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On the Magnetic Charge of an Electron and the Generalized Electrodynamic Forces

Klyushin Yaroslav

Saint-Petersburg State University of Civil Aviation, Russia

ABSTRACT

In the late 90s of XX century the author proposed a concept describing an electric charge as a rotating mass. Such a description makes it possible to exclude the Coulomb quantity from the system SI and to describe all electrodynamic quantities in mechanical dimensions. According to the dimensionality, a magnetic charge is an electric charge divided by the speed of light. In the 10s of the 21st century, the authors of this article proposed a generalization of the traditional system of Maxwell's equations in the context of a mechanical description of electromagnetic quantities. The generalized system has a solution for single charges. From these solutions, one can construct the energy and momentum of the interaction of two electrons. In this case, the electron is the unit of two principles: the electric and the magnetic. The gradient of the interaction energy gives the interaction force of the electric charges of the electrons, and the time derivative of the interaction momentum gives the interaction force of the magnetic charges of two electrons. A formal description of these forces completes the article.

***Corresponding author**

Klyushin Yaroslav, Saint-Petersburg State University of Civil Aviation, Russia. Tel.: +7-911-7542952;

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Introduction

The classical Maxwell equations can be considered as a theorem about the symmetry of electric and magnetic fields. However, this symmetry is broken by the absence of a magnetic charge. In Dirac proposed his idea of a magnetic monopole [1]. Since then, this idea has been the subject of lively discussion.

J.S. Schwinger paid much attention to the problem of magnetic charge. For example, in he presented his own understanding of the monopole in the context of photon sources [2]. In he constructed a quantum field theory which includes both electric and magnetic charges [3]. He also presented a different view from Dirac. He considered Dirac's view to be asymmetric and offered his own view, which he believed to be symmetric. His quantum field theory is relativistically invariant, but restricts the quantization condition more than Dirac's [4]. In another article he argues that relativistic renormalization of two charge types is an important part of electromagnetic field theory [5].

P. Goddard considers a gange group in which the magnetic charge appears as a coefficient and completely determines the topological quantum number of the solution [6].

The quantum mechanical problem of the motion of electric and magnetic charges in the field of a magnetic charge is considered in the article "Magnetic charge quantization and angular momentum" [7].

In the article by S.T. Bramwell, a modification of Maxwell's equations with electric and magnetic charges is proposed [8].

The approach proposed below is reminiscent of the idea of Bramwell [8]. The first step of this approach is to formulate the concept of an electric charge as a rotating mass [9]. This allows to rewrite all equations of electrodynamics in mechanical dimensions and to construct a system that generalizes the classical Maxwell equations and assumes the existence of a magnetic charge [10]. The description of all quantities in mechanical dimensions makes it possible to establish a direct relationship between the magnitude of the electric and the magnetic charge of the electron. The existence of two combined properties in an electron allows a generalization of the formula for the Lorentz force.

Magnetic and electric charges

It was assumed that electric charge is induced by rotating of the bigger circle of the torus [10]:

$$e = m\omega_1 = 7.072 \times 10^{-10}\ \frac{\text{kg}\cdot\text{rad}}{\text{s}}, \tag{1}$$

where e is the electric charge, m is the electron's mass, ω_1 is angular velocity of the torus bigger circumference.

The dimension of the charge (1) results from the assumption that the electric charge of the electron is a mass rotation. This assumption is considered in as hypnosis, i.e., as an assertion which must be confirmed experimentally [1]. The authors believe that the first step on the way to such confirmation is an experiment defining the wavelength of the electron and the frequency determined by it (1). This means that we exclude the dimension "Coulomb" from the system SI and obtain a mechanical system of the dimension for any electrodynamic value (see the transformation of the system SI into a mechanical one) [10]. The mechanical system for electrodynamic quantities is also used below.

The rotation (1) also produces a moment which is given by Planck's constant

$$\hbar_1 = 1.0546 \times 10^{-34}\ \frac{\text{kg}\cdot\text{m}^2\cdot\text{rad}}{\text{s}}. \tag{2}$$

The magnetic field is generated by the two-dimensional rotation of the smaller rings of the torus. This rotation induces the two-dimensional vector of the spin of the electron [9]:

$$\vec{\hbar}_{23} = (\hbar_2, \hbar_3), \tag{3}$$

where

$$\hbar_2 = \frac{1}{2}(1.0546 \times 10^{-34} \sin\omega_1 t)\ \frac{\text{kg}\cdot\text{m}^2\cdot\text{rad}}{\text{s}}, \tag{4}$$

$$\hbar_2 = \frac{1}{2}(1.0546 \times 10^{-34} \cos\omega_1 t)\ \frac{\text{kg}\cdot\text{m}^2\cdot\text{rad}}{\text{s}}. \tag{5}$$

Two-dimensional angular momentum (3) gives the magnetic properties of the electron, its spin. Three-dimensional angular momentum

$$\vec{\hbar} = (\hbar_1, \hbar_2, \hbar_3) \tag{6}$$

specifies the electric and magnetic properties of the electron [9].

