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Periodicity In Bitcoin Returns: A Time-Varying Volatility Approach

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ABSTRACT

We examine if the day-of-the-week effect is present in Bitcoin return series. The model specification in use accounts for conditional heteroscedasticity, which is captured in the form of a stochastic volatility process that allows for periodic time-varying parameters. We find periodicity in Bitcoin returns, which is evidence against the market efficiency of Bitcoin.

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Introduction

Over the last decade, there has been an increasing interest to understand the behaviour of a new type of financial asset that has been labelled cryptocurrency. Given its large capitalization, the market of cryptocurrencies has attracted much attention (governments, policy makers, banks, hedge funds, consumers) [1]. The most popular cryptocurrency is the Bitcoin [2].

It has been well documented that cryptocurrency return series exhibit time-varying volatility, also known as conditional heteroscedasticity. Conditional heteroscedasticity in the relevant empirical literature has been modelled as a GARCH-type specification or as a stochastic volatility process [3-12].

In this paper we turn our attention to the day-of-the-week effect on Bitcoin returns. Although there is a voluminous financial literature that has examined whether the expected return and volatility of a stock are uniformly distributed across the days of the week (thus, rejecting or accepting the efficient market hypothesis, proposed by, no study so far has examined something similar in the case of returns on crypto currencies [13-15]. The present paper aspires to fill this gap.

To our knowledge, the paper by is the only paper that has investigated the day-of-the-week pattern of price clustering in Bitcoin [16]. However, does not analyse potential abnormalities over the week in Bitcoin returns but in price levels. This is the reason for which this paper has ignored conditional heteroscedasticity, which is a main characteristic of cryptocurrency returns. Used a periodic autoregressive conditional duration to detect a day-of-the week periodicity in Bitcoin volumes. However, conditional heteroskedasticity has not been explored.

Accounting for conditional heteroscedasticity, we focus on whether the seasonality pattern in the form of day-of-the-week effect is present in the Bitcoin returns (and with what periodicity). To this end, we exploit the periodic autoregressive stochastic volatility model. It is an extension of the standard stochastic volatility model that allows the parameters in the stochastic volatility equation to vary periodically over time [17]. In this way, we can identify any periodically changing structure in the time series volatility of Bitcoin returns [18].

Our paper also contributes to the debate about whether the market of Bitcoin is efficient. The empirical findings about the efficient market hypothesis for Bitcoin are inconclusive [19-22]. If there is periodicity in Bitcoin returns, this is evidence against that hypothesis.

The paper is organized as follows. In Section 2 we describe the model and in section 3 we present the empirical results.

The Periodic Autoregressive Stochastic Volatility Model

The Gaussian Stochastic Volatility (SV) Model is Given by:

$$\begin{cases} y_t = \sqrt{h_t} \eta_t \\ \log(h_t) = \alpha + \beta \log(h_{t-1}) + \sigma e_t \end{cases}, \quad t \in \mathbb{Z}, \quad (1)$$

where y_t is the log-return and h_t is the conditional variance that follows a first-order autoregressive process, where $\alpha \in \mathbb{R}$ is the intercept, $|\beta| < 1$ is the slope parameter, and $\sigma^2 > 0$ is the variance.

Also, $\{(e_t, \eta_t)\} \stackrel{i.i.d}{\sim} N((0, 0)', I_2)$, where I_2 is the identity matrix of dimension 2.

