution (4), where d and f are the exponents, and therefore, in geometric sense, d and f are the degrees of freedom.

This assumption should be verified using the quantum-mechanical calculations. As applied to the wave function, d and f can be previously defined as the number of degrees of freedom for the configuration space.

The integer random variable X has the binomial distribution, if the probability of its possible values is calculated by the Bernoulli equation:

$$P_k = P(X = k) = \sum_{f=0}^d C_d^f p^f q^{(d-f)} = (p + q)^d, \quad (4)$$

$$(f = 0, 1, 2, \dots, d)$$

where p is the probability of event with the value 1 “success” and q with the value 0 “failure”; $p+q=1$; C_d^f is the binomial coefficient; X_1, \dots, X_n is the random variable from the finite sequence of independent random variables having the same Bernoulli distribution. The Bernoulli distribution is a special case of binomial distribution, and binomial distribution, in turn, is the most common form of discrete distribution.

In accordance with the principle of multilevel periodicity, the shells either of electrons or nucleons are located immediately in two configuration spaces R^{d_1} and R^{d_2} . f_1 and f_2 are the number of allowed degrees of freedom for the pairs of particles in the shell of a particular type (fig. 1, table 1). For example, from the point of view of this interpretation of quantum numbers, the p-orbital of electrons is located in two configuration spaces, in R^{d_1} and R^{d_2} , where $d_1 = 1$ and $d_1 = 2$. Configuration space of the shell is a superposition of two spaces. This will be shown with examples and calculations.

As is known from the experiments, a photon (Jung experiment, 1803), electron (experiments of Davison-Germer, 1927, Jensen, 1961) and other particles (fullerene-60, Zeilinger experiment, 1999) can behave like a wave or “corpuscle” depending on the presence or absence of additional external interaction (observer, measurement) [8, 9]. Both cases should be analyzed as applied to the binomial interpretation of quantum numbers.

In fig. 37, the diagram of a well-known double-slit experiment is shown [7]. In the experiment, it is shown that the electrons diffracting through two slits behave like waves, and the interference pattern is shown on the screen. It is interesting to note that the intensity of lines on the screen is distributed in accordance with one of the rows of the Pascal’s triangle or their sum and is proportional to the binomial coefficients. “Which line” is determined on the basis of the ratio of the distance between the slits and from the slits to the screen.

The magnetic quantum number ($m = \pm 1$) takes the following values: $m=0, \pm 1, \pm 2, \pm 3$. m shows the sign of the wave function for the electron orbital and its orientation in the coordinate system. The absolute value of m is changed similar to l and f : $|m|=f-1$ (l is the orbital quantum number). From the point of view of geometry, m ($m = \pm 1 \pm f$) corresponds to the half-axis of the direction f showing its sign.

In 1896, the Dutch physicist Peter Zeeman placed a device that is similar to a hydrogen lamp, but filled with hot sodium vapor, in a strong magnetic field. It was found that in the magnetic field, the number of lines in the emission spectra is increased. The spectra become complex, but it is shown that each p-line splits in a magnetic field into 3 new lines, each d-line splits into 5 lines, each f-line splits into 7 lines, and s-lines remain unchanged. The number of spectral lines for the electrons of different orbitals is equal to the sum of the binomial coefficients $C_{d_1}^f + C_{d_2}^f$, — the second diagonal row of the Pascal’s triangle (fig. 1), the second column at $f = 1$ (Table 1). For the electron orbitals, the number of spectral lines in the magnetic field is also equal to $d_1 + d_2$, as $d = C_d^f$ for this type of shells.

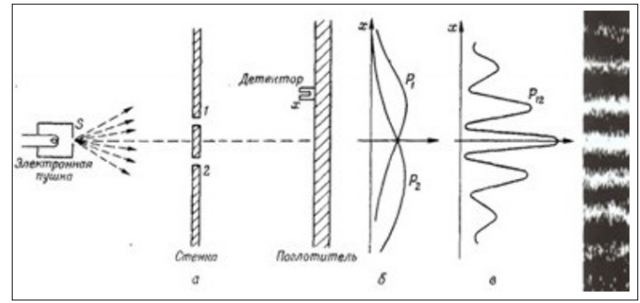


Figure 2: The interference experiment with electrons

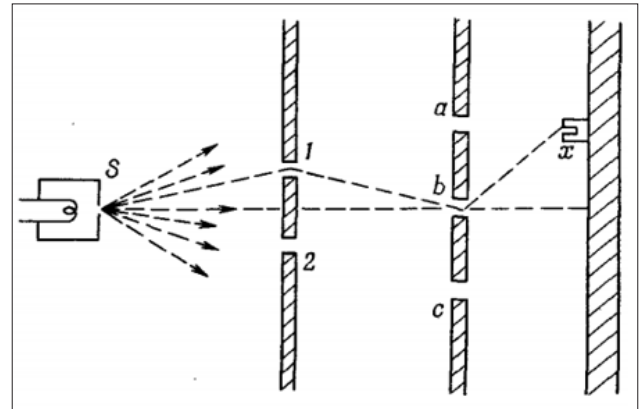


Figure 3: The interference experiment with two walls

The probabilistic aspect of this interpretation should be considered. In fig. 38, the interference experiment on the electron passage through the two walls demonstrating the composition laws of the probability amplitude from the Feynman lectures on physics is shown: “Here are the two walls: one with two slits 1 and 2, and the other one with three slits a, b and c. There is a detector behind the second wall at the “x” point, and the amplitude of the probability that the particle reaches “x” should be found. One way of solution is to calculate the superposition or interference of the waves passing through the slits; but it can be done differently saying that there are six possible ways. The electron may pass through slit 1, then through slit “a” and then to “x”, or it may pass through slit 1, then through slit “b” and then to “x”, etc. In accordance with the probability theory, the amplitudes of mutually exclusive paths are summed up so that the amplitude of transition from s to x should be written as the sum of six separate amplitudes. On the other hand, in case of successive events, each of them can be written as a product of three amplitudes. For example, one of them is the amplitude of transition from s to 1 multiplied by the amplitude of transition from 1 to “a” and by the amplitude of transition from “a” to “x”. Using our abbreviation, the full amplitude of transition from s to x can be written in the following form” [7]:

$$\langle x|s \rangle = \langle x|a \rangle \langle a|1 \rangle \langle 1|s \rangle + \langle x|b \rangle \langle b|1 \rangle \langle 1|s \rangle + \langle x|c \rangle \langle c|1 \rangle \langle 1|s \rangle + \langle x|a \rangle \langle a|2 \rangle \langle 2|s \rangle + \langle x|b \rangle \langle b|2 \rangle \langle 2|s \rangle + \dots \quad (5)$$

The expression for the composition of probability amplitudes is similar to the expansion into a polynomial or sum of several polynomials, where the product is the composition of a chain of independent events, and the sum is the variety of chains for such events. The probability of two independent events is equal to the probabilities of these events:

$$P(AB) = P(A) \cdot P(B) \quad (7)$$

The probability multiplication theorem can be generalized in case there is an arbitrary number of events. In general form, it is formulated as follows: the probability of the product of several events is equal to the product of probabilities for these events, and the probability of each subsequent event is calculated, if all the previous ones took place:

$$P(A_1 A_2 \dots A_n) = P(A_1) \cdot P(A_2 | A_1) \cdot P(A_3 | A_1 A_2) \cdot \dots \cdot P(A_n | A_1 A_2 \dots A_{n-1}) \quad (8)$$

In case of independent events, the theorem is simplified and expressed as follows:

$$P(A_1 A_2 \dots A_n) = P(A_1) \cdot P(A_2) \cdot P(A_3) \cdot \dots \cdot P(A_n), \quad (9)$$

that is, the probability of the product for independent events is equal to the product of probabilities for these events:

$$P\left(\prod_{i=1}^n A_i\right) = \prod_{i=1}^n P(A_i) \quad (10)$$

Therefore, in accordance with the Bernoulli's equation for the probability (4), as well as with the assumption that d and f have the meaning of the degrees of freedom, by the probability multiplication theorem, a particle is symmetrically distributed in space in several directions at the same time. The electron distribution with respect to several directions at the same time does not contradict the experimental data obtained as a result of electron passage through the two slits at the same time, when the electron behaves like a wave [8].

Using this explanation, the values for the parameters p^f and q^{d-f} in the Bernoulli's equation should be considered as applied to the geometric and combinatorial understanding of the binomial interpretation and the correspondence to the electron orbitals. From the point of view of geometry, in corpuscular representation of a particle for the filled shell, p^f is the total probability of a particle being close or symmetrical to the directions f in the configuration space R^d . Since the configuration space of the shell consists of the superposition of two spaces $d = d_1 + d_2$, the total number of the directions f within the shell is $f = f_1 + f_2$, and of the total number of the particles is $2(C_{n_1 k_1} + C_{n_2 k_2})$. q^{d-f} is the probability of the particle being close or symmetrical to the directions $d-f$ not covered by the orbital or period shell. For example, in case of the p_e -orbital: in accordance with the principle of multilevel periodicity, the sum of binomial coefficients corresponds to the p_e -orbital: $C_{10} + C_{12} = 1 + 2 = 3$. In other words, 3 pairs of electrons in the configuration space of $d_p = d_1 + d_2 = 1 + 2 = 3$ directions, where each of the orbital branches is distributed in space along each of three directions equally distributed in space, are the axes OX, OY, OZ (Table 1). At $f_1 = 0, d_1 = 1$. That is, for the first pair of electrons, the particles of the p_e -orbital are freely distributed, being not adjacent and not symmetrical to any direction ($f_1 = 0$), and can take any direction during bond formation ($d_1 = 1$). For the filled pe -orbital, each pair is adjacent or symmetrical to one ($f = 1$) of three directions ($d_p = d_1 + d_2 = 1 + 2 = 3$) for the configuration space. In general, the result corresponds to the modern understanding of e spatial distribution for the filled pe -orbital. For the de -orbitals, there will be such directions: $d_d = d_2 + d_3 = 2 + 3 = 5, f_1 = 1$ and $f_2 = 2$ (Table 1). That is, the first two pairs of electrons are distributed close or symmetrically to one direction each ($f_1 = 1$); the next three pairs of electrons are each distributed with respect to 3 directions out of 5 ($f_2 = 2$); for the f -orbital, the configuration space has 7 specified

directions: $d_f = d_3 + d_4 = 3 + 4 = 7$, and 14 electrons are distributed as follows: 6 e is symmetrical to 2 directions at the same time, $f_1 = 2$; 8 e is symmetrical to 3 directions at the same time, $f_2 = 3$. Such a distribution is possible when the electron behaves like a wave. If this understanding of the probabilities p^f and q^{d-f} (4) is true, then the form of p_e -orbitals can be represented using the coefficients d and f , just as it was done using the quantum numbers n, l, m . As the principle of multilevel periodicity covers the orbitals and periods of electrons and nucleons, it should be assumed that all the shell data can also be represented using the coefficients d and f . This understanding and assumption should be verified using calculations and constructions.

Using the classical problem of a particle in one-dimensional or three-dimensional potential well, the understanding of how the energy in the orbital is quantized in the direction from the nucleus is obtained depending on the quantum number n [10]. Using the binomial interpretation of the probability function, it is also possible to solve this problem and get the understanding of how the energy of the orbital or period shell is quantized both for electrons and nucleons. The solution to this problem should be considered using the binomial interpretation.

Within the rectangular potential well (fig. 5), the Schrödinger's steady-state equation is reduced to the following differential equation:

$$\Psi''(x) + k^2 \Psi(x) = 0 \quad (11),$$

where $k = \sqrt{2mE/\hbar^2}$

– is the wave number or spatial frequency.

The general solution of the differential equation (11) is found from the assumption that the probability function of finding a particle can be as follows: $\Psi(x) = e^{kx}$. After finding the roots of the characteristic equation, the general solution is obtained: $\Psi(x) = A \sin(kx) + B \cos(kx)$. Application of the boundary conditions $\Psi(0) = 0$ results in $B = 0$, thus: $\Psi(x) = A \sin(kx)$ [10].

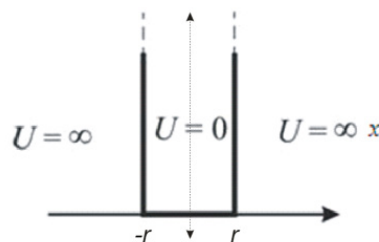


Figure 5: One-dimensional potential well with infinitely high walls

In the binomial case, instead of assuming that $\Psi(x) = e^{kx}$, it should be assumed that location of the particle in the shell follows the binomial distribution. The binomial distribution tends to be normal, and the normal distribution is an exponential function. Taking into account the frequency of application in various problems, the binomial distribution can be considered to be the most common type of discrete distribution. The binomial probability function should be used as follows:

$$\Psi(x) = p^f q^{(d-f)} \quad (12)$$

Since $\Psi(x)$ takes the values 1 (“success”) and 0 (“failure”) with the probabilities p and $q=1-p$ respectively, $p+q=1$ (4), the particle’s motion is cyclic, and the functions $\sin(kx)$ and $\cos(kx)$ are in antiphase, they can be applied to the description of states 1 (“success”) and 0 (“failure”). Applying the condition $p+q=1$ on the basis of the main trigonometric identity ($\sin^2(kx)+\cos^2(kx)=1$), it can be assumed that:

$$p = \sin^2(kx); q = \cos^2(kx). \quad (13)$$

When applying the condition, which follows from the Bernoulli’s equation, that the total probability is $(p+q)^d = 1^d$, the equation for the probability function can be found simply by applying the basic trigonometric identity ($\sin^2(kx) + \cos^2(kx) = 1$) being a circle equation ($x^2+y^2 = 1^2$, where: $x = \cos(kx)$; $y = \sin(kx)$):

$$\Psi_d(x) = (\sin^2(kx) + \cos^2(kx))^d = 1^d \quad (14)$$

It is also the condition for normalization. If it is considered that k is not equal to 1 (unit), then taking into account the integration coefficients, in 1D case at $d=1$, the expression takes the following form:

$$\Psi_d(x) = A^2 \sin^2(kx) + B^2 \cos^2(kx) = 1 \quad (15)$$

The expression (15) corresponds to the normalization condition for the wave function. The coefficients A and B are also squared, as the trigonometric functions have the degree 2.

