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On a form of Einstein's Equation in Relation to Quantum Mechanics

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Introduction

Establishing a physical theory accounting for the existence of particles seems to be a very ambitious project. However, we must remain hopeful that one day this wishful thinking could come true, as Louis de Broglie pointed out in his work "New Perspectives in Microphysics" dating from 1955:

"If, at the cost of an effort which would certainly be long and difficult, we managed to extend generalized relativity so as to bring the u waves of the various kinds of particles into the framework of space-time, we could establish the form non-linear equations satisfied by u waves, study what happens in singular regions and manage to understand the true nature of these spatiotemporal accidents which are the corpuscles and also the deep meaning of the quantum of action which is certainly linked in an essential way to the granular and wave structure of matter and radiation. We would thus obtain (this is not yet for tomorrow!) a magnificent synthesis of the conceptions of generalized Relativity and Quanta theory. »

The present essay contributes a small stone to the edifice planned by the eminent researcher by showing a possible path to the unification of Quantum Mechanics and General Relativity by positing an appropriate relativistic field equation intrinsically linking the geometry of space -time to the kinematics of the movement of matter.

Global Field Equation

Space-time is Riemannian in nature by signature (+ - - -) corresponding to Cartesian coordinates ($t = x^0, x = x^1, y = x^2, z = x^3$).

We pose as a fundamental postulate of the theory a relation linking the spatiotemporal curvatures to the four-acceleration at each point in the Universe:

$$\frac{R_{ij}}{R} = \frac{\gamma_i \gamma_j}{\gamma^k \gamma_k} \qquad \text{(I)}$$

where R_{ij}, R and γ_i designate respectively the Ricci tensor, its contraction and the four-acceleration at the universe point considered.

The relation (I) consists of 9 independent equations for 9 unknowns which are: 6 independent components of the metric tensor gij due to the free choice of the landmark and, taking into account the relation $\gamma_i = u^j(u_{i,j} - u_{j,i})$, 3 independent components of the four-speed unit $u_i = dx_i / ds$ where $ds = \sqrt{dx^i dx_i}$.

We refer to a local landmark. Let's ask:

$$\mu_i = \gamma_i \sqrt{\frac{R}{\gamma^j \gamma_j}} \qquad \text{(II)}$$

(Components μ_i are real or pure imaginary depending on the sign under the radical), then:

$$R_{ij} = \mu_i \mu_j \qquad \text{(III)}$$

$T_{ij} = \dfrac{1}{\chi}(\mu_i \mu_j - \dfrac{1}{2}\delta_{ij}\mu^k \mu_k)$ constitutes the Einstein energy-momentum tensor.

Where χ denotes Einstein's gravitational constant and

$$\delta_{ij} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Its divergence is zero: $T^j_{i,j} = 0$, expressing the local conservation of energy-momentum, in particular for $i = 0$:

$$\frac{1}{2}\left[(\mu_0)^2 + (\mu_1)^2 + (\mu_2)^2 + (\mu_3)^2\right]_{,0} + \chi T^{\alpha}_{0,\alpha} = 0 \quad \text{with } \alpha = 1 \text{ to } 3$$

And by integration over the entire volume of a free particle at the limits of which T^{α}_{0} cancels:

$$\int_v \frac{1}{2}\left[(\mu_0)^2 + (\mu_1)^2 + (\mu_2)^2 + (\mu_3)^2\right] dV = \text{constant} = \chi E \qquad \text{(IV)}$$

Half of the strictly positive quadratic form in square brackets (negative if μ_i is pure imaginary →antimatter?), summed over all physical space, is the expression of the total energy of the free particle.

The relationship (IV) shows that it is preserved over time. Furthermore, we check:

$$T^j_{i,j} = 0 \Leftrightarrow \mu_i \mu^j_{,j} + \mu^j(\mu_{i,j} - \mu_{j,i}) = 0$$
$$\Leftrightarrow$$
$$\mu^j(\mu_{i,j} - \mu_{j,i}) = 0 \quad \text{(V-a) and } \mu^j_{,j} = 0 \quad \text{if } \mu^i\mu_i \neq 0 \quad \text{(V-b)}$$

If the fundamental relation (I) is physically correct, the relations (V-a-b), joined to a normalization relation $\int_v \mu_0 dV = \chi hc$ (V-c), resulting from the divergence of the four-vector μ^i (article ref. [13] of the bibliography), where h,c designate respectively Planck's constant and the celerity of light, appear in the theory as the constitutive equations of the particles.

This remains to be proven by calculation. Solving equations (V-a-b) requires the translation of Cartesian spatial operators into spherical coordinates. This work will be undertaken following this article.

Let us first show how such relationships are compatible with Quantum Mechanics by highlighting the path which leads in theory to the Klein-Gordon equation for the free particle.

Klein-Gordon Equation

Quantum mechanics considers particles as point objects in physical space. We know that this a priori is at the origin of numerous difficulties in the development of Quantum Field Mechanics with the problem of divergences.