The use of mechanical dimensions to describe electrodynamic quantities makes it possible to describe the electric constant mechanically [10, §3.1]

$$\varepsilon_0 = 1.7251 \times 10^{8}\ \frac{\text{kg}\cdot\text{rad}^2}{\text{m}^3}. \tag{7}$$

and magnetic constant

$$\mu_0 = \frac{1}{\varepsilon_0 c^2} = 6.4498 \times 10^{-26}\ \frac{\text{m}\cdot\text{s}^2}{\text{kg}\cdot\text{rad}^2}. \tag{8}$$

The term magnetic charge is not defined in [10] (§2). This shall be made up for in the present work.

From the classical Maxwell equations, it follows that the electric fields **E** and the magnetic fields **B** are connected by the equation [11, §18.4]:

$$E = \pm cB, \tag{9}$$

where c is the speed of light.

The classical Maxwell equations do not predict the sign in (9) because div**B** = 0, i.e., the magnetic charge is assumed to be zero. The generalized Maxwell's equations give a minus in (9) [10] (§2.2).

The assumption of zero magnetic charge also raises the following question. The mathematical essence of Maxwell's equations requires that one of the fields $\vec{E}$ or $\vec{B}$ must be polar and the other must be an axial vector. This means that the speed of light c in (9) must be pseudoscalar, i.e., it must describe the tangential velocity [m·rad/s] but not the translational velocity [m/s]. This problem becomes essential in generalized electrodynamics. Its solution makes it possible to correctly define the magnetic charge.

At the time of writing the monograph, the mechanical structure of the electron was not yet understood [10]. It was assumed that the electric charge must be an ordinary scalar and the magnetic charge a pseudoscalar. When the mechanical model of the electron was constructed, it was found that the situation was reversed: the electric charge was pseudoscalar and the magnetic charge was an ordinary number, i.e., the correlation (9) is valid for the charges:

$$e = -\hat{c}b \Rightarrow b = -e/\hat{c} = -2.359 \times 10^{-18}\ \frac{\text{kg}}{m}. \tag{10}$$

where b is the magnetic charge and $\hat{c}$ is the tangential speed of light [m·rad/s]. To avoid misunderstanding, the tangential velocity is denoted by $\hat{c}$. The symbol c is left for the translational speed of light.

Generalized electrodynamic force

Let us verify that two electric charges (1) induce the Coulomb force

$$F_{Cl} = \frac{e_1 e_2}{4\pi\varepsilon_0 r^2}\left[\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\right]. \tag{11}$$

We have obtained the force as a result of the interaction between two electric charges defined in (1). It is obtained by substituting (10) into (11) instead of (1):

$$F_{b_1 b_2} = \frac{b_1 b_2}{4\pi\varepsilon_0 r^2}\left[\frac{\text{kg}}{\text{m}\cdot\text{rad}^2}\right]. \tag{12}$$

We have obtained mass density per radian squared instead of force.

Conclusion: The magnetic charges of the electrons do not interact in the static. The static force is defined only by the electric charges of the electrons.

In the case of moving electrons, the picture changes. Let us consider the case when the first electron moves with the translational velocity v_2 and the second electron remains stationary.

$$F_{v_1} = \frac{b_1 v_1 e_2}{4\pi\varepsilon_0 r^2}\left[\frac{\text{kg}\cdot\text{m}}{\text{s}^2\cdot\text{rad}}\right]. \tag{13}$$

We have obtained a force per radian. The magnetic charge of the moving electron interacts with the electric charge of the resting electron. This force is c/v_1 weaker than the Coulomb force. When both electrons are moving, two forces occur. The first

$$F_{v_1 v_2} = \frac{b v_1 b v_2}{4\pi\varepsilon_0 r^2}\left[\frac{\text{kg}\cdot\text{m}}{\text{s}^2\cdot\text{rad}^2}\right]. \tag{14}$$

This force is $c^2/v_1 v_2$ weaker than the Coulomb force. The second force in this group arises from the interaction of the electric charges of the moving electrons:

$$F_{e_1 v_1, e_2 v_2} = \frac{(e_1 v_1 - e_2 v_2)^2}{4\pi\varepsilon_0 c^2 r^2}\left[\frac{\text{kg}\cdot\text{m}}{\text{s}^2\cdot\text{rad}^2}\right]. \tag{15}$$

Force (14) disappears when at least one of the electrons is at rest. In this case, the force (15) reduces to (14). The Lorentz force, which is usually considered in today's physics, belongs to the class (14).

The force (13) can be called a "semi-dynamic" force. It occurs when one electron is static and only the second electron is moving. In present-day electrodynamics, it is assumed to be zero. The description of this force and its experimental verification can be found in Example 4 [10, §2.7].