The Gaussian periodic SV model (Aknouche, 2017) imposes S-periodicity on the parameters over time by setting

$t = nS + v$, for $n \in \mathbb{Z}, 1 \leq v \leq S$. The resulting specification reads:

$$\begin{cases} y_{nS+v} = \sqrt{h_{nS+v}} \eta_{nS+v} \\ \log(h_{nS+v}) = \alpha_v + \beta_v \log(h_{nS+v-1}) + \sigma_v e_{nS+v} \end{cases}, n \in \mathbb{Z}, 1 \leq v \leq S, \quad (2)$$

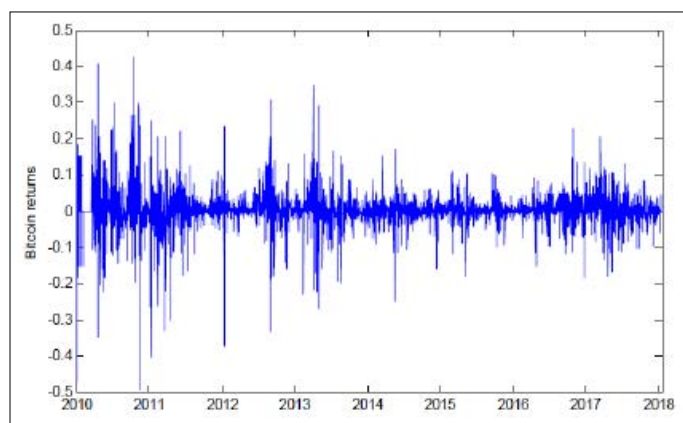
where the parameters $\alpha_v \in \mathbb{R}$, $|\beta_1, \dots, \beta_S| < 1$, and $\sigma_v^2 > 0$ ($1 \leq v \leq S$) are S-periodic over t, and $\{(e_{nS+v}, \eta_{nS+v})\}$ is defined as before. The model in (2) is named periodic autoregressive stochastic volatility (PAR-SV_S). It defines a periodic-time varying dependence structure, where the dependence between successive times is distanced by a multiple of the period S. If there is S-periodicity in the volatility of a daily return series (in our case Bitcoin), this suggests the presence of the day-of-the-week effect in that series. Notice also, that the PAR-SV_S reduces to the SV model for S = 1.

For the estimation of the SV and PAR-SV_S we adopt Bayesian methods as described in. The (reasonably flat) priors used in this paper are displayed in the Appendix of this paper.

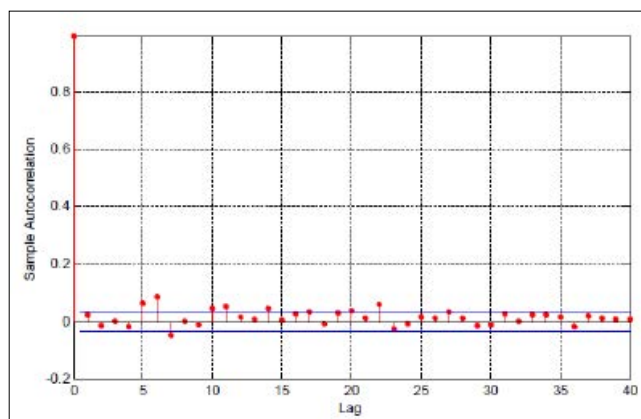
Empirical Analysis: The Bitcoin

• Data And Some Descriptive Statistics

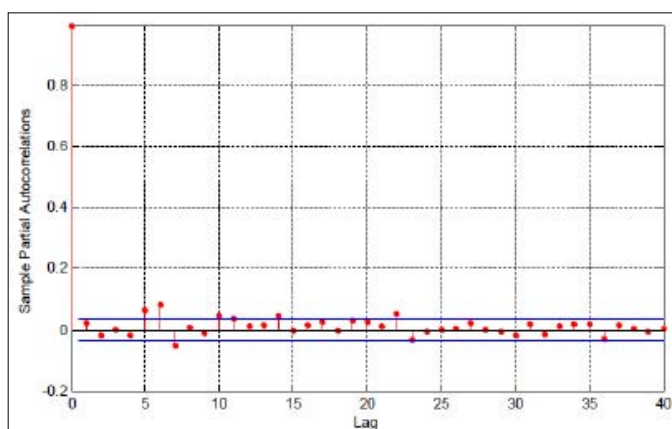
We use 3014 daily closing prices for the Bitcoin Coindesk Index, from July 18, 2010 to October 17, 2018. We transform the prices to returns, by taking the natural logarithm of the ratio of two successive closing prices. The time series plot of Bitcoin returns is given in Figure 1(a).