The integration constants should be found from the following conditions:

$$\text{For the “success” state: } \int_{-r}^r A^2 \sin^2(kx) dx = 1; A = \sqrt{\frac{2}{r}} \quad (16)$$

$$\text{For the “failure” state: } \int_{-r}^r B^2 \cos^2(kx) dx = 0; B = 0 \quad (17)$$

Thus, the probability of detecting a particle (“success”) in the binomial interpretation is $\Psi_f(x) = p^f$:

$$\Psi_f(x) = \left(\frac{2}{r}\right)^f \sin^{2f}(kx) \quad (18)$$

where k is the spatial frequency [rad/m] or wave number.

Solution for a particle in 1D potential well with infinitely high walls, obtained using the binomial interpretation for 1D case, is somewhat different from the classical one in the form, but not in the essence. In “binomial” case, the expression for the radial probability function is obtained not by solving the differential equation with complex roots, but directly from the Bernoulli’s probability equation taking into consideration the probability multiplication theorem. In classical case, energy quantization is shown by the function $\sin(kx)$; for the binomial interpretation, quantization can be shown by the exponential function $\sin^2(kx)$, which matches the points of the boundary conditions more asymptotically (fig. 6). In other words, the probability change can be described more accurately. No additional module function application (abs) is required to provide positive probability at negative kx angles. In addition, the normalization condition for binomial interpretation is the Newton binomial itself $(p+q)^d=1^d$ for $p+q=1$. In classical case, the normalization condition is as follows:

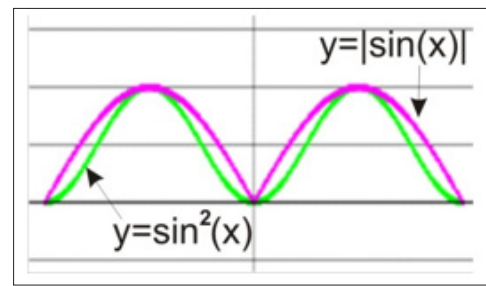


Figure 6: Probability functions for a particle in a potential well

$$A^2 \int_V \Psi \Psi^* dV = 1 \quad (19),$$

where Ψ is the wave function, Ψ^* is its complex conjugation, V is the volume being studied.

It is believed that the wave function itself has no physical meaning, but physical meaning is attributed to the square of its module $|\Psi(x_1, x_2, \dots, x_n, t)|^2$. For the binomial interpretation in the Bernoulli’s equation, physical meaning of the probability function (4) follows from the probability multiplication theorem for independent or sequential events (and/or). The function Ψ itself is the mathematical description of wave behavior in space, where the probability and normalization condition depend on the degree of freedom for the system d of $(p+q)^d=1^d$.

In the multidimensional case, the Bernoulli’s equation or Newton binomial $(p+q)^d = p^d + C_d^f p^f q^{d-f} + \dots + q^d$ is the expansion into the sum of summands, where, in accordance with the addition theorem on probability, each one is the description of probabilities for incompatible events (in accordance with the scheme or/or). As applied to quantum objects, the multidimensional case of the Bernoulli’s equation reproduces the principle of quantum superposition. The principle involves an idea that if the system is in the state described by the wave functions Ψ_1 and Ψ_2 , it can also be in the state described by the wave function $\Psi = C_1 \Psi_1 + C_2 \Psi_2$ for any complex C_n .

Obviously, it can be said about the superposition (addition) of any number of quantum states, i.e. about the existence of quantum state for the system, which is described by the wave function:

$$\Psi_\Sigma = C_1 \Psi_1 + C_2 \Psi_2 + \dots + C_n \Psi_n \quad (20)$$

Therefore, due to the Bernoulli’s equation for describing the probability of quantum events, two expressions from the quantum theory are reproduced: the normalization condition and the principle of quantum superposition.

Then, in accordance with the principles of multilevel periodicity and quantum superposition, the superposition of the following functions corresponds to each shell of electrons or nucleons:

$$\Psi_{nlm} = R_{nl}(r) Y_{lm}(\theta, \varphi) \quad (21)$$

where $\Psi_{d,f,2} = p^f q^{d-f}$, C_d^f is the binomial coefficient, and the numbers d and f correspond to the shell. In the geometric sense, the Bernoulli’s equation corresponds to the system with d degrees of freedom or to the d -dimensional configuration space (d from “dimension”) for the given f . One of the shell types (orbital or period) for both electrons and nucleons corresponds to f (Table 1).

The probability of finding the particle at any point in space depends on both the radial and angular parts of the wave function:

$$\Psi_{nlm} = R_{nl}(r) Y_{lm}(\theta, \varphi) \quad (22)$$

As it is shown in the example of a particle in one-dimensional potential well, an idea of energy quantization in the radial direction, i.e. from the nucleus, can be gained using the binomial interpretation. It is interesting to note that the binomial interpretation of quantum numbers also allows obtaining the angular components of the wave functions. The classical case should be considered first. As it is known, a spherical function of the degree n is a harmonic function being a homogeneous polynomial presented as follows:

$$S_n(x, y, z) = \sum_{i,j,k} C_{i,j,k} x^i y^j z^k, \quad i + j + k = n, \quad (23)$$

where $C_{i,j,k}$ are the constants. Using the spherical coordinate system:

$$\begin{aligned} x &= \rho \sin \theta \cos \lambda, \\ y &= \rho \sin \theta \sin \lambda, \\ z &= \rho \cos \theta, \end{aligned} \quad (24)$$

where λ is the horizontal angle (longitude), θ is the vertical angle (latitude), ρ is the radius vector of the direction for the current point P , it's obvious that:

$$S_n(x, y, z) = \rho^n \sum_{i,j,k} C_{i,j,k} \sin^{i+j} \theta \cos^k \theta \cos^i \lambda \sin^j \lambda. \quad (25)$$

A function having the following form:

$$Y_n(\theta, \lambda) = \sum_{i,j,k} C_{i,j,k} \sin^{i+j} \theta \cos^k \theta \cos^i \lambda \sin^j \lambda \quad (26)$$

is called a spherical function. For the three-dimensional case, 10 coefficients are obtained, part of the coefficients C_{ijk} in (23) is dropped and there remain only 3 coefficients which are independent and correspond to the x, y, z coordinates in the Laplace equation [11]. As can be seen, the expression (23) for the spherical function corresponds to the trinomial extension $(x+y+z)^n$, which, in the combinatorial sense, is similar to the Bernoulli's equation (4) for $(p+q)^n$ and differs in the number of terms and variables. The non-use of coordinates at this stage of calculation allows obtaining general solution for the probability in the one-dimensional case

for different systems and combinations of coordinates. Application of the binomial expansion for the spherical functions instead of (23) is possible, where the variables are:

1. Angles, because two angles are enough to set the direction in 3D space;
2. Cartesian coordinates in the projection plane of the wave function Ψ .

Using the Schrödinger equation for the steady-state conditions, the calculations should be made for the angular component of the spatial frequency k in the general form, for the one-dimensional case and all the shell types of the electrons and nucleons. The one-dimensional case is analyzed to facilitate the understanding on the basis of the experience that most of the solutions for the differential equation will be complex, and for them, in the classical case, when representing atomic orbitals in 3D space, the imaginary part is reduced to the real part by artificial algebraic manipulations, and also for a number of reasons outlined below. k can be calculated for two cases of probability functions, when:

1. $p = \sin(k\alpha)$; $q = \cos(k\alpha)$ – for the condition $p^2 + q^2 = 1^2$, to determine the spatial arrangement of d and f , since such a condition corresponds to the equation of the unit circle in the rectangular coordinate system, which means that the solution will be spatially more accurate.
2. $p = \sin^2(k\alpha)$; $q = \cos^2(k\alpha)$ – for the condition $p + q = 1$, for a more accurate determination of the spatial probability distribution on the basis of the previous problem (fig. 6).

The first case should be considered. In accordance with the Bernoulli's equation and the principle of multilevel periodicity, the general expression of the probability function for the orbitals and periods of electrons and nucleons in the 1st case is as follows:

$$\Psi_d^f = 2(C_{d1}^{f1} \Psi_{df1} + C_{d2}^{f2} \Psi_{df2}) = 2(C_{d1}^{f1} p^{f1} q^{d1-f1} + C_{d2}^{f2} p^{f2} q^{d2-f2}) \quad (27)$$

Since the expression (27) of the probability function for the orbitals and periods of electrons and nucleons consists of two summands and each summand is included in the expression for two shells, the Schrödinger equation should be solved and k should be found for each of the summands, in order to then substitute into expressions for different shells:

$$\Psi_d^f = C_d^f p^f q^{d-f} = C_d^f \sin^f(k\alpha) \cos^{d-f}(k\alpha) \quad (28)$$

By finding the derivatives Ψ_d^f , differentiation is performed with respect to the angle α :

$$\begin{aligned} \Psi_d^{f'} &= C_d^f k (\sin^{f-1}(k\alpha) \cos^{d-f+1}(k\alpha) - (d-f) \sin^f(k\alpha) \cos^{d-f-1}(k\alpha)) \quad (29) \\ \Psi_d^{f''} &= C_d^f k^2 (\underbrace{f(f-1) \sin^{f-2}(kx) \cos^{d-f+2}(k\alpha)}_{D_1} - \underbrace{f(d-f+1) \sin^f(kx) \cos^{d-f}(k\alpha)}_{E_1} - \\ &\quad \underbrace{(d-f)(f+1) \sin^f(k\alpha) \cos^{d-f}(kx)}_{F_1} + \underbrace{(d-f)(d-f-1) \sin^{f+2}(k\alpha) \cos^{d-f-2}(k\alpha)}_{G_1}) \end{aligned} \quad (30)$$

The variable should be replaced for Ψ_d^f (28) and Ψ_d^m (30), where:

$$\begin{aligned} \sin^f(k\alpha) = c; \quad \sqrt[f]{c} = \sin(k\alpha); \quad D_1 = f(f-1); \quad E_1 = f(d-f+1); \\ \cos^{d-f}(k\alpha) = e; \quad \sqrt[d-f]{e} = \cos(k\alpha); \quad F_1 = (d-f)(f+1); \quad G_1 = (d-f)(d-f-1). \end{aligned} \quad (31)$$

As a result of Ψ_d^f (28) and Ψ_d^m substitution (30) into the Schrödinger equation (11), taking into account the change of variables, the following is obtained:

$$C_d^f k^2 \left(D_1 \frac{e^2}{c^2} - E_1 c e - F_1 c e + G_1 \frac{c^2}{e^2} \right) + k^2 c e = 0 \quad (32)$$

$$\frac{C_d^f D_1 e^4 - (C_d^f E_1 + C_d^f F_1 - 1) c^3 e^3 + C_d^f G_1 c^4}{c^2 e^2} = 0 \quad (33)$$

As a result of simplification, the general form of the characteristic equation for finding $k\alpha$ from the steady-state Schrödinger equation in the binomial interpretation of the quantum numbers d and f is obtained:

$$C_d^f D_1 e^4 - (C_d^f E_1 + C_d^f F_1 - 1) c^3 e^3 + C_d^f G_1 c^4 = 0 \quad (34)$$

The characteristic equation (34) for $k\alpha$ in the trigonometric form is as follows:

$$C_d^f D_1 \cos^{4(d-f)}(k\alpha) - (C_d^f E_1 - C_d^f F_1 + 1) \sin^{3f}(k\alpha) \cos^{3(d-f)}(k\alpha) + C_d^f G_1 \sin^{4f}(k\alpha) = 0 \quad (35)$$

The coefficients $C_d^f, D_1, E_1, F_1, G_1$ are determined using d and f from (2) and (31). Thus, for the binomial interpretation of quantum numbers, in the given characteristic equation for the probability of finding a particle for all the shell types, the angular component of the spatial frequency or wave number k is completely determined by the coefficients d and f . Since the principle of multilevel periodicity (PMP) covers the shells of electrons and nucleons, in this case it is possible to construct nucleon orbitals. Since this is being done for the first time, it is necessary to check the correctness of the chosen approach on electron shells.

In Table 2, the results of calculating the coefficients and characteristic equations (34) for all the types of shells are presented.

Table 2. Coefficients, probability functions and characteristic equations for finding the angular component of the spatial frequency $-\kappa\alpha$, for the binomial interpretation of quantum numbers d and f.

