ISSN: 2755-0214

Journal of Economics & Management Research

SCIENTIFIC Research and Community

Review Article

Open Access

Optimal Investment and Financing Decisions Under Cost Uncertainty

I-Shin Wu

PhD student in National Taiwan University

ABSTRACT

In this paper, we employ the real options approach to study the impact of cost uncertainty on a firm's optimal investment and financing decisions. The firm owns a perpetual right to an irreversible investment project, and the firm's goal is to determine the optimal investment timing and level of coupon payments when the investment cost jumps upward at a random time. We find that the optimal investment threshold and level of coupon payments decrease with the jump intensity and the magnitude of the jump. The cost uncertainty gives the firm a stronger incentive to accept projects with risky cash flows. Furthermore, we examine the conflict of interests between the share- and bond-holders. We show that the debt overhang distortion and the asset substitution incentive decrease with the cost uncertainty. However, the effect of cost uncertainty on the incentive of asset substitution reverses in the region of low operating profits.

***Corresponding author**

I-Shin Wu, PhD student in National Taiwan University, Taiwan; Tel: +886919040910.

Received: February 28, 2023; **Accepted:** March 06, 2023; **Published:** March 15, 2023

Keywords: Cost Uncertainty, Real Options, Investment Timing, Debt Overhang, Asset Substitution

JEL Classification: C61, G31, G32

Introduction

When the investment is irreversible, it is optimal for the firm to refrain from investing until the uncertainty is resolved. Hence, the investment opportunity could be viewed as an American option, and the optimal investment decision is to choose the optimal timing of investment. Recently, several studies have emphasized the effect of structural changes on the firm's optimal investment and financing decisions under the real options approach. Most of these previous studies focus on the risk of macroeconomic conditions modelled by a regime switch in the diffusion and drift coefficients of the aggregate output or the operating cash flows, which are the key determinants of the value of investment options [1-6].

In addition to the drift and diffusion coefficients, the optimal exercise strategy depends on the investment cost, which is analogous to the exercise price of a call option. In the real world, the investment cost is often subject to a significant change. For example, a reduction in the investment tax credit or a stricter environmental standard could drastically increase the firm's net investment cost [7, 8]. investigate the effect of cost uncertainty on an all-equity firm's investment decision when the policy change causes an upward jump in the investment cost. To extend the existing literature, we propose a real options model to investigate the optimal investment and financing decisions of a levered firm under the jump risk in investment cost and the firm's optimization problem is to choose the optimal investment threshold and debt level to maximize the firm's value.

The main findings of this study are as follows. First, because the threat of an upward jump in investment cost would shrink the value of waiting, the optimal investment threshold and level of coupon payments fall with the cost uncertainty. Next, the risk of uncertain investment costs would give the firm a stronger incentive to accept the projects with risky cash flows. According to the nature of options, the firm could preserve the upside benefits of the project while avoiding the downside losses by waiting. Thus, projects with highly volatile cash flows would not be immediately accepted by the firm because the firm would bear the downside risk of cash flows once the firm decides to exercise the waiting option. However, when the investment cost uncertainty is considered, there is a trade-off between the risk of uncertain investment costs and the risk of the project's cash flows. If the firm attempts to refrain from investing immediately to avoid the downside risk of cash flows generated by the investment project, the firm is faced with the risk of uncertain investment costs before the investment takes place. Therefore, when the risk of uncertain investment costs is higher, the firm would be more willing to accept projects with highly volatile cash flows.

With debt financing, we examine the conflict of interests between the share- and bondholders. We show that the firm would choose a lower level of coupon payment at the time of investment in response to a higher risk of investment cost and therefore reduce the debt burden after the investment takes place. This lowered debt burden under cost uncertainty would alleviate the debt overhang distortion of and the incentive of asset substitution proposed by [9, 10]. However, we note that the cost uncertainty would strengthen the asset substitution incentive in the region of

DOI: doi.org/10.47363/JESMR/2023(4)172

extremely low operating profits because the low debt burden under cost uncertainty would lead to a lower default trigger. This would defer the occurrence of default when the operating profits are low and thus increase the benefits of asset substitution.

The Model

Optimal Investment and Financing Decisions without Cost Uncertainty

There is a risky investment project with stochastic operating profits P. Once the project has started, the shareholders receive the stochastic operating profits P per unit time and pay a fixed stream of coupon payments R to the bondholders. We assume that P follows a non-negative geometric Brownian motion process:

$$dP = \mu P dt + \sigma P d\varepsilon, \tag{1}$$

where μ is the expected growth rate of the operating profits, σ is the instantaneous volatility, and ε is a standard Brownian motion.