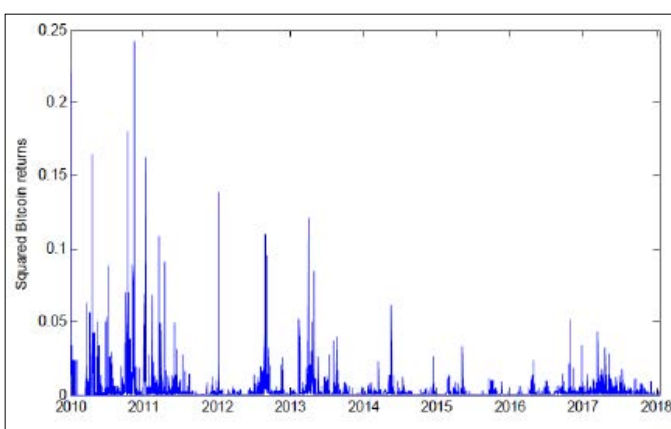
(a) Time series plot of Bitcoin returns.



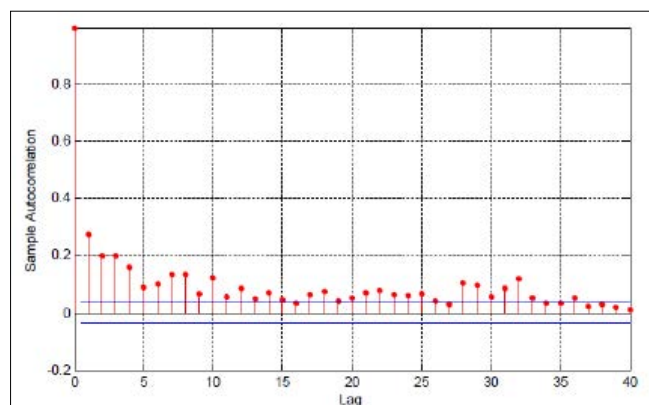
(b) Sample autocorrelation function of the Bitcoin returns.



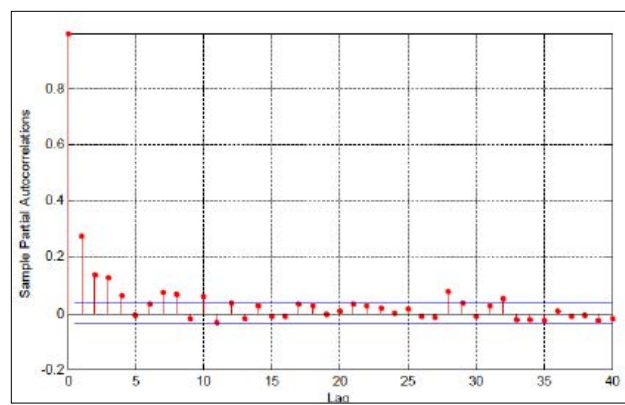
(c) Sample partial autocorrelation function of the Bitcoin returns.



(d) Time series plot of the squared Bitcoin returns.



(e) Sample autocorrelation function of the squared Bitcoin returns.



(f) Sample partial autocorrelation function of the squared Bitcoin returns.

Figure 1: Empirical results. Descriptive plots for Bitcoin.

Based on the simple and partial autocorrelation functions of Bitcoin returns (Figures 1(b) and 1(c), respectively), these returns seem to follow a white noise process. It is difficult to detect a possible periodicity in their conditional distributions, as almost all autocorrelations are insignificant.

However, if, for example, these returns follow a SV (or a GARCH) model, the squared returns will have an autoregressive conditional duration (ACD) representation, which is no longer uncorrelated and a possible periodicity could appear in their autocorrelations. Therefore, we plotted the squared returns of Bitcoin, along with their simple and partial autocorrelation functions (Figures 1(d) and 1(e), respectively).