| Shell | C_d^f | d | f | d-f | Trigonometric form of the angular function Ψ : $\Psi_d^f = C_d^f p^f q^{d-f} = C_d^f \sin^f(k\alpha) \cos^{d-f}(k\alpha)$ | CD ₁ | CE ₁ | CF ₁ | CE ₁ +CF ₁ -1 | CG ₁ | Characteristic equation | |
|------------------------------|------------------------|-------------------------|----|-----|---|--|--|-----------------|-------------------------------------|-----------------|-------------------------|-----------------------------|
| e orbitals | | | | | | | | | | | | |
| se | | 1 | 1 | 1 | 0 | $\sin(k_2\alpha)$ | 0 | 1 | 0 | 0 | 0 | |
| | pe | 1 | 1 | 0 | 1 | $\cos(k_2\alpha)$ | 0 | 2 | 1 | 2 | 0 | $-2c^3e^3=0$ |
| de | | 2 | 2 | 1 | 1 | $2\sin(k_6\alpha)\cos(k_6\alpha)$ | 0 | 4 | 4 | 7 | 0 | $-7c^3e^3=0$ |
| | fe | 3 | 3 | 2 | 1 | $3\sin^2(k_{10}\alpha)\cos(k_{10}\alpha)$ | 6 | 6 | 9 | 14 | 0 | $6e^4-14c^3e^3=0$ |
| | | 4 | 4 | 3 | 1 | $4\sin^3(k_{14}\alpha)\cos(k_{14}\alpha)$ | 24 | 8 | 16 | 23 | 0 | $24e^4-23c^3e^3=0$ |
| e periods | | | | | | | | | | | | |
| 2e | 8e | 1 | 2 | 0 | 2 | $\cos^2(k_2\alpha)$ | 0 | 3 | 2 | 4 | 2 | $-4c^3e^3+2c^4=0$ |
| 18e | | 3 | 3 | 1 | 2 | $3\sin(k_8\alpha)\cos^2(k_8\alpha)$ | 0 | 9 | 12 | 20 | 6 | $-20c^3e^3+6c^4=0$ |
| | | 32e | 6 | 4 | 2 | 2 | $6\sin^2(k_{18}\alpha)\cos^2(k_{18}\alpha)$ | 12 | 18 | 36 | 53 | 12 |
| | | | 10 | 5 | 3 | 2 | $10\sin^3(k_{32}\alpha)\cos^2(k_{32}\alpha)$ | 60 | 30 | 80 | 109 | 20 |
| Nucleon orbitals (np) | | | | | | | | | | | | |
| sn 2 | pn 10 | 1 | 3 | 0 | 3 | $\cos^3(k_2\alpha)$ | 0 | 4 | 3 | 6 | 6 | $-6c^3e^3+6c^4=0$ |
| | | 4 | 4 | 1 | 3 | $4\sin(k_{10}\alpha)\cos^3(k_{10}\alpha)$ | 0 | 16 | 24 | 39 | 24 | $-39c^3e^3+24c^4=0$ |
| dn 28 | fn 60 | 10 | 5 | 2 | 3 | $10\sin^2(k_{28}\alpha)\cos^3(k_{28}\alpha)$ | 20 | 40 | 90 | 129 | 60 | $20e^4-129c^3e^3+60c^4=0$ |
| | | 20 | 6 | 3 | 3 | $20\sin^3(k_{60}\alpha)\cos^3(k_{60}\alpha)$ | 120 | 80 | 240 | 319 | 120 | $120e^4-319c^3e^3+120c^4=0$ |
| Nucleon periods (np) | | | | | | | | | | | | |
| 2 np | 12 np | 1 | 4 | 0 | 4 | $\cos^4(k_2\alpha)$ | 0 | 5 | 4 | 8 | 12 | $-8c^3e^3+12c^4=0$ |
| 40 np | | 5 | 5 | 1 | 4 | $5\sin(k_{12}\alpha)\cos^4(k_{12}\alpha)$ | 0 | 25 | 40 | 64 | 60 | $-64c^3e^3+60c^4=0$ |
| | | 100 np | 15 | 6 | 2 | 4 | $15\sin^2(k_{40}\alpha)\cos^4(k_{40}\alpha)$ | 30 | 75 | 180 | 254 | 180 |
| | | | 35 | 7 | 3 | 4 | $35\sin^3(k_{100}\alpha)\cos^4(k_{100}\alpha)$ | 210 | 175 | 560 | 734 | 420 |

As in the radial case, the functions Ψ_d^f for the angular component of the spatial frequency k are obtained from the Bernoulli's equation of probability. Using these functions, the general form for the projections of orbitals and periods for electrons and nucleons is enough to be obtained. While implementing the orthogonality condition for the directions of the coordinate system, for construction it is enough to set each of the functions to one of the coordinates in the plane. The 2D projections are considered because of the disadvantages of the spherical coordinate system (see below). Since the correspondence of the function for a particular axis is chosen arbitrarily, the condition of direction independence in space should be implemented. Therefore, to obtain the projection of the orbital for a given plane, the spatial conjugation of functions, for which the axes are interchanged, should be shown (fig. 7). Considering that the electron energy is determined by the Coulomb interaction and the distance from the nucleus, and the orbital size (scale) is determined by the main quantum number, it is enough to construct the angular part of the wave function Ψ_{-d}^f to reveal the form of the shell.

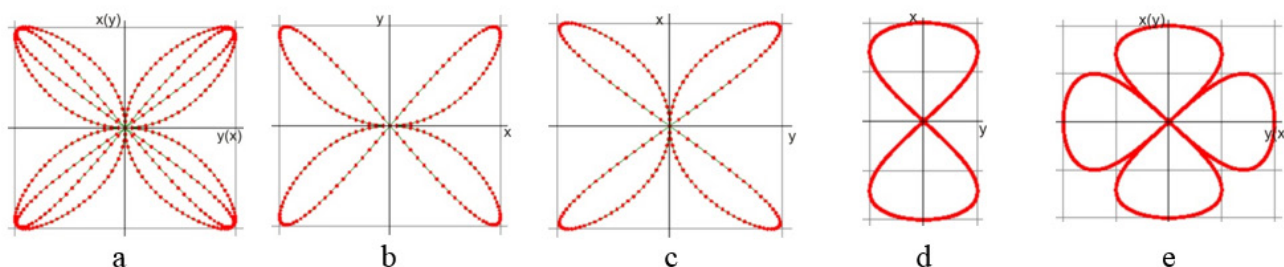
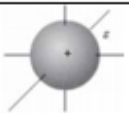

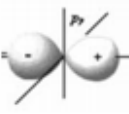
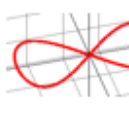
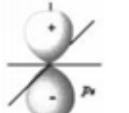
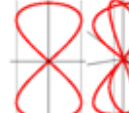



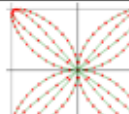

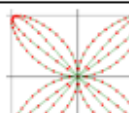
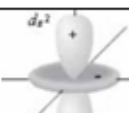
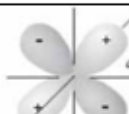
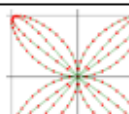
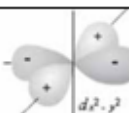


Figure 7: The d_{exy} - electron orbital (a); spatial conjugations of the d_e -orbital (XY-b; YX-c) for the Ψ_d^f functions (40) (see tables 2 and 3); the p_{exy} - electron orbital (XY-d) and spatial conjugations of the p_e -orbital (XY and YX-d).

The coefficients A are calculated by applying the normalization condition (19) in accordance with the solutions of the characteristic equation for the given Ψ_d^f . Spatial conjugation for the p_e -orbital is the orbital projection along the perpendicular axes. It corresponds to the "tendency" of the system to the state of maximum multiplicity in accordance with the Hund rule.

Table 3: Comparison of the electron angular function types, which are of greatest interest in chemistry, for the classical case (l, m) and binomial interpretation of quantum numbers (d, f) [12].




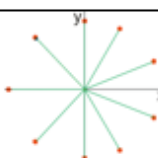
| l | m | Orbital | $Y_{lm}(\theta, \varphi)$ | Form | d | f | $\Psi_d^f(x, y); (x, z); (z, y)$ | Form |
|-----------|---------------|--|---|---|---|---|---|---|
| 0 | 0 | s | $\frac{1}{2\sqrt{\pi}}$ |  | 1 | 0 | $\begin{cases} x = A_x \cos(k\alpha) \\ y = A_y \sin(k\alpha) \end{cases}$ (36) |  |
| | | | | | 1 | 1 | | |
| 1 | $\pm 1^*$ | p_y | $\frac{\sqrt{3}}{2\sqrt{\pi}} \sin\theta \cdot \sin\varphi$ |  | 1 | 0 | $\begin{cases} x = A_x \cos(k\alpha) \\ y = A_y 2\sin(k\alpha)\cos(k\alpha) \end{cases}$ (37) |  |
| | | | 1 | 2 | | | | |
| | 0 | p_z | $\frac{\sqrt{3}}{2\sqrt{\pi}} \cos\theta$ |  | 1 | 0 | $\begin{cases} x(y) = A_x \cos(k\alpha) \\ z = A_y 2\sin(k\alpha)\cos(k\alpha) \end{cases}$ (38) |  |
| | | 1 | 2 | | | | | |
| | $\pm 1^*$ | p_x | $\frac{\sqrt{3}}{2\sqrt{\pi}} \sin\theta \cdot \cos\varphi$ |  | 1 | 0 | $\begin{cases} y = A_x \cos(k\alpha) \\ x = A_y 2\sin(k\alpha)\cos(k\alpha) \end{cases}$ (39) |  |
| | | 1 | 2 | | | | | |
| 2 | $\pm 2^*$ | d_{xy} | $\frac{\sqrt{15}}{4\sqrt{\pi}} \sin^2\theta \cdot \sin^2\varphi$ |  | 2 | 1 | $\begin{cases} x = A_x 2\sin(k\alpha)\cos(k\alpha) \\ y = A_y 3\sin^2(k\alpha)\cos(k\alpha) \end{cases}$ (40) |  |
| | | | 3 | 2 | | | | |
| | $\pm 1^*$ | d_{yz} | $\frac{\sqrt{15}}{4\sqrt{\pi}} \sin^2\theta \cdot \sin\varphi$ |  | 2 | 1 | $\begin{cases} z = A_x 2\sin(k\alpha)\cos(k\alpha) \\ y = A_y 3\sin^2(k\alpha)\cos(k\alpha) \end{cases}$ (41) |  |
| | | | 3 | 2 | | | | |
| | 0 | d_{z^2} | $\frac{\sqrt{5}}{4\sqrt{\pi}} (3\cos^2\theta - 1)$ |  | | | | |
| $\pm 1^*$ | d_{xz} | $\frac{\sqrt{15}}{4\sqrt{\pi}} \sin^2\theta \cdot \cos\varphi$ |  | 2 | 1 | $\begin{cases} z = A_x 2\sin(k\alpha)\cos(k\alpha) \\ x = A_y 3\sin^2(k\alpha)\cos(k\alpha) \end{cases}$ (42) |  | |
| | | 3 | 2 | | | | | |
| $\pm 2^*$ | $d_{x^2-y^2}$ | $\frac{\sqrt{15}}{4\sqrt{\pi}} \sin^2\theta \cdot \cos^2\varphi$ |  | | | | | |

* In the classical case, the quantum number m is indefinite, as the reducible real-value functions are obtained artificially by a linear combination of imaginary functions with the selected values of m [12].

As it is shown in Table 3, the form of the probability functions for the s_e , p_e , d_e -orbitals, obtained due to the quantum numbers l, m and spherical coordinate system (spherical harmonics are the classical case), coincides with the projections of the probability functions for the same shells obtained due to the binomial interpretation of the quantum numbers d and f constructed in a Cartesian coordinate system.

The solutions for characteristic equations that arise when solving the steady-state Schrödinger equation for various shells of electrons and nucleons in the binomial interpretation should be analyzed. First, the solutions for electron orbitals should be analyzed (Table 4).

Table 4: Coefficients, probability functions and characteristic equations for finding the angular component of the spatial frequency $k\alpha$ for the binomial interpretation of quantum numbers

| Shells | Ψ function $\Psi_d^f = C_d^f p^f q^{d-f} = C_d^f \sin^f(k\alpha) \cos^{d-f}(k\alpha)$ | The characteristic equation: taking into consideration the replacement of variables (table 2) and in trigonometric form | Unit circle solutions |
|--|---|--|--|
| e orbitals | | | |
| se | $\sin(k\alpha)$ (43) | |  d = 1 |
| | $\cos(k\alpha)$ (44) | $-2c^2 e^3 = 0;$ $-2 \cos^3(k\alpha) = 0$ (45) | |
| Solutions for the characteristic equation*: $\alpha = \frac{\pi n}{k} - \frac{\pi}{2k} \quad k \neq 0, n \in \mathbb{Z}$ (46) | | | |
| pe | $2\sin(k\alpha)\cos(k\alpha)$ (47) | $-7c^3 e^3 = 0;$ $-7 (\sin^3(k\alpha) \cos^3(k\alpha)) = 0$ (48) |  d = 2 |
| | Solutions for the characteristic equation: $\alpha = \frac{\pi n}{k}; \alpha = \frac{\pi n}{2k} \quad k \neq 0, n \in \mathbb{Z}$ (49) | | |
| de | $3\sin^2(k\alpha)\cos(k\alpha)$ (50) | $6e^4 - 14c^3 e^3 = 0$ $6 \cos^4(k\alpha) - 14 (\sin^2(k\alpha) \cos^3(k\alpha)) = 0$ (51) |  d = 3 |
| | Solutions for the characteristic equation: $\alpha = \frac{3.14159n - 1.5708}{k}, \alpha = \frac{6.28319n - 2.39398}{k}, k \neq 0, n \in \mathbb{Z}$ (52) $\alpha = \frac{6.28319n}{k} + \frac{0.747611}{k}, \alpha = \frac{6.28319n}{k} - (0.373806 + 0.656751i) \frac{1}{k},$ $\alpha = \frac{6.28319n}{k} - (0.373806 - 0.656751i) \frac{1}{k}, \alpha = \frac{6.28319n}{k} + (2.76779 - 0.656751i) \frac{1}{k},$ | | |
| fe | $4\sin^3(k\alpha)\cos(k\alpha)$ (53) | $24e^4 - 23c^3 e^3 = 0$ $24 \cos^4(k\alpha) - 23 (\sin^6(k\alpha) \cos^3(k\alpha)) = 0$ (54) |  d = 4 |
| | Solutions for the characteristic equation: $\alpha = \frac{0.5(12.5664n - 3.14159)}{k}, \alpha = \frac{0.5(12.5664n + 3.14159)}{k}, k \neq 0, n \in \mathbb{Z}$ $\alpha = \frac{2(3.14159n - 0.544347)}{k}, \alpha = \frac{2(3.14159n + 0.544347)}{k},$ $\alpha = \frac{2(3.14159n - (1.5708 + 0.466222i))}{k}, \alpha = \frac{2(3.14159n + (1.5708 + 0.466222i))}{k},$ $\alpha = \frac{2(3.14159n - (0.212012 - 0.431144i))}{k}, \alpha = \frac{2(3.14159n + (0.212012 - 0.431144i))}{k},$ $\alpha = \frac{2(3.14159n - (0.212012 + 0.431144i))}{k}, \alpha = \frac{2(3.14159n + (0.212012 + 0.431144i))}{k},$ $\alpha = \frac{2(3.14159n - (1.139 - 0.299819i))}{k}, \alpha = \frac{2(3.14159n + (1.139 - 0.299819i))}{k},$ $\alpha = \frac{2(3.14159n - (1.139 + 0.299819i))}{k}, \alpha = \frac{2(3.14159n + (1.139 + 0.299819i))}{k},$ (55) | | |

*- All the solutions were obtained using Wolfram Mathematica [13]

In accordance with the principle of multilevel periodicity (PMP), the number of particles for each shell consists of two summands Ψ_{df} (subshells) (21). The equations for all the summands have solutions; for the s_e and p_e orbitals there are only real roots. All other summands have both real and imaginary roots.