Denoting $E(P)$ as the value of equity, the shareholders optimally choose a default trigger P_d such that $E(P_d)=0$. When the market is complete, that is, the risk of the operating profits could be spanned by the tradable financial securities, $E(P)$ could be derived as

$$E(P)=\begin{cases}\left(\frac{P}{\delta}-\frac{R}{r}\right)(1-\tau)+\frac{1-\tau}{1-\beta}\frac{R}{r}\left(\frac{P}{P_d}\right)^{\beta} & \text{if } P>P_d,\\ 0 & \text{if } P=P_d,\end{cases} \tag{2}$$

where r is the risk-free interest rate, δ is the convenience yield, τ is the corporate tax rate and

$$\beta \triangleq \frac{1}{2}-\frac{r-\delta}{\sigma^2}-\sqrt{\left(\frac{1}{2}-\frac{r-\delta}{\sigma^2}\right)^2+\frac{2r}{\sigma^2}}<0. \tag{3}$$

The optimal default trigger P_d is obtained as

$$P_d=\frac{\delta\beta}{\beta-1}\frac{R}{r}. \tag{4}$$

The unlevered firm value $V^u(P)$ is the equity value when R=0, that is:

$$V^u(P)=E(P)|_{R=0}=\frac{P(1-\tau)}{\delta}. \tag{5}$$

The bondholders receive a stream of continuous coupon payments R per unit time until default. Once the shareholders decide to default on the debt, the ownership of the firm is transferred to the bondholders. The bondholders receive the unlevered firm value net of the bankruptcy cost and the shareholders receive nothing. We assume that the bankruptcy cost is b ($0<b<1$) times the unlevered firm value evaluated at the default trigger. The solution of $D(P)$ is given as follows:

$$D(P)=\begin{cases}\frac{R}{r}+\left[(1-b)V_d-\frac{R}{r}\right]\left(\frac{P}{P_d}\right)^{\beta} & \text{if } P>P_d,\\ (1-b)V_d & \text{if } P=P_d,\end{cases} \tag{6}$$

where $V_d \triangleq V^u(P_d)=\frac{\beta(1-\tau)}{\beta-1}\frac{R}{r}$.

The value of the levered firm, $V(P)$, is the sum of E and D, that is:

$$V(P) \triangleq D(P)+E(P)=\underbrace{\frac{P}{\delta}(1-\tau)}_{V^u(P)}+\underbrace{\frac{\tau R}{r}\left[1-\left(\frac{P}{P_d}\right)^{\beta}\right]}_{ETS(P)}-\underbrace{bV_d\left(\frac{P}{P_d}\right)^{\beta}}_{EBC(P)}. \tag{7}$$

Equation (7) shows that the levered firm value is the sum of the unlevered firm value ($V^u(P)$) and the present value of the expected tax-shield benefits ($ETS(P)$), net of the expected bankruptcy costs ($EBC(P)$).

We assume the firm owns the perpetual right over the investment project and that the shareholders have the option to invest in this project at any time by paying the investment cost I in a lump sum. Denoting $F(P;\hat{P},R)$ as the option value where $\hat{P}$ is the investment threshold, we have:

$$F(P;\hat{P},R)=\mathrm{E}\left[e^{-rT}\left(V(\hat{P})-I\right)\right], \tag{8}$$

where $T \triangleq \inf\{t>0|P\geq\hat{P}\}$. The following proposition summarizes the optimal investment and financing decisions:

Proposition 1. The optimal investment threshold $\hat{P}^*$ and coupon payment R^* are given as follows:

$$\hat{P}^*=\xi I, \quad [9]$$

$$R^*=\kappa\hat{P}^*, \quad [10]$$

where

$$\xi \triangleq \frac{\delta\alpha}{\alpha-1}\left[1-\tau+\frac{\tau\kappa}{r}\frac{\delta\beta}{\beta-1}\right]^{-1}, \quad [11]$$

$$\alpha \triangleq \frac{1}{2}-\frac{r-\delta}{\sigma^2}+\sqrt{\left(\frac{1}{2}-\frac{r-\delta}{\sigma^2}\right)^2+\frac{2r}{\sigma^2}}, \quad [12]$$

$$\kappa \triangleq \frac{r(\beta-1)}{\delta\beta}\left(1-\beta-b\beta\frac{1-\tau}{\tau}\right)^{\frac{1}{\beta}}. \quad [13]$$

The optimal option value is thus given by the following:

$$F^*(P) \triangleq F(P;\hat{P}^*,R^*)=\frac{I}{\alpha-1}\left(\frac{P}{\hat{P}^*}\right)^{\alpha}. \quad [14]$$

Proof. See the Appendix.