In general, a force occurs between moving electrons. The force (14) arises as an interaction between magnetic charges of two electrons and the forces (15) between two electric charges of two moving electrons. This means, in particular, that the Lorentz force formula is not sufficient to describe all electrodynamic forces. The general formula for these forces is obtained in the framework of generalized electrodynamics. Let us consider and explain one of three equivalent formulas for the forces in generalized electrodynamics, namely (2.4.2) in [10].

$$\begin{aligned}
\vec{F}_{21} = {} & \frac{q_1 q_2}{4\pi\varepsilon_0 r^3}\vec{r}_{21} + \frac{q_1 q_2}{4\pi\varepsilon_0 r^3 c^2}\cdot \\
& \cdot\Bigg\{\Bigg[\overbrace{2\vec{r}_{21}(\vec{v}_1\cdot\vec{v}_2)}^{1} - \overbrace{\vec{v}_1(\vec{r}_{21}\cdot\vec{v}_2)}^{2} - \overbrace{\vec{v}_2(\vec{r}_{21}\cdot\vec{v}_1)}^{3} - \overbrace{\frac{3\vec{r}_{21}}{r^2}\big((\vec{r}_{21}\times\vec{v}_1)(\vec{r}_{21}\times\vec{v}_2)\big)}^{4}\Bigg] + \\
& + \Bigg[\overbrace{\vec{r}_{21}\left(\vec{v}_1-\vec{v}_2\right)^2}^{5} - \overbrace{\left(\vec{v}_1-\vec{v}_2\right)\left(\vec{r}_{21}(\vec{v}_1-\vec{v}_2)\right)}^{6}\Bigg] - \\
& - \frac{3\vec{r}_{21}(\vec{v}_1-\vec{v}_2)}{r^2}\Bigg[\overbrace{\vec{r}_{21}\left(\vec{r}_{21}(\vec{v}_1-\vec{v}_2)\right)}^{7} - \overbrace{\left(\vec{v}_1-\vec{v}_2\right)r^2}^{8}\Bigg] + \\
& + \Bigg[\overbrace{\vec{r}_{21}\left(\vec{r}_{21}(\vec{a}_1-\vec{a}_2)\right)}^{9} - \overbrace{\left(\vec{a}_1-\vec{a}_2\right)r^2}^{10}\Bigg] + \overbrace{\frac{1}{c}\left(\vec{v}_2-\vec{v}_1\right)\left(\vec{r}_{21}(\vec{v}_1\times\vec{v}_2)\right)}^{11} + \\
& + \overbrace{\frac{1}{c}\vec{r}_{21}\left[(\vec{r}_{21}\times\vec{v}_2)\vec{a}_1 - (\vec{r}_{21}\times\vec{v}_1)\vec{a}_2\right]}^{12} + \overbrace{\frac{3\vec{r}_{21}}{r^2 c}\left[\vec{r}_{21}(\vec{v}_1-\vec{v}_2)\right]\left[\vec{r}_{21}(\vec{v}_1\times\vec{v}_2)\right]}^{13}\Bigg\}.
\end{aligned} \tag{16}$$

The interacting bodies here are electrons. $\vec{F}_{21}$ is the force acting from electron 2 on electron 1. The symbol of electron q is used to denote a set of electrical and magnetic properties. It is convenient when it is not necessary to consider electric and magnetic charges separately. If we want to consider separately the forces defined by the interaction of electric and magnetic charges, the formula (16) is as follows:

$$\vec{F}_{21} = \frac{e_1 e_2 \vec{r}_{21}}{4\pi\varepsilon_0 r^3} +$$
$$+\frac{b_1 b_2}{4\pi\varepsilon_0 r^3}\left\{2\vec{r}_{21}(\vec{v}_1\cdot\vec{v}_2)-\vec{v}_1(\vec{r}_{21}\cdot\vec{v}_2)-\vec{v}_2(\vec{r}_{21}\cdot\vec{v}_1)-\frac{3}{r^2}\vec{r}_{21}\left((\vec{r}_{21}\times\vec{v}_1)(\vec{r}_{21}\times\vec{v}_2)\right)\right\}+$$
$$+\frac{b_1 e_2 + b_2 e_1}{4\pi\varepsilon_0 r^3 c^2}\left\{\vec{r}_{21}\left[(\vec{r}_{21}\times\vec{v}_2)\vec{a}_1-(\vec{r}_{21}\times\vec{v}_1)\vec{a}_2\right]+\frac{3}{r^2}\vec{r}_{21}\left[\vec{r}_{21}(\vec{v}_1-\vec{v}_2)\right]\left[\vec{r}_{21}(\vec{v}_1\times\vec{v}_2)\right]+\right.$$
$$\left.+\left(\vec{v}_2-\vec{v}_1\right)\left(\vec{r}_{21}(\vec{v}_1\times\vec{v}_2)\right)\right\}+$$
$$+\frac{e_1 e_2}{4\pi\varepsilon_0 r^3 c^2}\cdot\left\{\left[\vec{r}_{21}\left(\vec{v}_1-\vec{v}_2\right)^2-\left(\vec{v}_1-\vec{v}_2\right)\left(\vec{r}_{21}(\vec{v}_1-\vec{v}_2)\right)\right]-\right.$$
$$-\frac{3}{r^2}\vec{r}_{21}(\vec{v}_1-\vec{v}_2)\left[\vec{r}_{21}\left(\vec{r}_{21}(\vec{v}_1-\vec{v}_2)\right)-\left(\vec{v}_1-\vec{v}_2\right)r^2\right]+$$
$$\left.+\left[\vec{r}_{21}\left(\vec{r}_{21}(\vec{a}_1-\vec{a}_2)\right)-\left(\vec{a}_1-\vec{a}_2\right)r^2\right]\right\}. \quad (16a)$$