Despite considerable improvements to the theory in recent years (renormalization), it still does not constitute a general theory of physics due, in particular, to the fact that it does not account for the values of the charge and the mass of the electron.

However, there is no doubt that the particle is an extremely small material object in physical space.

Thus, within the framework of our theory, if our goal is not to describe the internal dynamics of the particle, we have the possibility of assimilating it to a point in the field of μ_i where all its mass energy is concentrated there, animated by a speed u_i. We know that the proper mass of a free particle is expressed by (article ref. [13] of the bibliography):

$$m = -\frac{1}{2\chi c^2}\int_v \mu^i \mu_i \, dV$$

So, we can ensure that the field of μ_i satisfy the relation (V-a), by positing:

$$\mu_{i,j} - \mu_{j,i} = \frac{mc}{\hbar}\varepsilon_{ijkl}u^k\mu^l \Leftrightarrow \mu^{i,j} - \mu^{j,i} = \frac{mc}{\hbar}\varepsilon^{ijkl}u_k\mu_l \quad \text{(VI)}$$

Note: $\varepsilon^{ijkl} = (-1)^q \delta^{im}\delta^{jn}\delta^{ko}\delta^{lp}\varepsilon_{mnop}$ where q is the number of signs – of the signature of the space. Here $q = 3$.

By following derivation j, taking into account equation (V-b):

$$\Box\mu_i = \frac{mc}{\hbar}\varepsilon_{ijkl}u^k\mu^{l,j} + \frac{mc}{\hbar}\varepsilon_{ijkl}u^{k,j}\mu^l$$

However, having regard to [II]:

$$\frac{mc}{\hbar}\varepsilon_{ijkl}u^{k,j}\mu^l = \frac{mc}{\hbar}\varepsilon_{ijkl}s^{,j}\gamma^k\mu^l = 0$$

As a result:

$$\Box\mu_i = \frac{1}{2}\frac{mc}{\hbar}\varepsilon_{ijkl}u^k(\mu^{l,j} - \mu^{j,l})$$

Taking into account [VI]:

$$\Box\mu_i = \frac{1}{2}\frac{m^2c^2}{\hbar^2}\varepsilon_{ijkl}\varepsilon^{ljmn}u^k u_m \mu_n = -\frac{1}{2}\frac{m^2c^2}{\hbar^2}\varepsilon_{ljki}\varepsilon^{ljmn}u^k u_m \mu_n$$

$$\Box\mu_i = -\frac{m^2c^2}{\hbar^2}\left(\delta^m_k\delta^n_i - \delta^n_k\delta^m_i\right)u^k u_m \mu_n$$

where: $u_i u^i = 1$ and $u_k \mu^k = 0$

This results in the Klein-Gordon equation in the case of the free particle:

$$\Box\mu_i + \frac{m^2c^2}{\hbar^2}\mu_i = 0 \qquad \text{(VII)}$$

with the particularity that the wave function is a four-vector with real components satisfying the normalization relation (IV), which can be written here:

$$\int_v \frac{(\mu_0)^2 + (\mu_1)^2 + (\mu_2)^2 + (\mu_3)^2}{2\chi E}dV = \int_v \rho dV = 1$$

Or ρ appears in the framework of QM as a density of probability of presence of the particle.

These few calculations show a possibility of unifying quantum theories and General Relativity based on a single hypothesis: the relation (I).

Note that, since $\mu^i\mu_i \simeq 0$, everywhere around the particle considered as point where all its energy is concentrated there, the previous relation is written, taking into account the normalization relation (V-c) and the Planck-Einstein relation $E = h\nu$:

$$\int_v \frac{(\mu_0)^2}{\chi E}dV = \frac{\chi hc\bar{\mu}_0}{\chi h\nu} = 1 \Rightarrow c\bar{\mu}_0 = \nu$$

Or $\bar{\mu}_0$ has the value of the component μ_0 at the location of the particle whose frequency of the associated wave is **ν**.

Conclusion

The relations (V-b) and (VI) constitute an alternative method to that using Paul Dirac's bispinors to account for the Klein-Gordon equation. It can be the basis of a complete description of particles in a unified, but approximate theory. Indeed, we show that the Klein-Gordon equation is only valid in the case of a needle representation of the particle. It is therefore the fact that the particle is quasi-point which ensures the relevance of the Klein-Gordon equation and which makes it possible to confuse the energy-momentum density of the particle with a density of something virtual: the probability of presence of the particle. Indeed, Quantum Mechanics considers the particle as an object where all the energy is concentrated at one point. Ignoring the internal dynamics of the particle, Quantum Mechanics is therefore an incomplete theory. The 4 first order differential equations (V-a-b), 3 of which are non-linear, are here the basis which could be that of an exact approach to a unified theory. It is remarkable to note that nonlinear equations of General Relativity are capable of accounting for a linear equation of Quantum Mechanics which is its foundation [1-13].

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