Optimal Investment and Financing Decisions with Cost Uncertainty

The current investment cost I may shift to a higher level, which is denoted as $I_h \triangleq \eta I$ ($\eta>1$), and the shifting process of the investment cost follows a Poisson process. Let $\lambda>0$ denote the transition intensity of the Poisson process; then, there is a probability $\lambda\Delta t$ that the current investment cost I would shift to the higher investment cost I_h during an infinitesimal time interval Δt.

Before the investment cost changes, that is, when the investment cost is I, the option value is denoted as $L(P;\hat{P},R)$, and it satisfies the following differential equation:

$$\frac{1}{2}\sigma^2P^2L_{PP}+(r-\delta)PL_P-rL+\lambda(H^*-L)=0, \quad [15]$$

where H^* is the optimal option value after the investment cost shifts to I_h, which is given by the following:

$$H^{*}(P) \Box F^{*}(P)|_{I=I_h} = \frac{I_h}{\alpha-1}\left(\frac{P}{\hat{P}_h^{*}}\right)^{\alpha}, \qquad [16]$$

where $\hat{P}_h^{*} = \xi I_h$.rding to the boundary conditions of $L(0;\hat{P}, R)$ =0 and $L(\hat{P};\hat{P},R)$=V($\hat{P}$)- I, the solution of $L(P;\hat{P},R)$ is given by:

$$L(P;\hat{P},R) = \left[V(\hat{P}) - I - H^{*}(\hat{P})\right]\left(\frac{P}{\hat{P}}\right)^{\omega} + H^{*}(P), \qquad [17]$$

where

$$\omega \triangleq \frac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2(r+\lambda)}{\sigma^2}} > \alpha. \qquad [18]$$

The optimal investment threshold and coupon payment under cost change risk are denoted as $\hat{P}_l$* and R_l*, respectively. $\hat{P}_l$* and $\hat{P}_l$* satisfy the first-order conditions of the following:

$$\left.\frac{\partial L}{\partial \hat{P}}\right|_{\hat{P}=\hat{P}_l^*,R=R_l^*} = \left(\hat{P}_l^*\right)^{-1}\left(\frac{P}{\hat{P}_l^*}\right)^{\omega}\{\omega I + (\omega-\alpha)\frac{I_h}{\alpha-1}\left(\frac{\hat{P}_l^*}{\hat{P}_h^*}\right)^{\alpha}$$

$$-(\omega-1)(1-\tau)\frac{\hat{P}_l^*}{\delta} - \frac{\tau R_l^*}{r}\left[\omega - \frac{\omega-\beta}{1-\beta}\left(\frac{\kappa\hat{P}_l^*}{R_l^*}\right)^{\beta}\right]\} = 0, \qquad [19]$$

and

$$\left.\frac{\partial L}{\partial R}\right|_{\hat{P}=\hat{P}_l^*,R=R_l^*} = \left.\frac{\partial V(\hat{P}_l^*)}{\partial R}\right|_{R=R_l^*} = \frac{\partial ETS(\hat{P}_l^*)}{\partial R} - \left.\frac{\partial EBC_R(\hat{P}_l^*)}{\partial R}\right|_{R=R_l^*} = \frac{\tau}{r}\left[1-\left(\frac{\kappa\hat{P}_l^*}{R_l^*}\right)^{\beta}\right] = 0. \qquad [20]$$

Equation (20) shows that the optimal financing decision under cost uncertainty is consistent with the trade-off theory of capital structure — that is, the optimal financing strategy is to balance the trade-off between the tax-shield benefits and the bankruptcy costs at the time of investment.

According to Eqs. (19) and (20), the optimal investment and financing decisions under cost uncertainty are demonstrated in the following proposition:

Proposition 2. The optimal investment threshold, $\hat{P}_l$*, and coupon payment, R_l*, under cost uncertainty are obtained as follows:

$$\hat{P}_l^* = \psi I, \quad [21]$$

$$R_l^* = \kappa \hat{P}_l^*, \quad [22]$$

where ψ is the solution of the following non-linear equation:

$$\frac{\omega-\alpha}{\alpha-1}\eta^{1-\alpha}\left(\frac{\psi}{\xi}\right)^{\alpha} - \frac{\omega-1}{\alpha-1}\alpha\left(\frac{\psi}{\xi}\right) + \omega = 0. \quad [23]$$

It is clear that when λ=0 (which implies that ω=α) or η=1 (which implies that I_h=I), the solution of Eq. (23) satisfies ψ = ξ; that is, the solution is reduced to what is obtained under no cost uncertainty. Proposition 3 summaries the effects of cost uncertainty on the optimal investment threshold.