Here e_1 and e_2 are electric charges of the electrons, b_1 and b_2 are magnetic charges of the corresponding electrons. The magnetic and electric charges have opposite sign in the accordance with the definition (10).

Example

The forces acting on charges can be considered as a pair of forces with a lever as the radius vector ohm. For this pair, the well-known statement in mechanics about the equality and oppositeness of the moments of force applies. Unlike in the case of a lever, however, this moment also exists along the lever (radius vector between the charges). This manifest itself as Newton's third law, i.e., radial forces. These forces stretch or compress the radius vector (lever).

Let us look more closely at the analogy with mechanics. The two radial terms in the first curly bracket, the Ampère force, are the forces that stretch or compress the rubber stick. These forces satisfy Newton's third law. Two speed terms describe the situation with the lever. This pair of forces fulfils the law of symmetry of the moment of force.

What does modern physics have to say about this? The Lorentz force consists of a radial and a velocity term in the first vector bracket. It does not satisfy Newton's law or the law of the lever. Whittaker adds another velocity term to the Lorentz force. His force already fulfils the law of the lever. So, all magnetic forces have already been described. It is just that their inner connection has not been recognized, and the Whittaker formula is hardly known today.

Let us use the example of the magnetic forces.

Let's write down the trigonometric form that corresponds to (16).

Let θ_1 be the angle between $\vec{r}_{21}$ and $\vec{v}_1$; θ_2 is the angle between $\vec{r}_{21}$ and $\vec{v}_2$; θ_3 is the angle between $\vec{v}_1$ и $\vec{v}_2$; θ_4 is the angle between $\vec{r}_{21}$ and $(\vec{v}_1-\vec{v}_2)$; θ_5 is the angle between $\vec{r}_{21}$ and $(\vec{a}_1-\vec{a}_2)$; θ_6 is the angle between $\vec{r}_{21}$ and $(\vec{v}_1\times\vec{v}_2)$; θ_7 is the angle between $(\vec{r}_{21}\times\vec{v}_2)$ and $\vec{a}_1$; θ_8 is the angle between $(\vec{r}_{21}\times\vec{v}_1)$ and $\vec{a}_2$; θ_9 is the angle between $(\vec{r}_{21}\times\vec{v}_1)$ and $(\vec{r}_{21}\times\vec{v}_2)$. Then

$$\vec{F}_{21} = \frac{e_1 e_2}{4\pi\varepsilon_0 r_{21}^3}\vec{r}_{21} +$$
$$+\frac{b_1 b_2}{4\pi\varepsilon_0 r_{21}^3}\{-\vec{v}_1 v_2 r_{21}\cos\theta_1 - \vec{v}_2 v_1 r_{21}\cos\theta_2 + \vec{r}_{21} v_1 v_2(2\cos\theta_3 - 3\sin\theta_1\sin\theta_2\cos\theta_9)\} +$$
$$+\frac{b_1 e_2 + b_2 e_1}{4\pi\varepsilon_0 r_{21}^3 c^2}\left\{\frac{(\vec{v}_2-\vec{v}_1) r v_1 v_2\cos\theta_6\sin\theta_3}{c} + \frac{\vec{r}_{21} r_{21}[a_1 v_2\sin\theta_2\cos\theta_7 - a_2 v_1\sin\theta_1\cos\theta_8]}{c} +\right.$$
$$\left.+\frac{3\vec{r}_{21}}{c}[(v_1-v_2)(v_1\cdot v_2)]\cos\theta_4\cos\theta_6\right\} +$$
$$+\frac{e_1 e_2}{4\pi\varepsilon_0 r_{21}^3 c^2}\{[\vec{r}_{21}(\vec{v}_1-\vec{v}_2)^2(1-3\cos^2\theta_4) + 2(\vec{v}_1-\vec{v}_2)\cdot r_{21}|\vec{v}_1-\vec{v}_2|\cos\theta_4] +$$
$$+[\vec{r}_{21} r_{21}|\vec{a}_1-\vec{a}_2|\cos\theta_5 - (\vec{a}_1-\vec{a}_2) r_{21}^2]\} \quad (16b)$$