From the autocorrelations of the squared returns, there is an indication of periodicity, as it can be observed that important picks appear in lags, which are multiples of 7 (7, 21, 28 in Figure 1(f)). There exist of course other more important picks on other lags but they are not so persistent at their multiples.

In Table 1, we provided some descriptive statistics for the Bitcoin data, using the full sample and by each day of the week separately. The average return and the volatility (approximated by the absolute value) are somewhat different from one day to another. Furthermore, the returns exhibit negative skewness and high positive kurtosis that vary notably from Monday to Sunday. Since the distributions of returns over the days are not constant, we suspect that the day-of-the-week effect may characterize the Bitcoin return data.

Table 1: Empirical Results. Day-of-the-Week Effect in the Daily Bitcoin Returns(yt).

	Full series	Mean of yt	Mean of yt	Mean of y ²	Skewness	Kurtosis	Min	Max
		0.0037	0.0333	0.0033	-0.3452	15.0201	-0.4915	0.4246
1	Monday	0.0048	0.0346	0.0035	-0.8533	13.6092	-0.3483	0.3478
2	Tuesday	0.0068	0.0353	0.0035	0.9435	12.7229	-0.2188	0.4246
3	Wednesday	0.0043	0.0329	0.0032	-0.5759	11.9987	-0.3321	0.3086
4	Thursday	0.0044	0.0375	0.0040	-0.5852	13.2344	-0.4700	0.2908
5	Friday	0.0025	0.0349	0.0034	0.4227	9.3506	-0.2610	0.2987
6	Saturday	0.0032	0.0312	0.0036	-1.1543	24.9235	-0.4915	0.4055
7	Sunday	7.4187e-05	0.0264	0.0020	-0.8474	17.4367	-0.3724	0.2360

• Estimation Results

We design our empirical analysis, by assuming that the conditional distribution of Bitcoin returns is characterized by a periodicity of up to magnitude 7. This approach is different from the one usually used for non-cryptocurrency returns (such as stocks, exchange rates, etc.), which are characterized by a periodicity of up to magnitude 5, due to the non-trading days at each week (week-end).

Therefore, we first estimate seven PAR-SVS models, corresponding to each $S \in \{1, \dots, 7\}$.

Then, we conduct model comparison, using the Deviance information Criterion (DIC) in order to identify the period of the best fitting PAR-SVS model [23].

We run the MCMC samplers for 2500 iterations after a burn-in of 2500 draws. To monitor convergence and mixing of the samplers, we use (and report) the relative numerical inefficiency (RNI) and the numerical standard error (NSE); see, for example [24].

Based on the DIC values (Table 2), PAR-SV₇ is the best model, being followed by the SV model. In Table 3, the results (posterior means and standard deviations) for the SV model show that there is high persistence (0.8626) in the estimated volatilities, which are plotted in Figure 2. In Table 4, the parameters for the PAR-SV₇ model are all significant and different from one day to another especially for the α_v 's and β_v 's, a fact that supports the use of periodic SV modeling tools. The existence of periodicity in Bitcoin returns is also an evidence that they are market inefficient. Finally, Figure 3 plots the estimated volatilities from the PAR-SV₇ model and Figure 4 portrays the difference in volatilities between the PAR-SV₇ and the SV models.

Table 2: DIC Values and Monodromy Parameters (Bitcoin Returns).

	PAR-SV ₁ \equiv SV	PAR-SV ₂	PAR-SV ₃	PAR-SV ₄	PAR-SV ₅	PAR-SV ₆	PAR-SV ₇
DIC (Std)	-9946.7109 (2.2431)	-9883.0069 (2.5607)	-9918.1148 (2.8753)	-9922.8003 (3.0197)	-9905.4721 (2.9816)	9902.2034 (2.8903)	-9950.8321 (2.6844)
Rank of DIC values	2	7	4	3	5	6	1
Monod.	0.8626	0.7841	0.6480	0.6229	0.6024	0.5775	0.4621

Notes: The monodromy (Monod.) parameters $\prod_{v=1}^S \beta_v$ are quite large, indicating strong volatility persistence. Std stands for standard deviation. In computing the standard errors of DIC, we have replicated the algorithm 100 times.