Proposition 3. The effects of cost uncertainty on the optimal investment decision are summarized as follows:

1. The cost uncertainty would reduce the optimal investment threshold and coupon payment, that is, $\hat{P}_l$*<$\hat{P}$* and $\hat{P}_l$*<R^R*.
2. The optimal investment threshold and coupon payment decrease with the jump intensity, λ, and the magnitude of jump, η.

   Proof. See the Appendix. ⍰

Proposition 3 indicates that the cost uncertainty would result in a premature investment decision. Because the optimal level of coupon payments under the trade-off theory is commensurate with the operating profits when the investment takes place, the cost uncertainty also reduces the optimal level of coupon payments.

Numerical Analyses and Implications

In this section, we use some numerical examples to investigate the optimal investment and financing decisions under cost uncertainty. The benchmark parameters are as follows: the risk-free rate is r=0.05, the convenience yield of the operating profits is δ=0.02, the volatility of the operating profits is σ=0.25, the corporate tax rate is τ=0.3, bankruptcy costs is b=0.35 of the unlevered firm value evaluated at the default trigger, and the current investment cost is I=20.

Incentive to Accept Projects with Volatile Operating Profits

We first examine the effect of cost uncertainty on the firm's incentive to accept projects with volatile operating profits in the sense of [11]. We plot the investment threshold as a function of the volatility of the operating profits σ to investigate the effect of cost uncertainty on the incentive to accept risky projects. In Fig. 1(a), the lines represent the investment thresholds under I_h=24 for λ=0, 0.3, and 1. In Fig. 1(b), the lines represent the investment thresholds under λ=0.3 for I_h=22, 24, and 26. The firm would exercise the investment option when P lies above the investment thresholds.

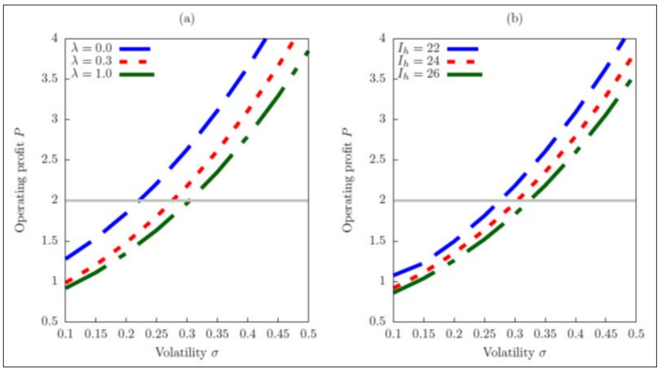

Figure 1: The lines plot the optimal investment thresholds under cost uncertainty. The firm would exercise the investment option when the operating profit P lies above the line. Panel (a) plots the investment thresholds under I_h=24 for λ=0, 0.5, and 1. Panel (b) plots the investment thresholds under λ=1 for I_h=22, 24, and 26.

Figure 1(a) shows that, given a certain level of operating profits, say P=2, the firm would immediately exercise all investment opportunities with σ≤0.22 when λ=0. However, when λ=0.3 (resp. λ=1), the firm would exercise all investment opportunities with σ≤0.28 (resp. σ ≤0.30). The firm would accept riskier investment opportunities when there is a higher probability of change in the investment cost. Similarly, Fig. 1(b) shows that, given P=2, the firm exercises the investment opportunities with σ ≤0.28 when λ=1 and I_h=22. However, when I_h=26, the firm immediately accepts all the investment opportunities with σ ≤0.32. Thus, the incentive to accept projects with high risk is also positively related to the level of I_h.

According to the nature of options, firms could preserve the upside benefits of the project while avoiding the downside losses by waiting. Therefore, firms prefer waiting rather than immediately accepting the project when the project's cash flows are highly uncertain. When the investment cost uncertainty is considered, there is a trade-off between the risk of uncertain investment costs and the risk of the project's cash flows. Firms would bear the downside risk of cash flows once they decide to accept the investment project. Contrarily, if firms decide to wait, they are faced with the risk of uncertain investment costs. Therefore, firms are more willing to accept projects with highly volatile cash flows when the risk of investment costs is higher.