Let two electrons move along parallel straight lines l_1 and l_2 with identical and codirectional velocities $\vec{v}_1 = \vec{v}_2 = \vec{v}$. The angle θ_3 between $\vec{v}_1$ and $\vec{v}_2$ is zero, so $\cos\theta_3 = 1$. The angle between vectors $(\vec{r}_{21}\times\vec{v}_1)$ and $(\vec{r}_{21}\times\vec{v}_2)$ is zero, so $\cos\theta_9 = 1$. The angles θ_1 and θ_2 are equal and both depend on the position of electrons q_1 and q_2 on lines l_1 and l_2. Therefore, $\sin\theta_1 = \sin\theta_2 = \sin\theta$, $\cos\theta_1 = \cos\theta_2 = \cos\theta$.

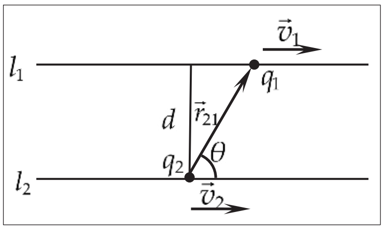

Figure 1: Two electrons move along parallel straight lines with the same velocity.

All forces except magnetic are equal to zero. Magnetic force

$$\vec{F}_{21} = \frac{b_1 b_2}{4\pi\varepsilon_0 r_{21}^3}\{-2\vec{v} v r_{21}\cos\theta + \vec{r}_{21} v^2 (2 - 3\sin^2\theta)\} \quad (17)$$

If $\theta = 0$ (Figure 2)

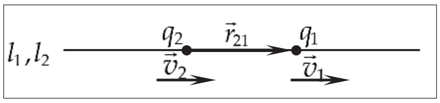

Figure 2: Two electrons move along one straight lines with the same velocity, second charge behind the first.

those the distance between lines l_1 and l_2 is equal to zero. The direction of the radius-vector $\vec{r}_{21}$ coincides with the direction $\vec{v}_1$ and $\vec{v}_2$, the electron q_1 is ahead of q_2. Then (16b) takes the form

$$\vec{F}_{21} = \frac{b_1 b_2}{4\pi\varepsilon_0 r_{21}^3}\{-2\vec{v} v r_{21} + 2\vec{r}_{21} v^2\} \quad (18)$$

The directions of $\vec{v}$ and $\vec{r}_{21}$ coincide, so

$$\vec{F}_{21} = 0 \quad (19)$$

Conclusion: There is no force interaction between the electron q_2 and the electron q_1, which are ahead of q_2.

Later we will talk about the Coulomb interaction between electrons, but we will not consider it. Here we are talking only about dynamic interaction.

Let now $\theta = 90°$.

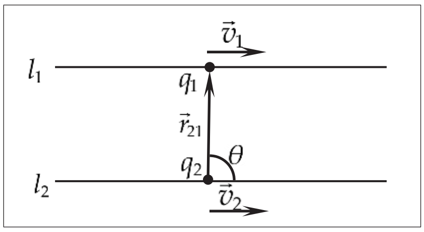

Figure 3: Two electrons move with equal velocity on parallel straight lines opposite each other.

$$\vec{F}_{21} = \frac{b_1 b_2}{4\pi\varepsilon_0 r_{21}^3}\{\vec{r}_{21} v^2 (2-3)\} = -\frac{b_1 b_2}{4\pi\varepsilon_0 r_{21}^3}\vec{r}_{21} v^2 \quad (20)$$

The force is directed against $\vec{r}_{21}$, the electron q_1 is attracted by q_2, as are all electrons q1 on this normal in parallel lines. The dynamic force contradicts the Coulomb force.

Now let θ = 180°, the first electron is behind the second (Figure 4).

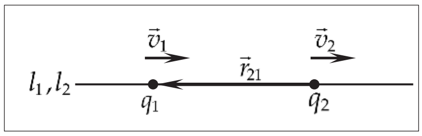

Figure 4: Two charges move along the same straight line at the same speed, with the second charge ahead of the first.

$$\vec{F}_{21} = \frac{b_1 b_2}{4\pi\varepsilon_0 r_{21}^3}\{-2\vec{v} v r_{21} - 2\vec{r}_{21} v^2\} = -\frac{b_1 b_2}{4\pi\varepsilon_0 r_{21}^3} 4\vec{r}_{21} v^2 \quad (21)$$

Radial and velocity forces have combined to help the Coulomb sweep out stragglers; attractive forces overcome the Coulomb when $4v^2 > c^2$, i.e., when v is greater than half c. The electrons stick together in clusters along l_2. This is known from experiments.

Consider separately the radial forces $\vec{F}_r$ and the velocity forces $\vec{F}_v$ that make up $\vec{F}_{21}$. The radial forces are given in parentheses in (16b). They are zero if

$$2 - 3\sin^2\theta = 0. \quad (22)$$

In the interval (54.7°, 125.3°) $\vec{F}_r$ is negative, it attracts the charges q_1 which are on the straight line l_1 in this interval, which contradicts Coulomb's assumption. In the intervals (0°, 54.7°] and [125.3°, 180°) it is positive and supports Coulomb's (Figure 5).