For all the s_e , p_e and d_e summands, the real part of the roots for the characteristic equation is symmetric with respect to the origin of coordinates (see figures in Table 4). Between the symmetric roots $k\alpha$, the straight lines intersecting at the origin can be drawn. It is shown that in the configuration space of this shell, there are directions of axial symmetry in the central field of the atom. The number of directions for the unit circle (Table 4) is equal to the binomial coefficient C_d^f or number d corresponding to this shell and equation. Upon reaching the number of directions $d \geq 3$ (Table 1), i.e. more than the number of dimensions for 3D space, the central symmetry is broken. It should be assumed that the roots of the Schrödinger equation have central symmetry at d being equal to the number of coordinates analyzed in the equation. Also, it should be assumed that the location of the axes in the coordinate system with $d(n) = 4$ is different from the Cartesian coordinate system [6]. Therefore, the central symmetry is broken. This result can be taken into account as a confirmation of the assumption that d and f have the geometric sense for the degrees of freedom.

The angular part of the spatial frequency k depends on the direction α . The dependence of k on α is nonlinear. However, for all the summands and shells, it has a similar hyperbolic and layered nature (fig. 8), where the hyperbole family is distanced from the origin as n is increased. n is the analogue of the main quantum number for the binomial interpretation. For electrons and nucleons, the principal quantum number should be divided into n_e and n_n by the correspondence of radius and scale of the shell distance. On the basis of the graph, there is a mutually proportional fluctuation of the values for the ratio of k and α . Near $\alpha = 0$, the value of the spatial frequency k is increased significantly, and theoretically it can be equal to infinity. At the same time, the value of the angle α for the spatial cycle of particle fluctuations is finite. Therefore, the graph for the dependence of α on k jumps abruptly through the ordinate axis at the point: $\alpha = n\pi$, where $k \rightarrow 0$ in the one-dimensional case and

$\alpha = n\pi/2$ in the two-dimensional case. The value of the vector k has spatial anisotropy. It follows that to calculate the spatial frequency, it is important to select the coordinate system, location of the axes and, at the same time, differentiation directions when choosing the operator. For objective estimation, it is important that the coordinate system corresponds to the structure of the configuration space.

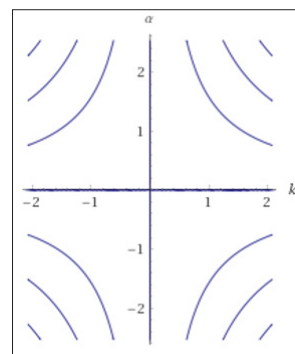


Figure 8: Example for the dependence of α/k , angular part of the spatial frequency k on the direction α and number n for the summand: (60), for the equations of the p_e and d_e orbitals.

Thus, for the binomial interpretation, the particle position, without taking into account the spin, can be set using 3 quantum numbers: n is the analogue of the main quantum number; d is a number showing the total number of degrees of freedom in the configuration space of the subshell for the orbital or period; taking into account that f varies from shell to shell: $f = 0 \dots d$, then f is a number showing the number of allowed degrees of freedom in the configuration space of a particular subshell for the orbital or period (Table 1). It should be assumed that the periodic change in the atomic shells is a consequence of the atomic configuration space evolution from f to d . As the number is increased, the division of periods into a larger number of orbitals should also be considered as a consequence of the configuration space evolution. Thus, if the atom is considered to be the system with varying degrees of freedom, the atomic electron shells exist in the configuration space with the dimensions from 0 to 5, and nucleons from 2 to 7.

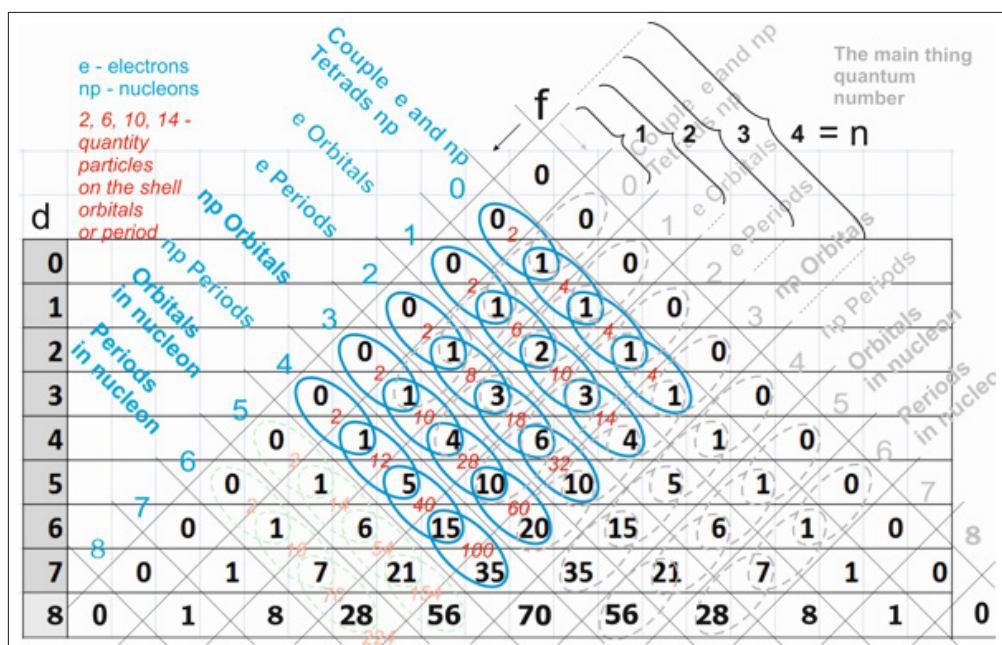


Figure 9: The n , d , f values in the binomial interpretation of quantum numbers and their correspondence to the shells of electrons and nucleons in accordance with the principle of multilevel periodicity.

In fig. 9, the correspondence of n , l , f quantum numbers of the binomial interpretation to the various atomic shells is shown. The d values divide the Pascal triangle into horizontal rows. Each row corresponds to the configuration space with d degrees of freedom. The f values divide the triangle into diagonal columns. Each such column and number f has its own type of shell: $f=0$ is for the pairs of electrons and nucleons; $f=1$ is for the electronic orbitals; $f=2$ is for the electronic periods; $f=3$ is for the nucleon orbitals; $f=4$ is for the nucleon periods. The analogue of the principal quantum number for the binomial interpretation of $n=1-4$ shows a hierarchy for the periods of electrons or nucleons depending on the size and number of orbitals.

The state $d=0; f=0$ corresponds to the state of a pair of particles just as to the shell subtype. Taking into account that in the binomial interpretation, in accordance with the principle of multilevel periodicity, each shell consists of two subshells, the formation not only of pairs, but also of particle tetrads as the smallest filled shell is specific for $f=0$. The effects of α -partial correlations are shown systematically for the nuclei with $A < 40$, i.e. until the end of small s_n and p_n orbitals and the α -process as a whole (section 13, the “ $5n\alpha$ ” factor) [14]. In this case, for the nucleus, it is also important to remember the nuclear cluster model (J. A. Wheeler, 1937). In accordance with this model, the nucleus consists of $A/4$ number of α -particle clusters (due to the anomalous binding energy of 20 MeV) and nucleon remnants. Within the principle of multilevel periodicity, the pair and tetrad of particles should be attributed to a separate shell type, along with the orbitals and periods. It should be assumed that there is also a tetrad group for electrons, which is shown more intensively within the shell of two periods: 4 (2+2); 16 (8+8); 36 (18+18); 64 (32+32), where the total number of particles is multiple of 4, and the number of tetrads is proportional to the square of natural numbers: 1, 4, 9, 16.

Disadvantages of the Classical Spherical Coordinate System. Errors in Electron Orbital Modeling. Alternative Spherical Coordinate Systems

The comparison with the projections of the electron orbitals was not chosen by chance. The reason was found out in the process of modeling the shells. The fact is that when setting 3D mutually perpendicular vibrations, the modern spherical coordinate system (fig. 11) has two significant disadvantages, that lead to errors and inaccuracies in constructions:

1. The vertical angle is not a completely independent variable for setting rotation or vibrations. Position of the plane for the vertical angle depends on the value of the horizontal angle and is not an independent coordinate system. The range of vertical angle variations ($0-\pi$) is not equivalent to the range of horizontal angle variations ($0-2\pi$). This, for example, makes it very difficult to set perpendicular vibrations for the plane different from the horizontal angle plane or for the direction of the horizontal angle, etc.;
2. In this coordinate system, it is impossible to set 3 independent phase vibrations along the XYZ axes or vector rotation in the XOZ and YOZ planes.

The modern spherical coordinate system has been developed on the basis of the experience on space perception from a flat surface, and is successfully used in geodesy, navigation, artillery and other applied ground areas. However, it is unsuitable for modeling cyclic processes on a large macro or micro scale of the spherically organized systems (universe, atom). Using the classical spherical

coordinate system, the observed vibrations can be reproduced. When simulating vibrations, one plane of the horizontal angle and the Z axis being the origin of coordinates for the vertical angle are primary, and the remaining axes and planes are, by default, secondary. For example, when modeling the ecliptic plane in the Solar System, this priority can be justified by convenience, but how such a priority can be justified in the atomic space? The priority of a single specified axis is partially confirmed in instrument engineering, where a one-dimensional algorithm for comparing the measured signal with the reference one is often used. However, due to the two-dimensional algorithm, for example, in NMR signal interpretation, a new level of understanding has been brought. Algorithms for measuring or thinking are being developed along with the knowledge on the laws of nature. Using the classical spherical coordinate system, it is impossible to simulate vibrations in the field of spherical potential, where all the axes and planes are equal.

It should be assumed that there is inaccuracy in the form of the s_e, p_e, d_e, f_e -orbitals obtained due to the classical spherical coordinate system. In fig. 8, it is shown that the form of shells for the d_e and f_e orbitals is different for the case of symmetry in relation to the selected Z axis in comparison with other axes. Therefore, there is a reasonable question: "What physical parameter caused such an exclusivity of the Z axis?". For the Laplace operator of the Cartesian system, all the coordinates are equal. Gravity? It is not taken into account at orbital simulation. It should be assumed that this discrepancy is caused by the fact that Z direction is specified in the classical spherical coordinate system.

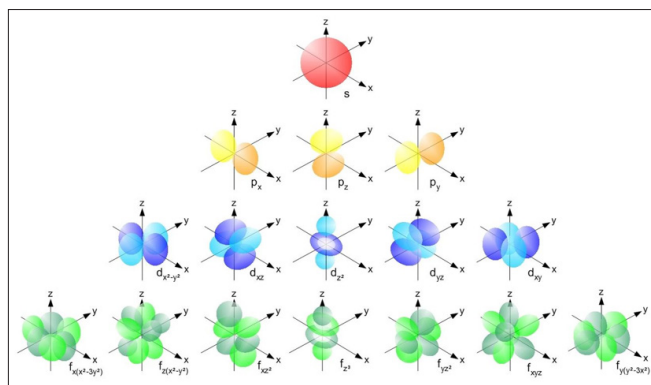
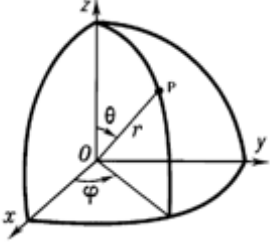
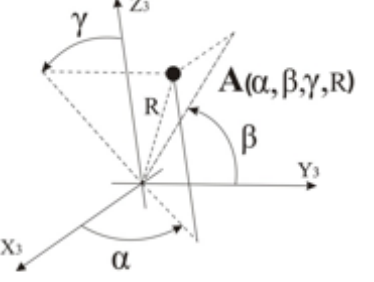
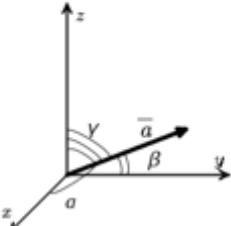
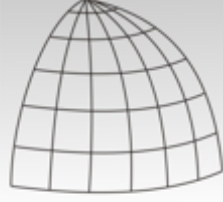
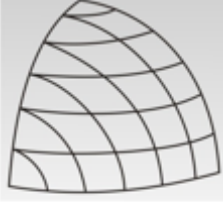
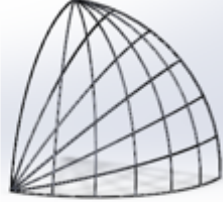


Figure 10: The form and spatial arrangement of s-, p-, d- and f-orbitals

To overcome such a discrepancy in the spherical coordinate system, it is proposed to use spatial conjugation, where the horizontal angle planes change and coincide with the XOY, XOZ, YOZ planes in turn, the selected reference axes of the vertical angle coincide with the coordinate axes in turn, and the resulting direction vector from the origin to the point is the sum of three vectors in each of the subsystems. In the sense of calculation convenience, an important role is played by the coincidence of the range of angles and the values of the Lamé coefficients for the Laplace operator. In the classical spherical coordinate system, these ranges are different (horizontal angle: $0-2\pi$; vertical angle: $0-\pi$). Taking into consideration all of the above mentioned, it is possible to propose other spherical coordinate systems having no specified direction or plane.