The Moral Hazard Incentives after the Investment Project Takes Place Debt Overhang Problem

For a financially distressed firm, the shareholders would be reluctant to issue new equities to invest in new projects that have a positive net present value because most of the increased value of the new projects would be reaped by the existing bondholders, not the shareholders themselves. According to Pennacchi et al. (2014), the incentive of debt overhang is measured by the following:

$$\frac{\partial E}{\partial V^{u}} - 1 = -\left[\frac{r(\beta-1)P}{\delta\beta R}\right]^{\beta-1}, \quad [24]$$

which represents the net increment of equity value as the asset value (or, equivalently, the unlevered firm value) increases by one unit. A negative value of $\partial E/\partial V^{u-1}$ implies that the shareholders would receive less than what they invest and, hence leading to a distortion of debt overhang. Furthermore, a greater absolute value of $\partial E/\partial V^{u-1}$ implies a stronger distortion of debt overhang.

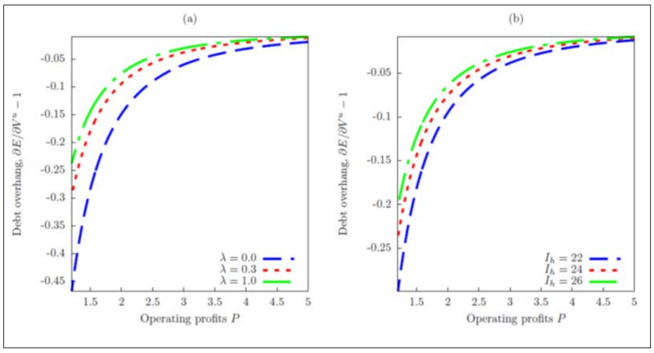

Figure 2: Panel (a) plots the incentive of debt overhang measured by $\partial E/\partial V^{u-1}$ for λ=0, 0.3, and 1 under I_h=24. Panel (b) displays the incentive of debt overhang for I_h=22, 24, and 26 under λ=1.

Figure 2 plots the extent of debt overhang as a function of the level of operating profits, P, and examines the effects of cost uncertainty on the extent of debt overhang. The debt overhang problem is more severe when P is low. When P is low, the probability of default is high, and the profits generated by the newly invested projects would effectively reduce the probability of default. Therefore, undertaking new investment projects would benefit existing bondholders. However, when P is high, there is a low-level probability of default, and the probability of default would not be significantly improved when new projects are undertaken. In this case, because the payments to the existing bondholders are fixed, the new investment would not increase the value of existing bonds, and most of the increased value would belong to the shareholders. In Fig. 2(a), we plot the extent of debt overhang under I_h=24 for λ=0, 0.3, and 1. When there is a higher probability of cost to jump upward (i.e., a higher level of λ), the resulting extent of debt overhang is lower. As proved by Proposition 3, the optimal level of coupon payments decreases with λ; that is, the firm's debt burden is low when λ is high. Hence, the problem of debt overhang is less severe when λ is high. Fig. 2(b) shows the extent of debt overhang under λ=1 for I_h=22, 24, and 26. Similarly, the higher the level of I_h, the lower the firm's debt burden is and, as a result, the lower the extent of the debt overhang will be.

Inefficiency of Asset Substitution

The equity of a levered firm is similar to a call option owned by the shareholders because of its limited liability feature, and the shareholders have an incentive to increase the risk of assets by using riskier projects to substitute for the existing projects. If the high-risk project is successful, the shareholders receive most of the benefits because the income stream of bondholders is fixed. However, if the high-risk project fails, most of the potential downside losses are transferred to the bondholders because the shareholders have only limited liability to the firm's financial burden. This inefficiency of raising a project's risk to increase the equity value at the expenses of the bondholders' interests is referred to as the asset substitution problem. Following the literature, the asset substitution incentive is measured by the partial derivative of the equity value with respect to the volatility of the project's operating profits, that is, $\partial E/\partial\sigma$. When $\partial E/\partial\sigma>0$, an increase in the risk of operating profits would result in a higher equity value, which implies that the shareholders have an incentive in wanting asset substitution, and a greater value of $\partial E/\partial\sigma$ means a stronger incentive of asset substitution [12-14]. Figure 3 plots the effect of cost uncertainty on the asset substitution incentive. Panel (a) of Fig. 3 plots the asset substitution incentive as a function of P under I_h=24 for λ=0, 0.3, and 1, and Panel (b) plots the results under λ=1 for I_h=22, 24, and 26. We show that, when P is high, the asset substitution incentive falls with λ and I_h. When λ and/or I_h become higher, the firm's optimal level of coupon payments becomes lower. With a lower debt burden, the shareholders are more capable of bearing the downside losses when the project fails; therefore, the increased downside losses of the riskier project would not be transferred to the bondholders. However, the effect of cost uncertainty on the asset substitution incentive reverses when there are low operating profits, where the level of P is close to the default trigger. In this region, any slightly increased risk would greatly increase the probability of default. Thus, the asset substitution incentive falls rapidly as P drops to the default trigger. Because the level of coupon payments and the resulting default trigger would decrease with λ and I_h, a higher cost change risk would defer the occurrence of default and thus increase the benefits of asset substitution in the region of low cash flows.