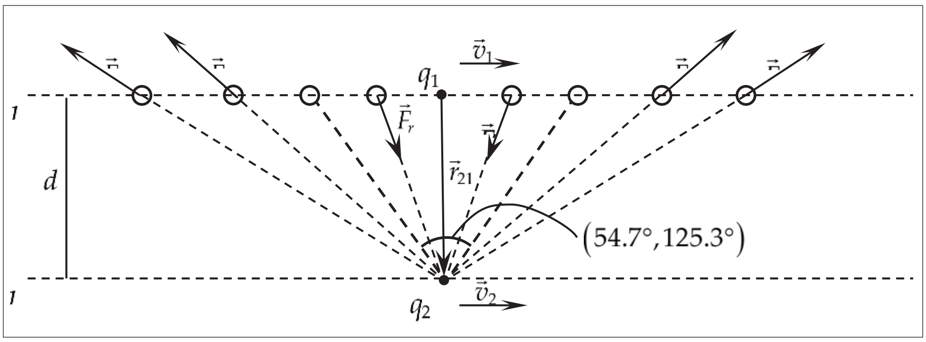

Figure 5: The distribution of forces when two charges move along parallel lines with the same speed.

If we let the distance d between the lines l_1 and l_2 tend to zero, we get that in the boundary between the charges q_1 and q_2 only the normal force $\vec{F}_r^n$ acts, the tangential radial force disappears and cancels each other. If we integrate this local force over the entire length of the line l_1 and l_2, we conclude that the current lines l_1 and l_2 attract each other. Since the curly bracket indicating the magnetic field is preceded by a coefficient inverse to r^3, the attractive force between the segments of the streamlines decreases inversely $4\pi d^2\varepsilon_0$ as the lines move apart.

The forces $\vec{F}_v$ are directed against the velocity of the motion. They decrease by $4\pi d^2\varepsilon_0$ as the distance d between the lines increases. When θ increases from 0° to 90°, $\vec{F}_v$ decreases from its maximum value to zero. Here it is directed against the velocity of the charges. Then it changes sign and reaches its maximum at 180°.

If the velocities $\vec{v}_1$ and $\vec{v}_2$ are oppositely directed, then in our argument the sign simply changes. If l_1 and l_2 are isolated conductors of current, then they will repel each other. If they are currents of free electrons, then electrical forces come into play when the velocities change in this way. Instead of the difference of the velocities, the sums of the velocities appear here so that they are not equal to zero, and this assumption was made when considering all the examples.

Discussion

Let us assume that the surrounding system consists of a closed-circuit L' through which a direct current I' flows, and consider one of its current elements $I'd\vec{l}'$, which can be represented as a charge q' moving with a velocity $\vec{v}'$, under the condition $q' = I'dt$, $\vec{v}' = d\vec{l}'/dt$. Imagine a test charge q moving with velocity $\vec{v}$, which is also a current element $Id\vec{l} = q\vec{v}$. Then we obtain the Grassmann formula [12]:

$$d\vec{f} = \frac{I \cdot I'}{c^2 r^3} d\vec{l} \times (d\vec{l}' \times \vec{r}) = \frac{I \cdot I'}{c^2 r^3}\{(\vec{r} \cdot d\vec{l})d\vec{l}' - (d\vec{l} \cdot d\vec{l}')\vec{r}\} \quad (23)$$

If we substitute the value of the charge for the currents, we get

$$d\vec{f}_{21} = \frac{q_1 q_2}{c^2 r_{21}^3}\left\{\overbrace{\vec{v}_2(\vec{r}_{21} \cdot \vec{v}_1)}^{-3} - \overbrace{\vec{r}_{21}(\vec{v}_1 \cdot \vec{v}_2)}^{-1}\right\} \quad (23a)$$

The terms in (17a) are half of the first term and the second term in the magnetic part of (16) with reversed sign.

Marinov notes that the force $d\vec{f}$ with which the current element $I'd\vec{l}'$ acts on the current element $Id\vec{l}$ is neither equal nor opposite in sign to the force with which $Id\vec{l}$ acts on $I'd\vec{l}'$ [13]. This is contrary to Newton's third law. To eliminate this asymmetry of the Grassmann formula, Marinov assumed that the force with which $I'd\vec{l}'$ acts on $Id\vec{l}$ and the force with which $Id\vec{l}$ acts on $I'd\vec{l}'$ can be represented as follows:

$$d\vec{f} = (d\vec{f} - d\vec{f}')/2, d\vec{f}' = (d\vec{f}' - d\vec{f})/2$$

Then, instead of the Grassmann formula, the author obtains the Marinov formula:

$$d\vec{f} = \frac{I \cdot I'}{2c^2 r^3}\{d\vec{l} \times (d\vec{l}' \times \vec{r}) + d\vec{l}' \times (d\vec{l} \times \vec{r})\} = \frac{I \cdot I'}{c^2 r^3}\left\{\frac{1}{2}(\vec{r} \cdot d\vec{l}')d\vec{l} + \frac{1}{2}(\vec{r} \cdot d\vec{l})d\vec{l}' - (d\vec{l} \cdot d\vec{l}')\vec{r}\right\} \quad (24)$$