Table 3: Empirical Results for The SV Model (Bitcoin Returns).

	Mean	St. dev	NSE	RNI
α	0.8324	0.0514	0.0025	0.4080
β	0.8626	0.0097	0.0001	0.5261
σ^2	0.3162	0.0111	0.0005	0.2592

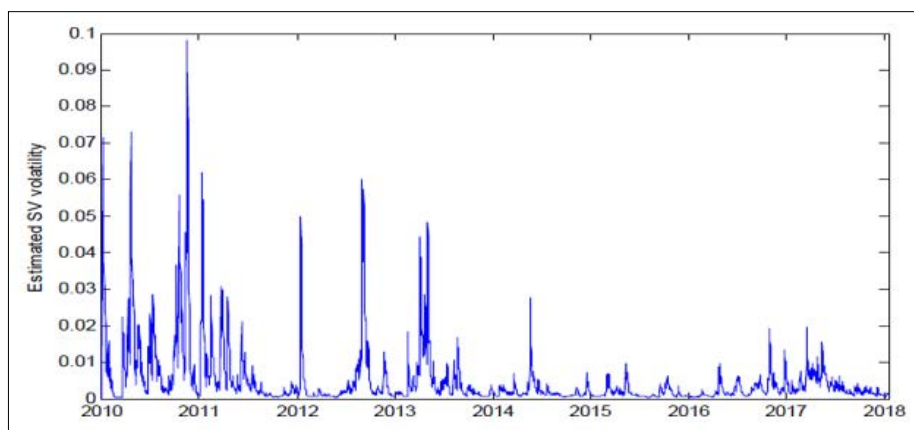


Figure 2: Empirical Results. Bitcoin Estimated Volatilities Induced by the SV Model.

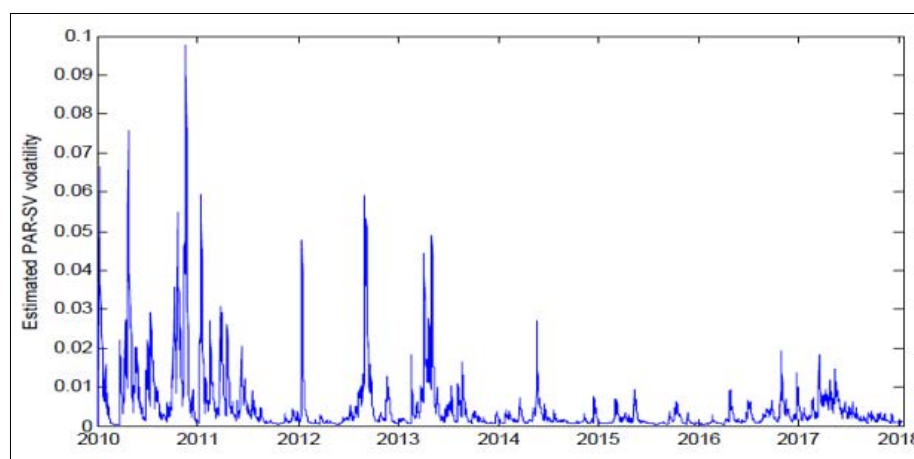


Figure 3: Empirical Results. Bitcoin Estimated Volatilities Induced by the PAR-sv7 Model.

Table 4: Empirical Results for the PAR-SV7 Model (Bitcoin Returns).