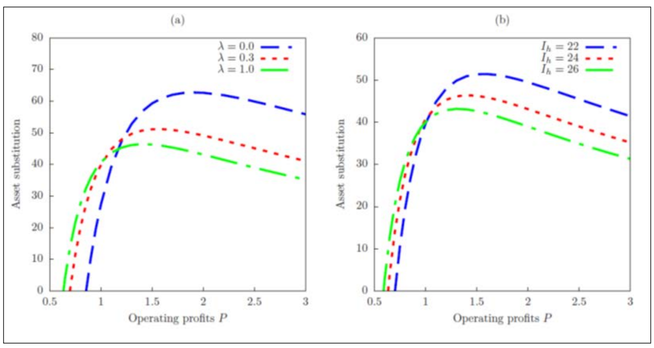

Figure 3: Panel (a) plots the incentive of asset substitution measured by $\partial E/\partial\sigma$ for λ=0, 0.3, and 1 under I_h=24. Panel (b) displays the incentive of asset substitution for I_h=22, 24, and 26 under λ=1.

Conclusions

In this paper, we investigated the optimal investment and financing decisions under cost uncertainty; here, the cost uncertainty is represented by an upward jump in the investment cost. The optimal investment threshold and level of coupon payments decrease with the jump intensity and the magnitude of the cost jump. The incentive to accept projects with volatile operating profits is positively related to the uncertainty of cost. We examine the effect of cost uncertainty on the conflict of interests between the share- and bond-holders. The debt overhang distortion and asset substitution incentive would be alleviated under cost uncertainty. However, the effect of cost uncertainty on the asset substitution incentive reverses when the level of operating profits is low.

Proofs of Propositions

The derivation of *E*(*P*) and *D*(*P*)

The equity value E(P) is defined as

$$E(P) = \mathrm{E}\left[\int_0^{T_d} e^{-rt}\,(1-\tau)(P-R)dt\right], [\mathrm{A}.1]$$

where "E" [·] is the expectation operator under the risk-neutral probability measure and $T_d \triangleq \inf\{t>0|P\leq P_d\}$ is a stopping time. $E(P)$ satisfies the following differential equation:

$$\frac{\sigma^2}{2}P^2E_{PP} + (r-\delta)PE_P - rE + (P-R)(1-\tau) = 0, [A.2]$$

where r is the instantaneous risk-free rate, δ is the convenience yield, τ is the corporate tax rate, and E_P and E_{PP} denote the first- and second-order derivatives of E with respect to P. The optimal default trigger satisfies the smooth-pasting condition:

$$E_P(P_d) = 0. [\mathrm{A}.3]$$

Solving Eqs. (A.2) and (A.3) gives the equity value:

$$E(P) = \begin{cases} \left(\frac{P}{\delta} - \frac{R}{r}\right)(1-\tau) + \frac{1-\tau}{1-\beta}\frac{R}{r}\left(\frac{P}{P_d}\right)^{\beta} & \text{if } P > P_d, \\ 0 & \text{if } P = P_d, \end{cases} [A.4]$$

The debt value satisfies Eq. (A.5) with the boundary condition of $D(P_d) = (1-b)V^u(P_d)$.

$$\frac{\sigma^2}{2}P^2D_{PP} + (r-\delta)PD_P - rD + R = 0. [\mathrm{A}.5]$$

Solving Eq. (A.5), we obtain the solution of $D(P)$

$$D(P) = \begin{cases} \frac{R}{r} + \left[(1-b)V_d - \frac{R}{r}\right]\left(\frac{P}{P_d}\right)^{\beta} & \text{if } P > P_d, \\ (1-b)V_d & \text{if } P = P_d, \end{cases} [A.6]$$

where $V_d \triangleq V^u(P_d) = \frac{\beta(1-\tau)}{\beta-1}\frac{R}{r}$.