If we put the value of the charge instead of the currents, we get

$$d\vec{f}_{21} = \frac{q_1 q_2}{c^2 r_{21}^3}\frac{1}{2}\left\{\overbrace{\vec{v}_1(\vec{r}_{12} \cdot \vec{v}_2)}^{-2} + \overbrace{\vec{v}_2(\vec{r}_{12} \cdot \vec{v}_1)}^{-3} - \overbrace{2\vec{r}_{12}(\vec{v}_1 \cdot \vec{v}_2)}^{-1}\right\} \quad (24a)$$

Formula (24a) is the first, second and third terms in the magnetic part (16) with reversed sign.
Two other formulas are known for the force with which one current element acts on another current element. The Ampere formula, which was established in 1823-25 [14]:

$$d^2\vec{f} = \frac{I \cdot I'}{c^2 r^5}\{3(\vec{r} \cdot d\vec{l})(\vec{r} \cdot d\vec{l}') - 2(d\vec{l} \cdot d\vec{l}')r^2\}\vec{r} \quad (25)$$

and the Whittaker formula, obtained by him in 1910 [15]:

$$d\vec{f} = \frac{I \cdot I'}{c^2 r^3}\{(\vec{r} \cdot d\vec{l}')d\vec{l} + (\vec{r} \cdot d\vec{l})d\vec{l}' - (d\vec{l} \cdot d\vec{l}')\vec{r}\} \quad (26)$$

Substituting the value of the charge instead of the currents, we obtain

$$d\vec{f}_{21} = \frac{q_1 q_2}{c^2 r_{21}^3}\left\{\overbrace{\frac{3\vec{r}_{12}}{r_{21}^2}(\vec{r}_{12} \cdot \vec{v}_1)(\vec{r}_{12} \cdot \vec{v}_2)}^{4} - \overbrace{2\vec{r}_{12}(\vec{v}_1 \cdot \vec{v}_2)}^{-1}\right\} \quad (25a)$$

Formula (25a) is the first and fourth terms in the magnetic part of (16).

$$d\vec{f}_{21} = \frac{q_1 q_2}{c^2 r_{21}^3}\left\{\overbrace{\vec{v}_1(\vec{r}_{12} \cdot \vec{v}_2)}^{-2} + \overbrace{\vec{v}_2(\vec{r}_{12} \cdot \vec{v}_1)}^{-3} - \overbrace{(\vec{v}_1 \cdot \vec{v}_2)\vec{r}_{12}}^{-1}\right\} \quad (26a)$$

Formula (26a) is half of the first, second, and third terms in the magnetic part of (16).

Assis shows the derived force between two-point charges from field theory up to second order in v/c based on the work of Liénard, Wiechert and Schwarzschild, which was first obtained by O'Rahilly as [16, 17]

$$\vec{F}_{21} = q_1 \left\{ \frac{q_2}{4\pi\varepsilon_0 r^2} \left[\hat{r} \left(1 + \frac{\vec{v}_2 \cdot \vec{v}_2}{2c^2} - \frac{3}{2}\frac{(\hat{r}\cdot\vec{v}_2)^2}{c^2} - \frac{\vec{r}\cdot\vec{a}_2}{2c^2} \right) - \frac{r\vec{a}_2}{2c^2} \right] \right\} + q_1\vec{v}_1 \times \left\{ \frac{q_2}{4\pi\varepsilon_0} \frac{1}{r^2} \frac{\vec{v}_2 \times \hat{r}}{c^2} \right\} \tag{27}$$

In this formula q_1 is the test charge and q_2 is the source charge generating the electric and magnetic fields; $\hat{r} = \vec{r}/r$, and both $\vec{r}$ as well as $\hat{r}$ are pointing from q_2 to q_1. The velocity $\vec{v}_2$ and acceleration $\vec{a}_2$ denotes the acceleration of the point charge. If we introduce the notation used in our article, we obtain the following expression:

$$\vec{F}_{21} = \frac{q_1 q_2 \vec{r}_{21}}{4\pi\varepsilon_0 r_{21}^3} + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{21}^3 c^2} \left\{ \frac{1}{2} \left[\overbrace{\vec{r}_{21} v_2^2}^{5} - \overbrace{\frac{3\vec{r}_{21}}{r^2} (\vec{r}_{21} \cdot \vec{v}_2)^2}^{7} - \overbrace{\vec{r}_{21}(\vec{r}_{21} \cdot \vec{a}_2)}^{9} - \overbrace{r_{21}^2 \vec{a}_2}^{-10} - \overbrace{2\vec{r}_{21}(\vec{v}_1 \cdot \vec{v}_2)}^{-1} \right] + \overbrace{\vec{v}_2(\vec{r}_{21} \cdot \vec{v}_1)}^{-3} \right\} \tag{28a}$$

In O'Rahilly's formula, the last term in curly brackets is the third term of the magnetic forces (16) with the sign reversed. The last term in square brackets is the first term in magnetic force (16) with reversed sign. The first term in square brackets is the fifth term (the first term in square brackets of the electric forces) assuming that the velocity of the first charge is zero. This assumption is also valid for other forces represented by electric forces in (16).