Periodic SV Parameters	Mean	St. dev		NSE	RNI
1 Monday	α_1	-1.0069	0.1319	0.0025	0.0458
	$\beta_1 \sigma^2$	0.8421	0.0214	0.0004	0.0509
	1	0.2661	0.0215	0.0060	0.1934
2 Tuesday	α^2	-0.9573	0.1511	0.0012	0.3174
	$\beta_2 \sigma^2$	0.8398	0.0249	0.0033	0.0644
	2	0.3404	0.0287	0.0004	0.0359
3 Wednesday	α_3	-0.7087	0.1469	0.0214	2.5999
	$\beta_3 \sigma^2$	0.8788	0.0239	0.0032	2.2271
	3	0.3328	0.0312	0.0069	0.2151
4 Thursday	α_4	-0.6756	0.1464	0.0010	0.1903
	$\beta_4 \sigma^2$	0.8877	0.0241	0.0044	0.1281
	4	0.3230	0.0330	0.0008	0.1695
5 Friday	α_5	-0.2986	0.1654	0.0117	0.6854
	$\beta_5 \sigma^2$	0.9434	0.0258	0.0016	0.4957
	5	0.3501	0.0293	0.0006	0.0954
6 Saturday	α_6	-0.3996	0.1357	0.0012	0.2397
	$\beta_6 \sigma^2$	0.9309	0.0234	0.0023	0.6778
	6	0.3269	0.0320	0.0037	1.5641
7 Sunday	α^7	-0.2888	0.1564	0.0012	0.2375
	$\beta_7 \sigma^2$	0.9536	0.0257	0.0013	0.2116
	7	0.3043	0.0277	0.0020	0.6830

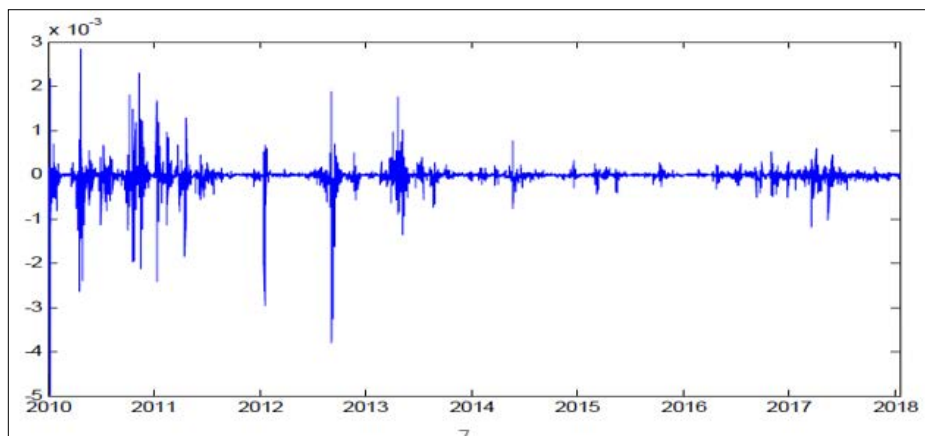


Figure 4: Empirical Results. The Difference Between Estimated Volatilities ($h^{\text{PAR-SV7}} - h^{\text{SV}}$).

Conclusions

We examined the day-of-the-week effect in Bitcoin and found that the hypothesis that Bitcoin returns exhibit periodicity in their conditional distributions is tenable. Also, this empirical finding does not support the efficient market hypothesis for the data in question.

Appendix

Table A.1: Prior Distributions for the Parameters of the SV Model (Which is Equivalent to PAR-SVs with S=1).

Priors	Prior for $\omega = (\alpha, \beta)'$, $\omega \sim N(\omega_0, \Sigma_0)$		Prior for σ^2 : $\frac{a\lambda}{\sigma^2} \sim \chi_a^2$
	ω_0	Σ_0	a
Hyperparameters	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0.05 & 0 \\ 0 & 0.5 \end{pmatrix}$	5

Notes: χ_a^2 denotes the chi-square distribution with a degrees of freedom.

Table A.2: Prior distributions of ω and σ^2 for the candidates PAR-SV_s, $2 \leq v \leq 7$.