Proof of Proposition 1

When $P<\hat{P}$, F satisfies the following differential equation:

$$\frac{1}{2}\sigma^2P^2F_{PP} + (r-\delta)PF_P - rF = 0. [A.7]$$

With the following boundary conditions:

$$F(0;\hat{P},R) = 0, [\mathrm{A}.8]$$
$$F(\hat{P};\hat{P},R) = V(\hat{P}) - I, [\mathrm{A}.9]$$

we solve Eq. (A.7) and obtain:

$$F(P;\hat{P},R) = \left[V(\hat{P}) - I\right]\left(\frac{P}{\hat{P}}\right)^{\alpha}, [A.10]$$

$$\alpha \triangleq \frac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1. [A.11]$$

The optimal investment threshold P̂* given the coupon payment R satisfies the smooth pasting condition $F_P(\hat{P}^*;\hat{P}^*,R) = V_P(\hat{P}^*)$, which leads to the following:

$$\hat{P}^*V_P(\hat{P}^*) = \alpha\left[V(\hat{P}^*) - I\right]. [A.12]$$

The optimal coupon payment R* satisfies the following first-order condition:

$$F_{\hat{P}}(P;\hat{P}^*,R^*)\frac{\partial \hat{P}^*}{\partial R}|_{R=R^*} + F_R(P;\hat{P}^*,R^*) = 0, [A.13]$$

and therefore we have the following:

$$\alpha\left[V(\hat{P}^*) - I\right]|_{R=R^*} = \hat{P}^*\left[V_P(\hat{P}^*) + V_R(\hat{P}^*)\right]|_{R=R^*}. [A.14]$$

According to Eq. (A.12), the first-order condition in Eq. (A.14) is simplified as follows:

$$0 = V_R(\hat{P}^*)\big|_{R=R^*} = ETS_R(\hat{P}^*) - EBC_R(\hat{P}^*)\big|_{R=R^*}. [A.15]$$

In Eq. (A.15), ETS_R and EBC_R represent the partial derivatives of the expected tax-shield benefits and expected bankruptcy costs with respect to R, respectively. Hence, Eq. (A.15) implies that the firm's optimal financing decision is to balance the trade-off between the marginal tax-shield benefits and the marginal bankruptcy costs of debt at the time of investment.

We substitute Eq. (7) into Eqs. (A.12) and (A.15) to derive the following:

$$\alpha I - (\alpha-1)(1-\tau)\frac{\hat{P}^*}{\delta} - \frac{\tau R}{r}\left[\alpha - \frac{\alpha-\beta}{1-\beta}\left(\frac{\kappa\hat{P}^*}{R}\right)^{\beta}\right] = 0, [A.16]$$

$$1 - \left(\frac{\kappa\hat{P}^*}{R^*}\right)^{\beta} = 0, [A.17]$$

where

$$\kappa \triangleq \frac{r(\beta-1)}{\delta\beta}\left(1 - \beta - b\beta\frac{1-\tau}{\tau}\right)^{1/\beta}. [A.18]$$

According to Eq. (A.17), the optimal coupon payment R* is solved as follows:

$$R^* = \kappa P^*. [A.19]$$

By substituting R^* into Eq. (A.16), the solution of the optimal investment threshold P^^* is given by the following:

$$\hat{P}^* = \xi I, [A.20]$$

where

$$\xi \triangleq \frac{\delta\alpha}{\alpha-1}\left[1 - \tau + \frac{\tau\kappa}{r}\frac{\delta\beta}{\beta-1}\right]^{-1}. [A.21]$$

Proof of Proposition 3

Substituting $\hat{P}_l^*=\psi I$ and $\hat{P}_l^*=\kappa\hat{P}_l^*$ into $\partial L/\partial\hat{P}$, we rewrite the partial derivative of the option value with respect to the investment threshold as follows:

$$\frac{\partial L}{\partial \hat{P}} = \frac{1}{\psi}\left(\frac{P}{\psi I}\right)^{\omega}\left[\frac{\omega-\alpha}{\alpha-1}\eta^{1-\alpha}\left(\frac{\psi}{\xi}\right)^{\alpha} - \frac{\omega-1}{\alpha-1}\alpha\left(\frac{\psi}{\xi}\right)+\omega\right]. [A.22]$$