Table 5. Spherical coordinate systems

| | | |
|--|--|--|
|  |  |  |
| <p>Fig. 11. The classical spherical coordinate system</p> | <p>Fig. 12. The orthogonal angular spherical coordinate system OASCS. (the Cartesian coordinate system in units of angles)</p> | <p>Fig. 13. The angular spherical coordinate system of guiding cosines – ASCSGC or CSGC.</p> |
| $\begin{aligned} x &= R \sin(\theta) \cos(\varphi); \\ y &= R \sin(\theta) \sin(\varphi); \\ z &= R \cos(\theta); \\ \theta &\in 0 - \pi; \varphi \in 0 - 2\pi \end{aligned} \quad (56)$ | $\begin{aligned} x &= R(x_\alpha + x_\gamma) = R(\cos(\alpha) + \sin(\gamma)); \\ y &= R(y_\beta + y_\alpha) = R(\cos(\beta) + \sin(\alpha)); \\ z &= R(z_\gamma + z_\beta) = R(\cos(\gamma) + \sin(\beta)); \alpha, \\ \beta, \gamma &\in 0 - 2\pi \end{aligned} \quad (57)$ | $\begin{aligned} x &= R \cos(\alpha); \\ y &= R \cos(\beta); \\ z &= R \cos(\gamma); \\ \alpha, \beta, \gamma &\in 0 - 2\pi \end{aligned} \quad (58)$ |
| <p>Coordinate lines</p> | | |
|  |  |  |
| <p>Fig. 14</p> | <p>Fig. 15</p> | <p>Fig. 16</p> |
| <p>The Laplace operator:</p> $\Delta f(q_1, q_2, q_3) = \text{div grad } f(q_1, q_2, q_3) = \frac{1}{H_1 H_2 H_3} \left[\frac{\partial}{\partial q_1} \left(\frac{H_2 H_3}{H_1} \frac{\partial f}{\partial q_1} \right) + \frac{\partial}{\partial q_2} \left(\frac{H_1 H_3}{H_2} \frac{\partial f}{\partial q_2} \right) + \frac{\partial}{\partial q_3} \left(\frac{H_1 H_2}{H_3} \frac{\partial f}{\partial q_3} \right) \right], \quad (59)$ <p>Where H_i are the Lamé coefficients for the given coordinate system, which can be obtained by writing the expressions for the lengths of S arcs for the infinitesimal elements of coordinate lines.</p> | | |
| <p>Lamé coefficients</p> | | |
| $\begin{aligned} dS_\varphi &= r \sin\theta d\varphi \rightarrow H_\varphi = r \sin\theta; \\ dS_\theta &= r d\theta \rightarrow H_\theta = r; \\ dS_r &= dr \rightarrow H_r = 1. \end{aligned} \quad (60)$ | $\begin{aligned} dS_\alpha &= r \sin\gamma d\alpha \rightarrow H_\alpha = r \sin\gamma; \\ dS_\beta &= r \sin\alpha d\beta \rightarrow H_\beta = r \sin\alpha; \\ dS_\gamma &= r \sin\beta d\gamma \rightarrow H_\gamma = r \sin\beta; \\ dS_r &= dr \rightarrow H_r = 1. \end{aligned} \quad (61)$ | $\begin{aligned} dS_\alpha &= r d\alpha (\beta, \gamma) \\ H_\alpha &= r; H_\beta = r; H_\gamma = r; \\ dS_r &= dr \rightarrow H_r = 1. \end{aligned} \quad (62)$ |
| <p>The Laplace operator for the given coordinate system:</p> | | |
| <p>For the coordinate format (φ, θ, R):</p> $\Delta f = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{\sin\theta r^2} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{\sin^2\theta r^2} \frac{\partial^2 f}{\partial \varphi^2} \quad (63)$ | <p>For the coordinate format (α, γ, R):</p> $\Delta f = \frac{1}{r^3 \sin\alpha \sin\gamma} \left(\frac{\partial}{\partial r} \left(r^2 \sin\alpha \sin\gamma \frac{\partial f}{\partial r} \right) + \frac{\partial f}{\partial \alpha} \left(\frac{\sin\gamma \partial f}{\sin\alpha \partial \alpha} \right) + \frac{\partial f}{\partial \gamma} \left(\frac{\sin\alpha \partial f}{\sin\gamma \partial \gamma} \right) \right) \quad (64)$ | <p>For the coordinate format (α, γ, R):</p> $\Delta f = \frac{1}{r^2} \left(\frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r} \right) + \frac{\partial^2 f}{\partial \gamma^2} + \frac{\partial^2 f}{\partial \alpha^2} \right) \quad (65)$ |

The orthogonal angular spherical coordinate system (OACS) $(\alpha, \beta, \gamma, R)$ is structurally a superposition for the cases of three classical spherical coordinate systems (CSCS) with respect to the three specified directions of the X, Y, Z space. That is, for OACS, in relation to the Z axis, the α angle is horizontal, and the γ angle is vertical. For the other axes, the status of the angles is changed. For the X axis, the β angle is horizontal, and the α angle is vertical. For the Y axis, the γ angle is horizontal, and the β angle is vertical. Thus, a conjugation of angles and equality of coordinate axes is achieved. The convenience of OACS is that the expression for each of the coordinates (57) coincides with the solution for the linear differential equations of the second order for the complex roots $(r \pm ia)$ having the following form: $e^{rx}(A\cos(ax)+B\sin(ax))$, and can be used as reduced coordinates for this type of solutions and generation of the corresponding formalism. OACS has equal range for all the $(0-2\pi)$ angles. OACS disadvantages are the following:

- the inconvenience lies in the fact that the Laplace operator has a more lengthy expression (64) for this coordinate system than for CSCS (63). However, if the angular velocities are equal, the expression (64) is simplified;
- in OACS, the coordinate lines of the angles are located on the circles of various radii, similar to the latitude in CSCS.

The angular spherical coordinate system of guiding cosines (CSGC) is constructed in accordance with the resection method (applied in geodesy), when position of a point is determined by two angles from the points with known coordinates (poles). In this CS there are 6 poles: 2 at the “north” and “south” poles and 4 at the equator located at 90° . The similarity of this system was used for old maps (in a plane), before the latitude and longitude format was introduced. The described CS consists of two values of longitude and radius, and OACS consists of two values of latitude and radius.

CSGC advantages are the following:

- the Laplace operator has the simplest expression (65) among all the considered spherical CS, including the classical one;
- in CSGC, the coordinate lines of both angles are always located on the circles of equal radii, in contrast to the CSCS and OACS; this is convenient for simulation and calculations;
- all the angles have equal range $(0-2\pi)$, in contrast to CSCS, where the ranges are different;

In addition, for the spherical CS, when finding the solution in a complex form, in this coordinate system (CSGC), many complex planes passing through one of the real axes can be simulated. Position of each complex plane in 3D space can also be simulated by dividing the complex part into two orthogonal components, i.e. in the format: $R_c + Im_1 + Im_2$ in accordance with the distance formula in 3D space, where:

$$R = \sqrt{Re^2 + Im_1^2 + Im_2^2} \quad (66)$$

The real part is measured along the axis under study. Each of the complex components is measured along the other two axes. Thus, it is possible to systematize complex solutions in 3D space and visualize them, which can greatly simplify understanding of the wave processes.

When simulating the unknown spatial structure, the conjugation is also required for CSGC with respect to three pairs of angles, when the direction vector to the point is the sum of three vectors, in order to avoid the situation with the specified direction or reference plane of the angles.

In the two spherical CS described above, all the angular coordinate lines have the same latitude-latitude (OACS) and longitude-longitude (CSGC) types. It should be assumed that for CSGC in geodesy, such a problem of the CSCS as the increase in distance distortions while approaching the zone boundaries formed by the discreteness of longitude (usually 3° or 6°) will be eliminated. To set the origin of coordinates and the direction of rotation in the described CS, the right-hand rule can be used to determine the direction of the magnetic flux in electrodynamics. That is, the reference direction of the α angle from the positive OX semiaxis in the XOY plane coincides with the direction of curling 4 fingers, and the perpendicular XOY plane, as well as the positive OZ semiaxis, coincides with the thumb pointing direction and etc. for other axes.

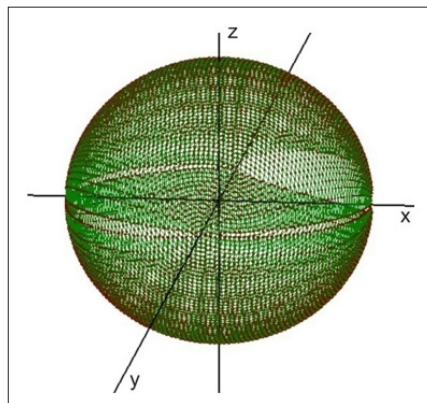


Figure 17 a: The example of spherical harmonic simulation in the spherical coordinate system of guiding cosines of CSGC (54).

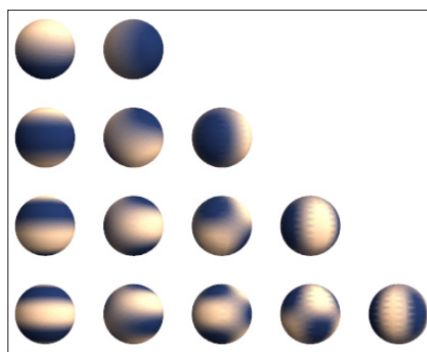


Figure 17 b: The real spherical functions $Y_{lm}, l = 1 \dots 4$ (from top to bottom), $m = 0 \dots 4$ (from left to right) [14]. The negative order functions $Y_{l,-m}$ are turned around the Z axis by $90/m$ degrees in relation to the positive order functions.

On the basis of OACS and CSGC, it is possible to derive the formulas of spherical harmonics from the trinomial (23) or binomial expansion, whose forms will be different from the existing ones. All the real parts of the spherical functions $Y_{lm}, l = 0 \dots 4$ (fig. 17 b) have symmetry only with respect to the specified Z axis. However, this symmetry does not coincide with the symmetry in relation to the X and Y axes, as the Z axis is specified for CSCS. This task is formulated at the level of capabilities and is an independent and rather extensive study that is not included in the scope of this work. In fig. 17 a, the example of spherical harmonic construction in the coordinate system of guiding cosines for the binomial expansion with the specified Z axis and XOY plane is shown.

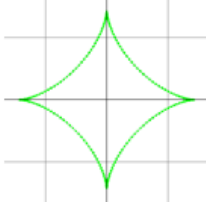
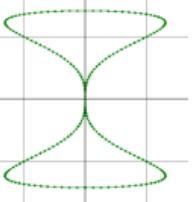
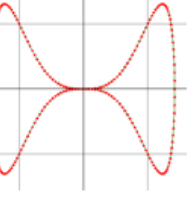
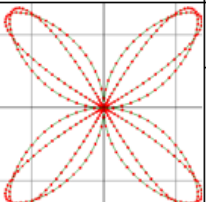
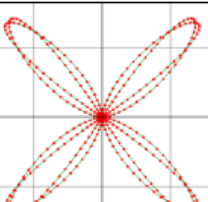
$$\begin{cases} x = R\cos(\alpha); \\ y = R\cos(\beta); \\ z = \pm\sqrt{R - \cos^2(\alpha) - \cos^2(\beta)} \end{cases} \quad (67)$$

Nuclear Orbital Models

A distinctive advantage of the binomial interpretation for n, d, f quantum numbers is that it can be used to simulate projections of the wave functions for orbitals and nucleon periods. To obtain the 2D projections, it is enough to set the probability functions for each of the subshells of the nuclear orbital (Table 2) to one of the coordinates in the plane (Table 6). Since the correspondence of the function for a particular axis is selected at random, the condition of direction independence in space should also be implemented. Therefore, to obtain the orbital projection for the given plane, the spatial conjugation of functions, for which the axes are interchanged, should be shown.

It is noticeable that the shape of 2D projections for nucleon orbitals is similar to the shape of projections for electron orbitals (Table 2), which is expected for the atom being the system with a single center, in the same manner as the shape of the crystal depends on the crystallization center configuration. The exception is the s_n-orbital, which is “turned inside out” in comparison with the electronic analogue. However, it is similar to the hyperbolic form of the k/α dependence (fig. 8) due to the application of cubic functions (68), and it is also a kind of sphere turned inside out.

Table 6: The construction of projections for nucleon orbitals in a single plane (XY).

| d | f | $\Psi_d^f(x, y)$ | Shape | d | f | $\Psi_d^f(x, y)$ | Shape |
|---|---|--|---|---|---|--|---|
| s_n-nucleon orbital (XY) | | | | p_n-nucleon orbital (XY and YX) | | | |
| 3 | 3 | $\begin{cases} x = A_x \cos^3(k\alpha) \\ y = A_y \sin^3(k\alpha) \end{cases}$ $\begin{cases} x = 0,714 \cos^3(k\alpha) \\ y = 0,714 \sin^3(k\alpha) \end{cases}$ (68) |  | 3 | 3 | $\begin{cases} x = A_x \cos^3(k\alpha) \\ y = A_y 4 \sin(k\alpha) \cos^3(k\alpha) \end{cases}$ |  |
| | | | | 4 | 3 | $\begin{cases} x = 0,714 \cos^3(k\alpha) \\ y = 0,505 * 4 * \sin(k\alpha) \cos^3(k\alpha) \end{cases}$ (69) p_n-nucleon orbital (XY and YX) |  |
| d_n- nucleon orbital (XY and YX) | | | | f_n- nucleon orbital (XY and YX) | | | |
| 4 | 3 | $\begin{cases} x = A_x 4 \sin(k\alpha) \cos^3(k\alpha) \\ y = A_y 10 \sin^2(k\alpha) \cos^3(k\alpha) \end{cases}$ |  | 5 | 3 | $\begin{cases} x = A_x 10 \sin^2(k\alpha) \cos^3(k\alpha) \\ y = A_y 20 \sin^3(k\alpha) \cos^3(k\alpha) \end{cases}$ |  |
| 5 | 3 | $\begin{cases} x = 0,505 * 4 \sin(k\alpha) \cos^3(k\alpha) \\ y = 0,369 * 10 \sin^2(k\alpha) \cos^3(k\alpha) \end{cases}$ (70) | | 6 | 3 | $\begin{cases} x = 0,369 * 10 \sin^2(k\alpha) \cos^3(k\alpha) \\ y = 0,285 * 20 \sin^3(k\alpha) \cos^3(k\alpha) \end{cases}$ (71) | |

Conclusions

The preliminary results should be summarized. As a result of calculations and constructions, it is shown that:

- The principle of multilevel periodicity (PMP) suggests a single algorithm for determining the size of electron and nucleon shells for both periodic systems of elements and isotopes.

- For the binomial interpretation of quantum numbers, the expression for radial probability function can be obtained not only by solving the differential equation, but also directly from the Bernoulli's equation of probability using the probability multiplication theorem. The Bernoulli's equation to a greater extent corresponds to the spatial (geometric) formalism, when the

exponent determines the degree of freedom for the system, and the same formula corresponds to the probability multiplication theorem. Therefore, it should be assumed that the distribution of particles in the shells can be caused to a greater extent by the properties of the configuration space inside the atom.

- If the atom is considered to be the system with varying degrees of freedom, then in accordance with the principle of multilevel periodicity, the shells of atomic electrons exist in the configuration space from 0 to 5 dimensions, and the shells of nucleons exist in the configuration space from 2 to 7 dimensions.