Priors	Prior for $\omega = (\alpha, \beta) :$ $\omega \sim N(\omega_0, \Sigma_0)$		Prior for $\sigma^2 :$ $\frac{a_v \lambda_v}{\sigma_v^2} \sim \chi_{a_v}^2$	
	$\omega_0 =$	$\Sigma_0 =$	$a = (a_2, ..., a_s)$	$\lambda = (\lambda_2, ..., \lambda_s)$
$S = 2$	$0_{4 \times 1}$	D_4	$10 \times 1_2$	$0.1 \times 1_2$
$S = 3$	$0_{6 \times 1}$	D_6	$10 \times 1_3$	$0.1 \times 1_3$
$S = 4$	$0_{8 \times 1}$	D_8	$10 \times 1_4$	$0.1 \times 1_4$
$S = 5$	$0_{10 \times 1}$	D_{10}	$10 \times 1_5$	$0.1 \times 1_5$
$S = 5$	$0_{12 \times 1}$	D_{12}	$10 \times 1_6$	$0.1 \times 1_6$
$S = 7$	$0_{14 \times 1}$	D_{14}	$10 \times 1_7$	$0.1 \times 1_7$

Notes:

D_k , $0_{k \times 1}$ and $1k$ denote respectively the diagonal matrix given by (1), the null vector with k components and the k -vector with all components equal to 1.

Online Appendix for: Periodicity in Bitcoin Returns: A Time-Varying Volatility Approach

Priors

Table 1: Prior Distributions for The Parameters of the SV Model (which is Equivalent to PAR-SV_s with S=1).

Priors	Prior for $\omega = (\alpha, \beta) :$ $\omega \sim N(\omega_0, \Sigma_0)$			Prior for $\sigma^2 :$ $\frac{a \lambda}{\sigma^2} \sim \chi_a^2$	
	ω_0		Σ_0	a	λ
Hyperparameters	0	0.05	0	5	0.2
	0	0	0.5		

Notes: χ_a^2 denotes the chi-square distribution with a degrees of freedom.

In Table 2, the diagonal matrix D_k ($k = 4, 10$) is defined to be

$$D_k(i, j) = \begin{cases} 0 & \text{if } i \neq j \\ 0.05 & \text{if } i = j \text{ is odd} \\ 0.5 & \text{if } i = j \text{ is even} \end{cases}, \quad 1 \leq i, j \leq k. \quad (1)$$

Table 2: Prior Distributions of ω and σ^2 for the Candidates PAR-SV_s, $2 \leq v \leq 7$.

Priors	Prior for $\omega = (\alpha, \beta) :$ $\omega \sim N(\omega_0, \Sigma_0)$		Prior for $\sigma^2 :$ $\frac{a_v \lambda_v}{\sigma_v^2} \sim \chi_{a_v}^2$	
	$\omega_0 =$	$\Sigma_0 =$	$a = (a_2, ..., a_s)$	$\lambda = (\lambda_2, ..., \lambda_s)$
$S = 2$	$0_{4 \times 1}$	D_4	$10 \times 1_2$	$0.1 \times 1_2$
$S = 3$	$0_{6 \times 1}$	D_6	$10 \times 1_3$	$0.1 \times 1_3$
$S = 4$	$0_{8 \times 1}$	D_8	$10 \times 1_4$	$0.1 \times 1_4$
$S = 5$	$0_{10 \times 1}$	D_{10}	$10 \times 1_5$	$0.1 \times 1_5$
$S = 6$	$0_{12 \times 1}$	D_{12}	$10 \times 1_6$	$0.1 \times 1_6$
$S = 7$	$0_{14 \times 1}$	D_{14}	$10 \times 1_7$	$0.1 \times 1_7$

Notes:

D_k , $0_{k \times 1}$ and $1k$ denote respectively the diagonal matrix given by (1), the null vector with k components and the k -vector with all components equal to 1.

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