From Eq. (A.22), it is clear that $\partial L/\partial\hat{P} > 0$ when $\psi\rightarrow 0^+$ and, when $\psi = \xi$, we have the following:

$$\frac{\partial L}{\partial \hat{P}}\Big|_{\psi=\xi} = \frac{1}{\xi}\left(\frac{P}{\xi I}\right)^{\omega}\left(\frac{\omega-\alpha}{\alpha-1}\eta^{1-\alpha} - \frac{\omega-1}{\alpha-1}\alpha+\omega\right) < \frac{1}{\xi}\left(\frac{P}{\xi I}\right)^{\omega}\left(\frac{\omega-\alpha}{\alpha-1} - \frac{\omega-1}{\alpha-1}\alpha+\omega\right) = 0. \quad [A.23]$$

The inequality in Eq. (A.23) is because of $\eta > 1$ and $\alpha > 1$. Because the partial derivative of option value with respect to the investment threshold is negative when $\hat{P} = \xi I$ and is positive when $\hat{P}\rightarrow 0^+$, the optimal investment threshold must lie between zero and ξI, that is, $0 < \psi < \xi$.

Next, because ψ is the root of Eq. (23), we can differentiate both sides of Eq. (23) to obtain the following:

$$\frac{\partial \hat{P}_l^*}{\partial \lambda} = \frac{\partial \psi}{\partial \lambda} I = \frac{\partial \psi}{\partial \omega}\frac{\partial \omega}{\partial \lambda} I = -\frac{2\psi\alpha I}{(\omega-\alpha)(\sigma^2\omega^2+2r+2\lambda)}\left(1-\frac{\psi}{\xi}\right), [A.24]$$

and

$$\frac{\partial \hat{P}_l^*}{\partial \eta} = \frac{\partial \psi}{\partial \eta} I = \frac{-\psi(\omega-\alpha)\eta^{-\alpha}\left(\frac{\psi}{\xi}\right)^{\alpha} I}{\alpha\left[\frac{\psi}{\xi}+\omega\left(1-\frac{\psi}{\xi}\right)\right]}. [A.25]$$

Because $\omega > \alpha > 1$ and $\psi < \xi$ when $\lambda > 0$ and $\eta > 1$, it is clear to show that $\partial\hat{P}_l^*/\partial\lambda < 0$ and $\partial\hat{P}_l^*/\partial\eta < 0$. Since $R_l^*=\kappa\hat{P}_l^*$, it is straightforward to show that $\partial\hat{P}_l^*/\partial\lambda < 0$ and $\partial\hat{P}_l^* / \partial\eta < 0$.

References

1. Guo X, Miao J, Morellec E (2005) Irreversible investment with regime shifts. Journal of Economic Theory 122: 37-59.
2. Hackbarth D, Miao J, Morellec E (2006) Capital structure, credit risk, and macroeconomic conditions. Journal of Financial Economics 82: 519-550.
3. Nishide K, Nomi EK (2009) Regime uncertainty and optimal investment timing. Journal of Economic Dynamics and Control 33: 1796-1807.
4. Chen H (2010) Macroeconomic conditions and the puzzles of credit spreads and capital structure. Journal of Finance 65: 2171-2212.
5. Arnold M, Wagner AF, Westermann R (2013) Growth options, macroeconomic conditions, and the cross-section of credit risk. Journal of Financial Economics 107: 350-385.
6. Jeon H, Nishihara M (2014) Macroeconomic conditions and a firm's investment decisions. Finance Research Letters 11: 398-409.
7. Hassett KA, Metcalf GE (1999) Investment with uncertain tax policy: Does random tax policy discourage investment? Economic Journal 109: 372-393.
8. Pawlina G, Kort PM (2005) Investment under uncertainty and policy change. Journal of Economic Dynamics and Control 29: 1193-1209.
9. Myers SC (1977) Determinants of corporate borrowing. Journal of Financial Economics 5: 147-175.
10. Jensen MC, Meckling WH (1976) Theory of the firm: Managerial behavior, agency costs, and ownership structure. Journal of Financial Economics 3: 305-360.
11. Mauer DC, Sarkar S (2005) Real options, agency conflicts and optimal capital structure. Journal of Banking Finance 29: 1405-1428.
12. Pennacchi G, Vermaelen T, Wolff C (2014) Contingent capital: The case of COERCs. Journal of Financial and Quantitative Analysis 49: 541-574.
13. Luo P, Yang Z (2017) Real options and contingent convertibles with regime switching. Journal of Economic Dynamics and Control 75: 122-135.
14. Wang H, Xu Q, Yang J (2018) Investment timing and optimal capital structure under liquidity risk. European Journal of Finance 24: 889-908.