- It is shown that the spherical function of the degree n (23) and the

binomial probability function have one type of expression and are the homogeneous polynomials: $(x + y + z)^n$ and $(p + q)^d$. Due to the spherical function of the degree n , the probability distribution for finding a particle (orbital) is constructed. In the expression for the spherical function (23), the value of the shell is determined by the coefficient $C_{(i,j,k)}$. It follows that the principle of multilevel periodicity, in which the binomial coefficients C_d^f are used for $(p+q)^d$, when determining the size and shape of the electron and nucleon shells, is presented in the same format of reasoning as the classical case. Therefore, it has a predictive potential not only in the empirical aspect as the periodic system of isotopes, but also in the theoretical aspect.

- The projections of the graphs for the probability functions of the s , p , d , f -orbitals obtained using the quantum numbers l , m in the spherical coordinate system (spherical harmonics are the classical case) coincide with the projections of the probability functions for the same shells obtained using the binomial interpretation of the quantum numbers d and f , in the Cartesian coordinate system. This shows the viability of this interpretation.

- Using the binomial interpretation of the quantum numbers n , d , f and the principle of multilevel periodicity, the projections of the wave functions for orbitals and nucleon periods are simulated. The form of 2D projections for the nucleon orbitals is similar to that of the projections for the electronic orbitals; the only exception is the sn -orbital ("turned inside out") (Table 2).

- The results of calculations and the assumption that the distribution of particles on shells is due to the properties of configuration spaces inside the atom makes it necessary to search for the causes of the emergence of such spaces, as well as their manifestations in nature (see Section 9: "Multidimensional interference model of intra-atomic space. Hypotheses of the principle multilevel periodicity").

- As a result of the analysis, it is shown that for CSCS, there is the specified Z axis and the plane of the horizontal XOY angle. Due to this, the system is anisotropic and this affects the simulation results. Two alternative variants of the spherical coordinate systems are proposed. The construction of 3D models for orbitals and periods in these coordinate systems is a rather extensive study. These models will be presented separately.

Multidimensional Interference Model of the Intra-Atomic Space. Hypotheses of Occurrence for the Principle of Multilevel Periodicity

Modern ideas on the atomic structure in the form of atomic theory were formed about 100 years ago. Many phenomena which suggest completing classical atomic models and finding new ones have been discovered over the years. The multidimensional interference model of the intra-atomic space presented in this article is a continuation of the theoretically-calculated and evidence-based part of the article on the principle of multilevel periodicity (PMP) and the periodic system of isotopes. In this article, the main focus is on the worldview aspects of the atomic structure and the possible causes of PMP. Understanding the principle of multi-level periodicity allows coming closer to answering the question: "What is the quark and what is the nucleon structure?"

Currently, science is actively searching for the models (interpretations) explaining quantum and other phenomena in nature and physics. In July 2011, three physicists from Austria and USA conducted a survey among thirty-three participants of the conference "Quantum Mechanics and the Nature of Reality". It was found that 42% supported the Copenhagen interpretation,

24% supported the theory of quantum information, and 18% supported the multi-world interpretation of quantum mechanics. 9% supported the Penrose interpretation of objectivity for wave function collapse [15].

On the basis of the experimental results and observations, the two working hypotheses explaining the principle of multilevel periodicity can be derived.

1. It should be assumed that there are interference waves in the interatomic space of the wave environment. Due to the sources of such waves, the configuration spaces are formed together with the set of shells for the orbitals and periods in the intra-atomic space. The propagation area of such a source is about 1-4 adjacent atoms (0.76 nm for carbon).
2. The numbers of the Pascal's triangle not only show the number of particles in atomic structures in accordance with the principle of multilevel periodicity, but also correspond to the number and dimension of spatial structures used for the construction of projections for multidimensional spaces [6]. Using the Pascal's triangle, the structure of multidimensional spaces can be understood and their projections can be constructed from 0 to n [6, 16]. This correspondence allows assuming that the projections of multidimensional spaces with the dimension of 1–7 can be used as the models of atomic structures for different levels: orbitals, periods of electrons and nucleons, crystal lattices.

| | | | | |
|-----------------------|---|----|----|-----|
| Orbitals of electrons | 2 | 6 | 10 | 14 |
| Periods of electrons | 2 | 8 | 18 | 32 |
| Orbitals nucleons | 2 | 10 | 28 | 60 |
| Periods nucleons | 2 | 12 | 40 | 100 |

Table 7: The property of mutual inclusion of the orbitals and periods for electrons and nucleons.

The first hypothesis should be considered. In the experiments on diffraction, when the particles behave like a wave, the intensity of interference fringes on the screen (multiplet) is proportional to the coefficients for one of the rows of the Pascal's triangle or to the sum of several ones (fig. 18, a). The binomial coefficients are proportional to the number of previous interactions of two interfering waves projecting onto a flat screen (fig. 18, b).

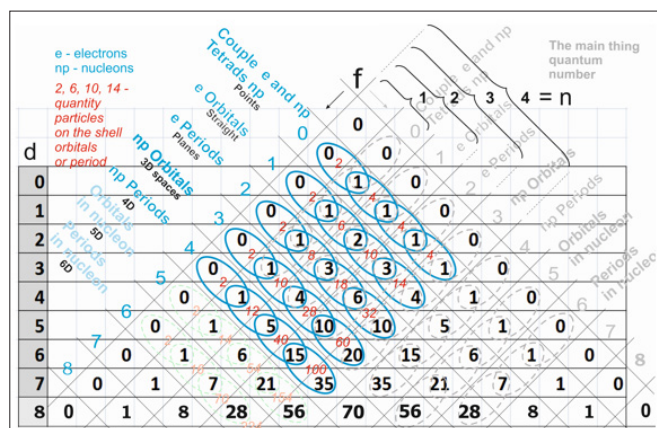


Figure 18 a: The principle of multilevel periodicity. n , d , f in the binomial interpretation of quantum numbers

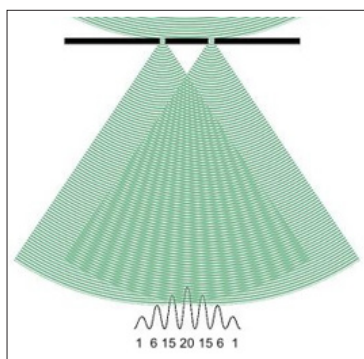


Figure 18 b: The Pascal's triangle reproduces the interference pattern.

Generalization of the observations described below shows that the intra-atomic space should be considered as a result of the interference of several wave sources:

- the signals obtained from atoms and molecules (NMR, AMR, etc.) are of multiplet nature;
- the numbers of the Pascal's triangle or their sums, on which PMP is based, simulate the intensity of fringes for the interference pattern of two sources;
- the value of the spatial frequency vector k is non-uniform, has spatial anisotropy and depends on the number of degrees of freedom and coordinate system;
- the number of degrees of freedom in the interference wave environment is related to the number of sources for such waves;
- the property of shell mutual inclusion (Table 7) and the property of charge period summation for magic numbers implies the existence of shells in a single configuration space or superposition of such spaces [14].

The interference region can be the model of configuration spaces created by the waves from different sources. In accordance with this approach to the intra-atomic environment, the shells of orbitals and periods are the superposition of the wave structures in the interference pattern, where the electrons (nucleons and other particles in the nucleus) are distributed among the extremes of this pattern. In the literature, the wave environment is known as the probability waves (de Broglie waves). It is believed that the idea of de Broglie waves is useful for making approximate conclusions on the scale of the wave properties for the particles. However, this idea does not reflect all the physical reality, so it is not fundamental for the mathematical apparatus of quantum mechanics, where the wave function is used, and in the quantum field theory, the field operators are used. The principle of multilevel periodicity and other observations show that interference can play a key role in the formation of intra-atomic and interatomic structures. Such a complementation allows simulating the given environment and constructing the interference model of atomic configuration spaces.

The double result of the experiments with two slits and an observer (experiments: Jung 1803, Davisson-Germer, 1927, Jensen, 1961, Zeilinger, 1999) for photons, electrons, and fullerenes suggests that there is a certain general wave environment that modulates different types of particles, creates the interference pattern and is destroyed in the presence of an "observer" [8, 9]. This sum of experiments shows that such an environment has unique information properties — instantaneous information transfer (quantum entanglement), delayed erasure, etc.) [17]. Due to the properties of reconstructing the interference pattern, it is called

the programming environment.

In accordance with the principle of multilevel periodicity, the shells of electrons and nucleons in the binomial interpretation are constructed in a single format of the quantum numbers d and f having geometric sense of the degrees of freedom. On this basis, it should be assumed that a new type of shell is formed by the interference pattern. While changing as follows: $f = 0 \dots d$, the interference pattern forms a new configuration space R_f together with a new shell in the environment with d degrees of freedom R_d as the degree of freedom f is increased. Within the hypothesis, the properties of the intra-atomic and interatomic wave environment can be studied on the basis of the following experiments and observations:

1. A modern understanding of the mechanism for one of the hyperpolarization methods (nuclear spin polarization) involves the following: the angular momentum of electrons is formed using a circularly polarized infrared laser beam; then, the angular momentum is transmitted from the electrons of alkali metals to the electrons of noble gases due to collisions, and further to their nuclei. Such a mechanism can be an example demonstrating the existence of wave environment for interaction between nuclei and electrons.
2. The NMR spectroscopy method records multiplet signals from the nuclei lying through 1-4 chemical bonds. The length of a single carbon-carbon bond C-C is about $0.154 \cdot 10^{-9}$ m, and the size of the nucleus is about 10-15 m, i.e., the waves diverging from the nucleus propagate quite far in the interatomic space, up to $0.154 \cdot 4 = 0.76$ nm, interact with each other and differ depending on the composition of the nucleus.
3. Images of the atomic structure obtained due to the ion microscope are a section of 3D interference pattern in the interatomic space (fig. 19). According to the authors, each bright point is an atom [18]; on the basis of the image, the sources of diverging interference waves lie in the interatomic space "away" from the electron shells (dark regions). Bright regions take part in atomic ionization; dark regions – no. What is it, if the bright region is an atom? Are these negative-energy particles from the Dirac Sea? It's hard to answer. However, it should be assumed that these regions may show the presence and location of the sources for diverging programming waves;
4. Fine splitting of spectral lines is also the interference pattern (fig. 20).

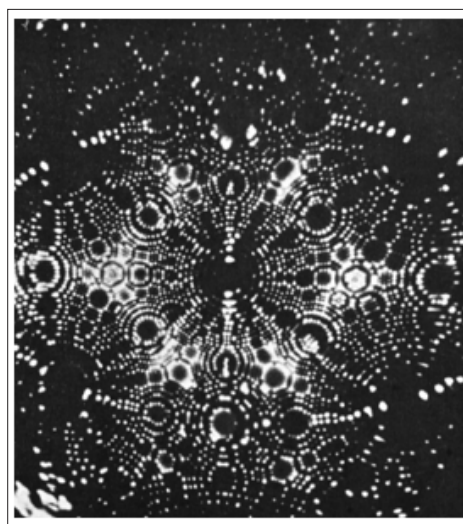


Figure 19: A tungsten needle with the radius of ~ 12 nm in a field ion microscope at the temperature of 21 K [18].

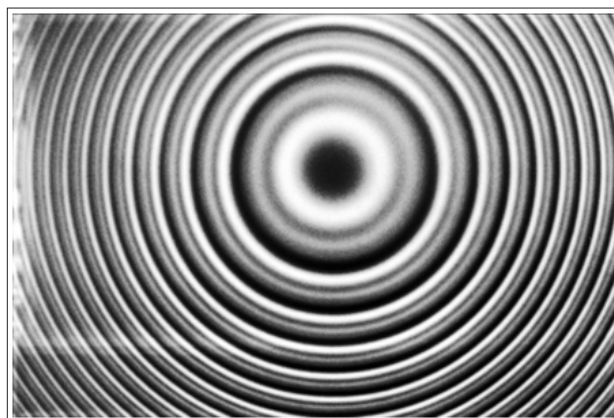


Figure 20: The interference pattern obtained in the Fabry-Perot interferometer from the light source in the form of strongly cooled deuterium; fine splitting [19].

Thus, the phenomena listed above show the existence of the interference wave environment with the propagation area of about 1-4 adjacent atoms (0.76 nm for carbon) in the interatomic space. It should be assumed that due to the sources of such waves, configuration spaces can be formed. In accordance with the image (fig. 19), there are many such sources, they are of different sizes, and their presence inside the electron shell should also be assumed. The wavelengths of such sources can be calculated within the range from the Planck length ($1,6 \cdot 10^{-35}$) to the wavelengths of fine structure and light.

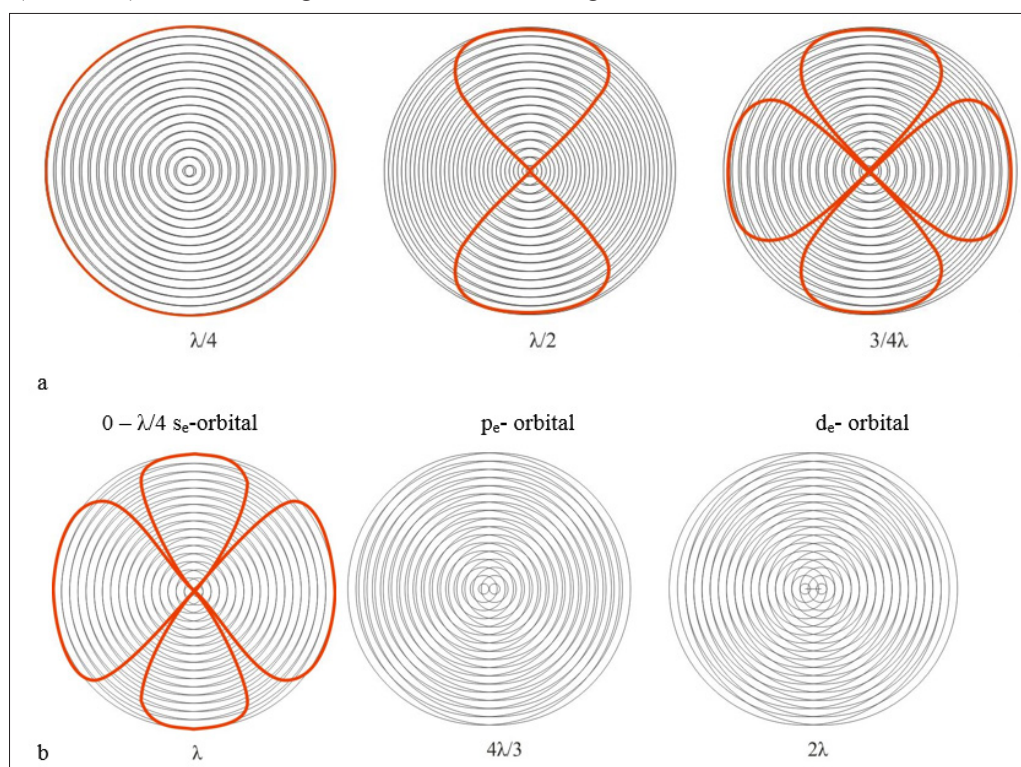


Figure 21: a and b. The 2D models of configuration spaces based on the interference of 2 sources. Projections of the s, p and d orbitals.