The second term is the seventh term (the product of the coefficient in the third square bracket and the second term in parentheses). The third term is the ninth term (the first term in square brackets describing the accelerated motion). The fourth term is the tenth term (the second term in the same brackets in (16)).
Here is the final formula for the Weber force with which two charges act on each other:

$$\vec{F} = \frac{q_1 q_2 \vec{r}}{4\pi\varepsilon_0 r^3} \left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r\ddot{r}}{c^2} \right) \tag{29}$$

Here $\vec{r}$ is the radius connecting the charges, $\dot{r}$ is the rate of change of the distance between the charges, and $\ddot{r}$ is the acceleration of the change of the distance between the charges. If we introduce the following notation $\vec{r}_{21} = \vec{r}_2 - \vec{r}_1$, we get Weber's formula acting from the side of the second charge to the first in our notation:

$$\vec{F}_{21} = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{21}^3} \vec{r}_{21} + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{21}^3 c^2} \left(\overbrace{\vec{r}_{21}(\vec{r}_{21}(\vec{a}_1 - \vec{a}_2))}^{9} - \overbrace{\frac{1}{2}\vec{r}_{21}(\vec{v}_1 - \vec{v}_2)^2}^{-5} \right) \tag{29a}$$

Weber's formula is part of the electric forces. This is the first term in the last square bracket and the first term in the first square bracket of the electric forces in (16).

Conclusions

The formula for the generalized electrodynamic force is based on the hypothesis of the existence of a magnetic charge of the electron. It generalizes the force formula proposed by Neumann (p. 82), Grassmann, Ampère, Whittaker (p. 91), Weber, Lorenz [10, 12-15, 18, (2.1.11)]. It explains the cluster effect observed in accelerators, the experiments of G.V.Nikolaev, P. Graneau and others [19,20].

All of these forces were determined based on experiments, so in some cases there are variations in the constant coefficients and signs that determine the direction of the force.

Let us concentrate on the magnetic forces. They were all determined in an experiment with current-carrying wires, i.e., under conditions where the electromagnetic forces listed in (16) do not occur. Formula (16) is the result of solving Maxwell's equations. These solutions describe the electric and magnetic fields generated by moving electrons. Do these fields agree with the experiment? Let us check this with the example of the Lorentz force, which describes the force due to the field generated by the second electron. We have:

$$\vec{F} = q_2\vec{v}_2 \times \vec{B}(v_2) = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{21}^2 c^2} [\vec{v}_1 \times (\vec{r}_{21} \times \vec{v}_2)] = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{21}^2 c^2} [\vec{r}_{21}(\vec{v}_1 \cdot \vec{v}_2) - \vec{v}_2(\vec{v}_1 \vec{r}_{21})] \tag{30}$$

We have obtained two terms of the magnetic forces in (16), which agree with the Grassmann force and the magnetic part of the O'Reilly force. But (16) contains two more symmetric terms $\vec{r}_{21}(\vec{v}_1 \cdot \vec{v}_2) - \vec{v}_1(\vec{v}_2 \cdot \vec{r}_{21})$. In (16) this often looks like $2\vec{r}_{21}(\vec{v}_1 - \vec{v}_2) - \vec{v}_2(\vec{v}_1 \cdot \vec{r}_{21}) - \vec{v}_1(\vec{v}_2 \cdot \vec{r}_{21})$.
Whittaker's formula is very close to this form, but instead of $2\vec{r}_{21}(\vec{v}_1 \cdot \vec{v}_2)$ it contains only $\vec{r}_{21}(\vec{v}_1 \cdot \vec{v}_2)$. In answering the question of the reason for the loss of one, as in Whittaker, or even two terms, as in Grassmann and O'Reilly, we come to the problem of describing electric and magnetic fields at the present time.

in which the first is moving, we already know from experiment. Formula (16) describes the interaction of two fields generated by two charges.

Let us look at the magnetic forces in (16) in a little more detail. They are two radial terms, the second of which corresponds to the Ampère force. Radial forces, of course, satisfy Newton's third law. But velocity forces generally do not satisfy this law, but the law of moment of forces [2, 10].

The authors plan to concentrate their future efforts on the study of electromagnetic forces, since they have apparently not yet been considered in electrodynamics.

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