ISSN: 2754-4753

Journal of Physics & Optics Sciences

SCIENTIFIC Research and Community

Short Communication

Open Access

Numerical Simulations for the Gross-Pitaevskii Equation

Taylanov NA[1] and Urinov SX[2]

[1]Jizzakh State Pedagogical University, Jizzakh, Uzbekistan

[2]Samarkand State University, Samarkand, Uzbekistan

ABSTRACT

In this study the split-step Fourier method for the numerical simulation of the Gross-Pitaevskii equation. Approximate numerical solutions of the Gross-Pitaevskii equation are obtained by using Matlab software. It is shown that the proposed method improves the computational effort significantly. This improvement becomes more significant especially for large time evolutions.

***Corresponding author**

Taylanov NA, Jizzakh State Pedagogical University, Jizzakh, Uzbekistan

Received: March 19, 2023; **Accepted:** March 23, 2023; **Published:** April 10, 2023

Keywords: Gross-Pitaevskii Equation, Split Step Method, Splitting, Partial Differential Equations, Approximate Numerical Solutions

Introduction

The Gross–Pitaevskii equation (GPE) has successfully described equilibrium Bose–Einstein condensates (BECs), including density profiles, vortex formation and hydrodynamics, quantum computing and quantum turbulence, cosmological phenomena such as black holes in condensates [1, 2]. The basis of the mathematical formulation of the problem of propagation of the Bose-Einstein condensate is the Gross-Pitaevskii (GP) equation [3]. Many techniques can be used in simulation of Gross-Pitaevskii equation: the Crank-Nicholson scheme, the hopscotch method, the pseudo–spectral split-step method, the Hamiltonian preserving method, and many others (see [4, 5]). One common numerical method for solving the GPE is to use the time-splitting spectral method [6]. In this article, we present a generalized finite-difference time-domain (G-FDTD) scheme, which is explicit, stable, and permits an accurate solution with simple computation for solving the above multi-dimensional dGPE [7, 8]. The idea of the GFDTD method is to first split the function $\psi(x, t)$ into real and imaginary components, resulting in two coupled equations.

Basic Equations

Consider the dGPE in two dimensions as follows:

$$i\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi + g|\psi|^2\psi, \qquad (1)$$

where $\hbar^2/2m = 1.0,\ g = 1.0,\ V = 0.5(k_x^2 + k_y^2)$,

and the initial condition was chosen to be

$$\psi(x,y,0) = \frac{2}{\sqrt{\pi}}(x + iy)\exp\left(-(x^2 + y^2)\right),\ -\infty < x, y < +\infty. \qquad (2)$$

The numerical solution was defined in the interval $-20 \le x, y \le 20$ and for the number of grid points in both x and y to be 200 with $\Delta t = 0.001$. In our computation, we chose three different meshes of 200×200, 300×300, and 400×400. Both numerical solutions along the x-axis (the numerical solutions along the y-axis are similar due to symmetry) at various times in $0 \le t \le 20$ were plotted in Figure 1 and Figure 2 shows the simulation of non-equilibrium Bose–Einstein condensation at various times in $0 \le t \le 20$. It can be seen from the figure that the non-equilibrium BEC has reached steady state by $t = 20$.

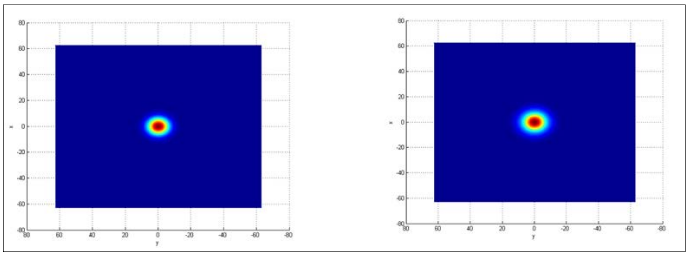

Figure 1: Simulation of Steady State Non-Equilibrium Bose–Einstein Condensation, where the G-FDTD Scheme was Employed with at (a) t = 1, (b) t =5.

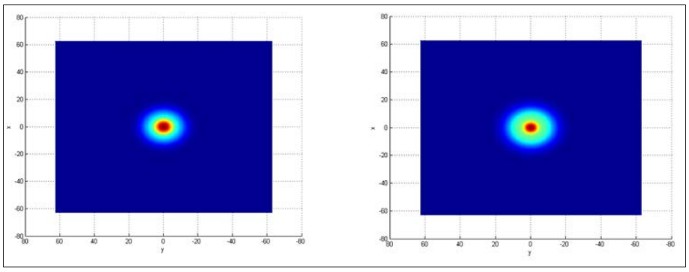

Figure 2: Simulation of Steady State Non-Equilibrium Bose–Einstein Condensation, where the G-FDTD Scheme was Employed with at (a) t = 10, (b) t= 20.

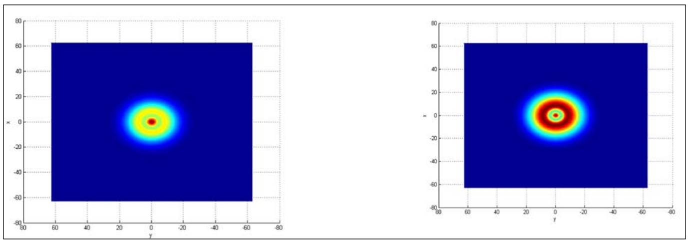

Figure 3: Simulation of Steady State Non-Equilibrium Bose–Einstein Condensation, where the G-FDTD Scheme was Employed with at (a) t = 25, (b) t=30.

According to the results presented in these figures, the present method offers high accuracy for the numerical solutions of the Gross-Pitaevskii equation. In the other hand, as can be seen from figures, a result obtained by the implicit exponential finite difference scheme has better than results obtained from the other numerical schemes.

Conclusion

In this study the split-step Fourier method for the numerical simulation of the Gross-Pitaevskii equation. Approximate numerical solutions of the Gross-Pitaevskii equation are obtained by using Matlab software. The applied here scheme can be used as an efficient tool in computational mathematics, namely in a class of nonlinear differential equations, which describe the theoretical quantum physics and engineering problems.

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