In figs. 21 a and b, the results of 2D simulation for the configuration spaces formed by two sources located at the distance expressed in the lengths of the interference waves are shown. Depending on the distance between the interference sources, in the interatomic space, the specified directions of the maximum occur successively: 1, 2, 3 etc. up to d (figs. 18 a and 21). If the distance between the sources of the interference waves r is equal to $0 - \lambda/4$, the region of the maximum similar to the sphere is formed; if r is within the range of $\lambda/4 - \lambda/2$, the region of the maximum similar to the p-orbital is formed; if r is within the range of $\lambda/2 - \lambda$, the region of the maximum similar to the d-orbital is formed (fig. 21 a). At $r > \lambda$, the interference region is formed with more than 4 lobes of the maximum (fig. 21 b). It should be assumed that in the interatomic and intra-atomic environment, the configuration space or its features can be formed. Within the hypothesis, electronic orbitals and period shells are the result of interaction between the extremes of the programming waves and the “electronic matter”, where the “observer” interacts with the programming waves. In accordance with the principle of multilevel periodicity, the same can be true for the nucleons in the configuration space with a large number of degrees of freedom, different range of programming waves and larger set of interacting pulses ensuring the formation of particles on a smaller dimensional scale.

In accordance with the Hund's rule, the electrons in an atom tend to maximum multiplicity. Therefore, it should be assumed that when determining the electron orbital dislocation, the electrons tend to the position with a unique combination of directions, i.e. the particle "goes over" spatial structures trying to occupy a unique free space. It should be assumed that such a "tendency" occurs due to the fact that the spatial structure is an active interaction region. To summarize the essence of this hypothesis briefly: "Atomic structures are formed by the interference of programming waves and appear as a result of interaction with the matter of electron, nucleon, etc."

Fig. 21 c. The interference model of the filled p-orbital of electrons. $\frac{1}{4}$ sphere in section, four wave sources.

To identify a 3D structure of the interference model for the interatomic space, 3D simulation should be performed. In fig. 21c, the interference model, where the filled p-orbital of electrons is simulated by the region of maxima is shown ($\frac{1}{4}$ sphere in section is shown together with the p-orbital branches adjacent to the section). The model is obtained geometrically by shifting three wave sources from the center along three axes of space by the distance equal to the length of the programming wave λ . Using the 3D structure, the correspondence of configuration spaces to simple substances should be simulated and found in accordance with the crystal lattice parameters. This topic is a fairly extensive field for experiments and simulation, and is shown here as an opportunity. This approach allows simulating known forms of orbitals and obtaining new configurations.

The "programming wave" hypothesis should be verified experimentally. In the experiments of the Zeilinger group, a laser beam was used as an "observer" [9]. If the interaction with laser photons can destroy the interference pattern of electrons and other particles, then perhaps photons from two or more sources can form such a pattern? If the coherent radiation sources are placed close to each other by transmitting the electron beam through the region of their interference, and the conditions for the lack of information are implemented for the experimenter, either there is an interference region or not, not only the interference pattern being destroyed by the beam (by the observer) can be assumed, but its formation as well. In this experiment, it is proposed to verify the assumption and hypothesis, and also to find out whether spatial programming of the microworld is possible, which may provide new opportunities in material science.

All the shells, orbitals or periods, either of electrons or nucleons, are of a four-step nature. That is, the periods of electrons or nucleons have four types of duration, which consist of 1-4 orbitals of four types. The reasons for this four-step nature remain to be seen. The following question should also be answered: "Is such a four-step nature the consequence of space-time dimensionality"?

In accordance with the property of mutual inclusion for the orbitals and periods of electrons and nucleons, when the number of particles in the shell of the next level is equal to the sum of shells for the previous one (Table 7), taking into account the interference model, the atomic shells should be considered as the multilevel pulse systems. Then the nucleus can be considered as the four-level pulse system from the orbitals and periods of the charge as well as from the orbitals and periods of the nucleons; the electron shells should be considered as the 1-3 level closed system. This assumption can be applied to the model of spin interactions or nuclear forces. For example, if the nuclear force

model is considered, where the cohesion of nucleons is a result of frequency coincidence for the mutual rotation (interaction) of matter, a strong interaction should be simulated at several levels of such rotation at the same time in accordance with the number of pulses. Figuratively speaking, a strong interaction can be similar to the car clutch mechanism, but in several planes of rotation at the same time, the number of which can be obtained from PMP (Table 7).

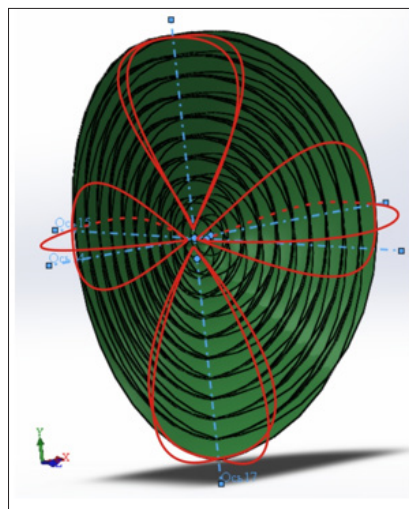


Figure 21 c: The interference model of the filled p-orbital of electrons. $\frac{1}{4}$ sphere in section, four wave sources

The second hypothesis is associated with the first one. It should be described starting from the following question: "If the principle of multilevel periodicity is true and confirmed in practice, and it is true that the electron shells are modulated by the nucleus, then what modulates the nuclear orbital periods? Is the scheme from biology true for the atom: the nucleus is in the cell, the nucleolus is in the nucleus, etc.? Where is the limit"? Quantum phenomena discovered in recent decades require the extension of current knowledge on space:

1. The quantum entanglement phenomenon implies the interdependence of quantum parameters (spin, polarization) even if these objects are separated in space beyond any known interactions [17]. Measuring the parameter of one particle results in instantaneous (higher than the speed of light) termination of the entangled state for another.
2. The quantum eraser experiment allows reconstructing the interference pattern of photons in the two-slit experiment, if the information about the particle path is "erased" [20, 12]. This demonstrates the presence of a certain information field between the particles and the observer, which affects the result of the experiment. And further, which is very strange, it doesn't matter was the erasing process performed before or after the photons reached the detector screen. This implies the influence from the future on the past or the transmission of information over time.
3. The tunnel effect is the phenomenon, where a microparticle overcomes potential barrier, when its total energy (which remains unchanged during tunneling) is lower than the height of the barrier. From the point of view of classical mechanics, the tunnel effect is the propagation of particles through insurmountable spatial barriers. It is possible only in a wave medium. What is this medium for which there are no barriers? It's hard to measure. An explanation among the known phenomena or current knowledge on space should be found.

In accordance with the principle of multilevel periodicity and the values of d and f in the binomial interpretation of quantum numbers, nuclear orbitals and periods are the objects of configuration spaces with the degrees of freedom from 3 to 7 ($d = 3-7$, Table 1). To explain the range of observed phenomena, a spatial model with varying degrees of freedom, the features of which can be observed in nature, is required. Such a model for the intra-atomic and interatomic space is proposed below.

Any theory based only on formulas without any support of visual models and observation experience is doomed to reproduce a partial picture and to long "wandering in the dark". In this regard, it is necessary to demonstrate another remarkable property of the Pascal's triangle. In terms of the tasks being solved, it is impossible not to note the fact that the principle of multilevel periodicity is fully consistent with the concept of multidimensional geometry described in the paper: "Geometry of multidimensional spaces. The rule of constructing 3D projections of multidimensional spaces" [6]. In this paper, the algorithm for constructing the structure of multidimensional spaces and their projections is shown. For this algorithm, similar to PMP, the Pascal's triangle is used.

Today, many theoretical physicists around the world investigate the issue of spatial multidimensionality. In mathematics, multidimensional space, as a rule, is an algebraic construction and is not shown graphically. In mathematical physics, several attempts have been made to graphically simulate multidimensional spaces. The most famous examples of graphical models for multidimensional spaces are: n -cells of a Hilbert space (fig. 22 a, b), simplexes (n -dimensional generalization of the triangle, fig. 22 c, d), the Calabi-Yau models (Riemannian manifolds used in the superstring theory, fig. 22 e) [21, 22]. Most models of multidimensional spaces are constructed using the parallel transfer of 3D objects (n -cells (fig. 22 a, b), 4D cube (fig. 22 a, b), hypercube or tesseract (fig. 22 b), etc.). A statement on 4- or n -dimensionality of a particular figure is made in accordance with the following logic: the trace of point motion is a straight line; the trace of straight line motion is a plane; the trace of plane motion is 3D space; the motion of 3D space is 4D space. Whence it follows that the face of a 4D cube is a 3D cube. When such faces are combined, a 4D cube (tesseract) or other more complex figures are formed.

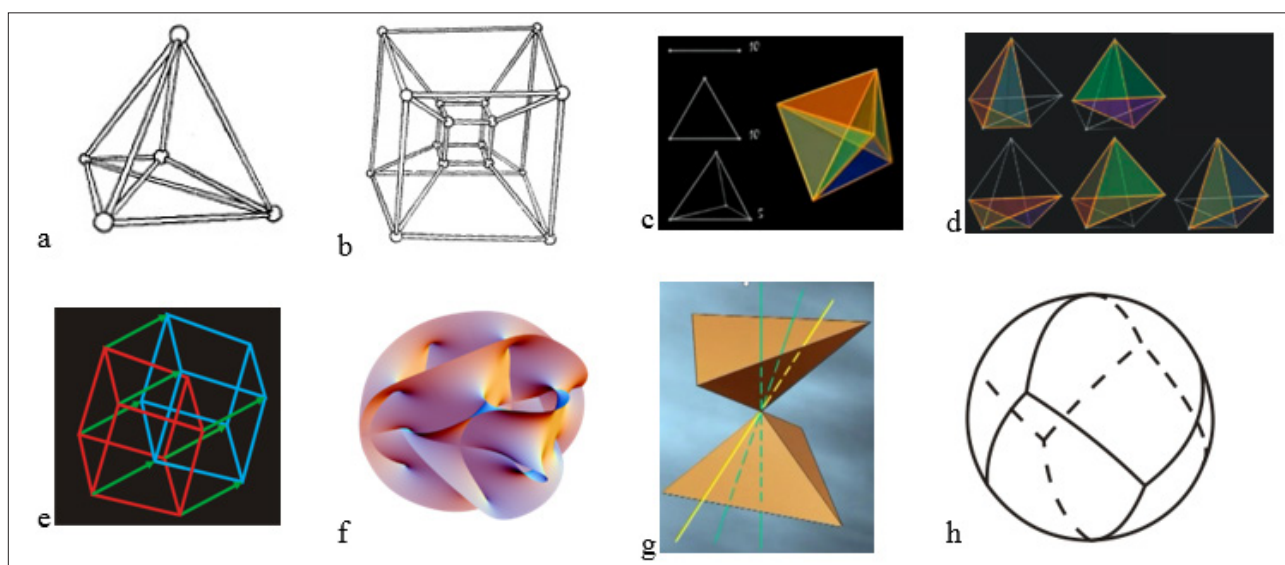


Figure 22: Analysis of multidimensional space visualization

- a, b. n -cells as polyhedra of 4D space (from left to right: 5, 8 "Hypercube") [21].
- c. The construction of 1, 2, 3, and 4D simplex is the simplest convex polyhedron of a given number of dimensions n .
- d. Five 3D faces of a 4D simplex.
- e. A "4D cube" or "Hypercube" is a 4D spatial model constructed by parallel transfer.
- f. A 3D projection of the 6D object in a Calabi-Yau space, obtained by Leonard Sasskind using Mathematica [22].
- g. Light cone in a 3D Finsler space [23].
- h. The Finsler model of 4D space "eight rhombuses on the surface of a ball" [24].

It is shown in that the parallel transfer of 3D figures does not give an idea on the structure of 4D space [6]. The models of Minkowski and Fintzler space (fig. 22 g, h) used in physics are rather algebraic models of these spaces; graphically and systematically they are not complete. Only the cone-shaped sector is simulated graphically. A superstring theory, being a physical theory that attempts to combine fundamental interactions into supersymmetry, faced the issue of landscape or false vacuum. In accordance with the string theory, the implementation of ten dimensions may result in many 4D worlds, the number of which is estimated at 10100-10500. Even if this issue is not taken into account, the string theory is extremely poor graphically and geometrically. From the image of 3D projection for the 6D string object of a Calabi-Yau space (fig. 22 e), it is difficult to obtain the structure of 6D space; it is even more difficult to find the correspondence in nature or among the known phenomena.

A distinctive feature of multidimensional geometry constructed on the basis of the Pascal's triangle is that the model is not constructed using the parallel transfer method; the model is simple and shows the structure of multidimensional spaces. The model shows the number of sectors for n -dimensional space and the composition of its projection. Also, it numerically corresponds to the principle of multilevel periodicity and the graphically real crystal structures. This correspondence allows assuming that the projections of

multidimensional spaces with the dimension of 1–7 can be used as the models of atomic structures for different levels: orbitals, periods of electrons, nucleons and crystal lattices.

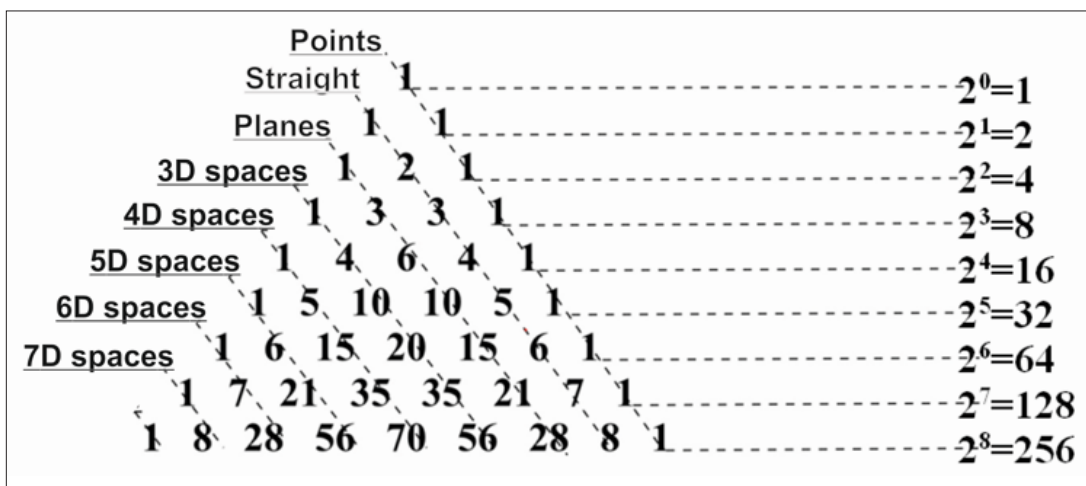


Figure 23: The Pascal's triangle is the key to constructing 3D projections of multidimensional spaces

The full projection of n-dimensional space is the totality of all its reflections in spaces with the dimensions from n to 0. The full projection of any n-dimensional space consists of as many points, lines, planes, spaces of three, four, five, etc. dimensions as is shown by the corresponding entry in the row of the Pascal's triangle. Row number corresponds to the dimension of the projected space, starting from 0 to n. The number of entry in the row corresponds to the dimensions of spaces for the projection elements (points, lines, planes, etc.), and the entry shows the number of each of the elements in (fig. 22).

The sum of binomial coefficients in the rows (fig. 23) is equal to the elements of the power series 2^n and shows the number of sectors for n-dimensional space. The construction of spatial projections from 0 to 7 should be considered in accordance with fig. 23. The projection of n-dimensional space consists of spaces with lower dimensionality. Each diagonal column corresponds to the dimension n of the projection element for the n-dimensional space, i.e. to the point, line, three, four, five-dimensional space, etc. The entries in the rows show the number of corresponding projection elements:

- $2^0 = 1$ (1) – 0D space: one point;
- $2^1 = 2$ (1 1) – 1D space: one point, one line, (two directions);
- $2^2 = 4$ (1 2 1) – 2D space: one point, two lines, one plane (four sectors);
- $2^3 = 8$ (1 3 3 1) – 3D space: one point, three straight lines, three planes, one 3D space (eight sectors – octants), (fig. 23). The projection of 3D space coincides with the Cartesian coordinate system.
- $2^4 = 16$ (1 4 6 4 1) – the projection of 4D space: one point, four lines, six planes, four 3D spaces, one 4D space, (fig. 25).

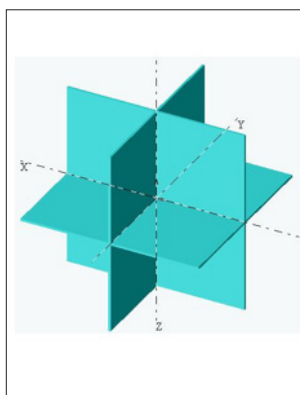


Figure 24: The 3D space model

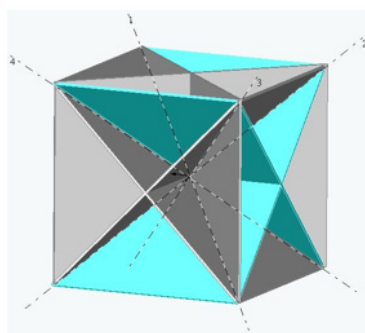


Figure 25: The 4D space model projection

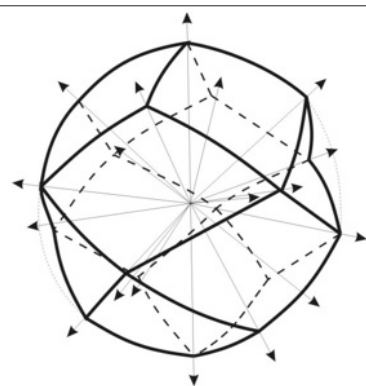


Figure 26: The 3D model of 4D space

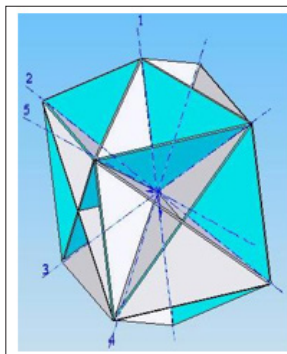


Figure 24: The projection of 6D space "Truncated octahedron."

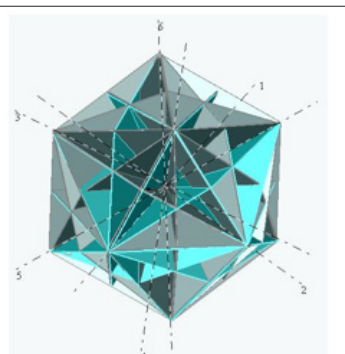


Figure 24: The projection of 7D space "Cuboctahedron".

In relation to the binomial interpretation of quantum numbers, the projection of n-dimensional space shows the structure of the configuration space in the shells of an atom, in accordance with the principle of multilevel periodicity, where $n = d$, and the degrees of freedom for the projection elements are equal to f and vary from 0 to n (d). The number of particles for the shells of electrons and nucleons is equal to the sum of particles for two subshells — the doubled sum of numbers from the adjacent rows of one diagonal column of the Pascal's triangle ((3) PMP, fig. 18 a).

In Fig. 29 a, the projections of multidimensional spaces are combined with the principle of multilevel periodicity and the change in the degrees of freedom for atomic shells is shown.

| Dimensionality of space d | Couple e and np | | | | | | | | Amount sectors d-dimensional of space |
|-----------------------------|---------------------------|---|----|----|-----------------------|----|----|---|---------------------------------------|
| | e Orbitals Straight lines | | | | np Orbitals 3D spaces | | | | |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| 2 | 0 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 4 |
| 3 | 0 | 3 | 3 | 1 | 0 | 0 | 0 | 0 | 8 |
| 4 | 0 | 4 | 6 | 4 | 1 | 0 | 0 | 0 | 16 |
| 5 | 0 | 5 | 10 | 10 | 4 | 1 | 0 | 0 | 32 |
| 6 | 0 | 6 | 15 | 20 | 15 | 6 | 1 | 0 | 64 |
| 7 | 0 | 7 | 21 | 35 | 35 | 21 | 7 | 1 | 128 |
| 8 | 0 | 8 | 28 | 56 | 70 | 56 | 28 | 8 | 256 |

Figure 29 a: The principle of multilevel periodicity and projections of multidimensional spaces in the Pascal's triangle. Geometric value for the quantum numbers d, f, n in the binomial interpretation.

The correspondence of this theory to the experience of crystallography should be demonstrated. In fig. 31, the shapes of elementary cells for the crystals of chemical elements are presented as well as fourteen Bravais lattices. Almost all the lattices have a cubic shape or its derivative. The exception is the hexagonal lattice (fig. 30 k) with a hexagon in the base. However, it can be divided into 3 separate lattices with a rhombus in the base, which are derived from the cube during deformation. Thus, all the lattices of elementary cells for the crystals of chemical elements can be reduced to a shape similar to or derived from a cube, where some sides or angles may differ from the others. If the units of all the cells of crystals for the chemical elements are merged, the resulting cell having the structure that coincides with the 3D projection of 7D space will be obtained (fig. 29 and 31). This shape also contains other projections: the projections of 3, 4, 5 and 6D space. Thus, the maximum dimensionality of the configuration space shown using the principle of multilevel periodicity for the nucleon periods coincides with the dimensionality of the projection for 7D space. It should be assumed that the configuration space inside an atom, at the level of the nucleus and electron shells, varies from 7 to 0 degrees of freedom, reproducing the shells of nucleons, electrons and the crystals of chemical elements.

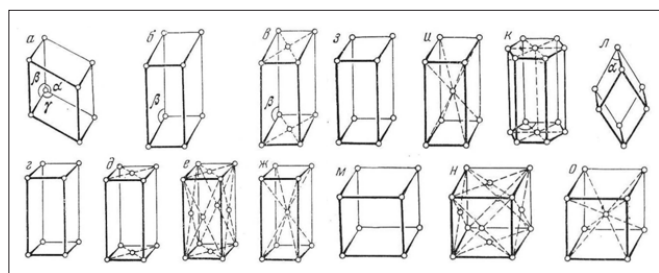


Figure 30: The shapes of elementary cells for chemical elements. Fourteen Bravais lattices

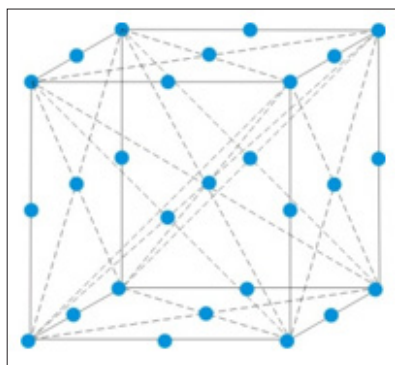


Figure 31: The projection of 7D space, wireframe geometry. The merged cell of crystals for chemical elements.

calculations, the corresponding experiment can be performed. With the knowledge of the parameters for the crystal lattice of matter, the projections of multidimensional spaces can be applied as the functional space model for the intra-atomic space at the preparation of operators and differentiation.

In accordance with the foregoing, the nucleon shells can also be considered as a result of the interference of configuration programming waves. In this relation, a reasonable question arises: "What is the source of such waves inside the nucleus?" In accordance with wave-particle duality, this question can be answered in two ways:

1. Inside the nucleus there is a smaller object, which is the source of such waves;
2. 3D space itself is a result of interaction of multidimensional spaces, and the atom should be recognized as a multidimensional complex object. Then what creates space, including multidimensional ones?

Fig. 33. The example of constructing the numerical pyramid with a triangle at the base. The pyramid faces are the Pascal's Triangles.

In the second case, the presence of a 3D pattern of matter can be explained by the following analogy: the intersection of 1D spaces (lines) is a point; the intersection of two planes is a straight line; two 3D spaces are a plane; the intersection of two 4D spaces is a 3D reality, etc. This scheme is very similar to the "male-female" scheme.

The model presented here is closer to the multi-world interpretation of quantum mechanics and the multiverse concept. The latter allows the existence of other spaces around 3D space, which is a result of the interaction of spaces with greater dimensionality. The principle of multilevel periodicity and multidimensional geometry can be the tools for simulation and further search.

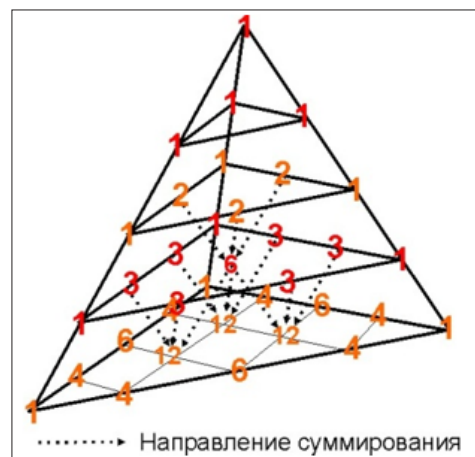


Figure 33: The example of constructing the numerical pyramid with a triangle at the base. The pyramid faces are the Pascal's Triangles.

The presence of anomalous magnitudes for the neutron magnetic moment ($-1.91\mu_n$) and the proton magnetic moment ($2.79\mu_n$) shows that the nucleons have a complex internal structure, i.e. there is electric current inside them [25]. In accordance with the principle of multilevel periodicity, it should be assumed that the following diagonal rows of the Pascal's triangle show the nucleon structure (fig. 18 a). As per the standard model, the atomic nucleus is simulated using quarks and partons. Using the principle of multilevel periodicity, it should be assumed that nucleons and atomic nuclei as a whole have a deeper periodic structure in accordance with diagonal columns following the columns 4 and 5 of the Pascal's triangle: the column 6 is the nucleon orbitals: 2, 14, 54, 154; the column 7 is the nucleon periods: 2, 16, 70, 224. The columns 1-5 of the Pascal's triangle are the interchanging columns defining electric and mass pairs of the orbital-period. It should be assumed that the columns 6 and 7 correspond to the nucleon periodicity, which is similar to the charge periodicity, and the quarks correspond to the orbital and periodic nucleon structures. Quarks, similar to orbitals or periods, do not exist in a free state. The quark model shows that in each generation, one quark has the charge of $+2/3$ (0.66), and the other one has the charge of $-1/3$ (-0.33). The ratio of adjacent coefficients for nucleon orbitals and periods also tends to $1/3$: $54/154 = 0.351$, $70/224 = 0.3125$ (average $0.33175 - 1/3$); the sum of smaller orbitals refers to a larger orbital: $(2+14+54)/154 = 2.2$, which is close to the ratio $2/3$ to $1/3$.

In accordance with the principle of multilevel periodicity, a nucleon can be considered as a stable "isotope" in the world of quarks, which is located at the end of the period, an analog of 208Pb. During further studies, the format of shells corresponding to the real structure of the nucleon will be defined.

If the principle of multilevel periodicity stands the test of experience and time, the further development of the theory for atomic periodic structures should be found in the system of polynomial coefficients (numerical pyramids, fig. 33). In accordance with the multidimensional interference model and the principle of multilevel periodicity, the atom inside is multidimensional. This is the reason for so much extracted energy, and only the knowledge of its structure allows using it correctly and efficiently.

Any scientific research in the field of the structure of matter, the universe or a person should be rewritten in a fascinating manner, in the form of a fairy tale about how the creator created the entire depth of the universe and how he wrote down its secrets in the smallest grain, drop or particle. From a philosophical point of view, all the variety of atoms of elements, isotopes and particles is the story of how nature packed in them all the stages of formation and the power of the multidimensional universe. It is expected that applied knowledge on the structure of the nucleus and atom as a whole will be expressed along with the understanding of evolutionary tasks and responsibility to the future [26, 27].

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