ISSN: 2754-4753

Journal of Physics & Optics Sciences

SCIENTIFIC Research and Community

Research Article

Open Access

Newton's Second Law in Special Theory of Relativity

Golovkin Boris Georgievich

The Public Institute of Natural and Human Sciences, Yekaterinburg, Russia

ABSTRACT

A formula for the second law is proposed, taking into account SRT and the delay of relativistic effects, according to which the acceleration of the body will decrease with an increase in the speed of the body, and at some critical speed ω≈235696.8871 *km/s* it will be equal to zero. When the speed of the body is greater than the critical mass of the body becomes negative.

***Corresponding author**

Golovkin Boris Georgievich, Researcher at the Public Institute of Natural and Human Sciences, Yekaterinburg, Russia; Tel: +79126414746.
E-mail: gbg1940@mail.ru

Received: October 06, 2022; **Accepted:** October 13, 2022; **Published:** October 20, 2022

Keywords: Critical Velocity, Einstein's Postulates, Newton's Second Law, Relativistic Effects, Negative Mass

Introduction

It has been experimentally established that accelerations in the motion of bodies are *proportional to the acting forces*:

$$a \bowtie F, \qquad (1)$$

and the directions of the resulting accelerations with the direction of the acting forces, and, at the same time, *the accelerations acquired by the bodies under the action of a given external force, are inversely proportional to the masses of the bodies*:

$$a \bowtie \frac{1}{m}. \qquad (2)$$

If we combine these two dependencies and agree on the units of all quantities, then Newton's second law will be obtained in the form:

$$\mathbf{F} = m\mathbf{a}. \qquad (3)$$

The force $\mathbf{F}=\sum \mathbf{F}_k$ is the resultant of the forces $\mathbf{F}_k$, acting on a body of mass m. Since the acceleration is determined by the ratio

$$\mathbf{a} = \frac{\mathbf{v_2}-\mathbf{v_1}}{t_2-\mathbf{t_1}}, \qquad (4)$$

then, substituting (4) into (3), we find the generally accepted formula for Newton's second law

$$\mathbf{F} = \frac{\mathbf{mv_2}-\mathbf{mv_1}}{t_2-\mathbf{t_1}} = \frac{\Delta(\mathrm{mv})}{\Delta t} \to \frac{d}{dt}(m\mathbf{v}) = m\mathbf{a}. \qquad (5)$$

Since in formula (5) no restrictions are imposed on the acceleration of the body, the speed of the body can also take on arbitrarily large values.

The discovery of the Special Theory of Relativity (SRT) required the appropriate refinement of the law [1].

The purpose of this work is to propose a formula for Newton's second law that takes into account SRT.

Newton's Second Law, Taking into Account Changes in Mass and Speed

According to the work of R.C. Tolman, if force is defined as the rate of increase in momentum, then equation (5) allows for changes in both mass and velocity [2]:

$$\mathbf{F} = \frac{d}{dt}(m\mathbf{v}) = m\frac{d\mathbf{v}}{dt} + \frac{dm}{dt}\mathbf{v}, \qquad (6)$$

Where the mass *m* of a moving body depends on its speed and rest mass m_0

$$m = m(v) = m_v = \frac{m_0}{\sqrt{1-v^2/c^2}}. \qquad (7)$$

After substituting (7) into (6), we obtain

$$\mathbf{F} = \frac{m_0}{\sqrt{1-v^2/c^2}}\frac{d\mathbf{v}}{dt} + \frac{d}{dt}\left(\frac{m_0}{\sqrt{1-v^2/c^2}}\right)\mathbf{v} = \frac{m_0}{\sqrt{1-v^2/c^2}}\frac{d\mathbf{v}}{dt} + \frac{\mathbf{v}}{c^2}\frac{d}{dt}\frac{m_0c^2}{\sqrt{1-v^2/c^2}} \qquad (8)$$

From the consideration of equations (6) and (8) it can be seen that the force acting on the body is equal to the sum of two vectors, one of which is directed towards the acceleration $\frac{d\mathbf{v}}{dt}$, and the other in the direction of the existing speed **v**,so that, in general, the force and the acceleration it produces do not act in the same direction.

In the case, important for further analysis, when acceleration, velocity and its change have the same direction, equality (8) can be written as

$$F = F_v + F_m = \frac{m_0}{\sqrt{1-v^2/c^2}}\frac{dv}{dt} + \frac{v}{c^2}\frac{d}{dt}\frac{m_0c^2}{\sqrt{1-v^2/c^2}} . \quad (9)$$

The terms of equality (9) can be represented as:

$$F_v = m_v \cdot \frac{dv}{dt} = m_v a = \frac{m_0 a}{\sqrt{1-v^2/c^2}} \quad (10)$$

$$F_m = v\frac{dm}{dt} = v\frac{d\frac{m_0}{\sqrt{1-v^2/c^2}}}{dt} = \frac{m_0v^2/c^2}{(1-v^2/c^2)^{1.5}} \cdot \frac{dv}{dt} = m_0 a\frac{v^2/c^2}{(1-v^2/c^2)^{1.5}} = m_v a\frac{v^2}{c^2-v^2} , \quad (11)$$

Where

$$a = a_0 f(v) , \quad (12)$$

$f(v)$ is a normalization function. Since the value a in equalities (10 - 12) plays the role of the acceleration of the body both at the initial moment of the action of the force at (moment of breakaway), and at any other moment when the speed of the body is already equal to v, and the breakaway refers already to this moment, then the acceleration a is essentially a constant. Therefore, the normalization function can be defined as:

$$f(v) = 1 , \quad (13)$$

Then

$$a = a_0 . \quad (14)$$

Substituting (7, 10 - 14) into (9), we get a convenient version of Newton's second law, taking into account changes in ace and speed

$$F = m_v a_0\left(1 + \frac{v^2}{c^2-v^2}\right) = m_v a_v , \quad (15)$$

Where

$$a_v = a_0\left(1 + \frac{v^2}{c^2-v^2}\right) = \frac{a_0}{1-v^2/c^2} \quad (16)$$

With an increase in the speed of the body, its acceleration under the action of the same force takes on even greater values, and when the speed of the body reaches the speed of light, it becomes infinite. If this conclusion were correct, then the speed of the body, under the influence of the corresponding forces, would become greater than the speed of light, which contradicts the postulates of Einstein and the fact that, in accordance with formula (7), the mass of the body becomes imaginary [1].

Newton's Second Law, Taking into Account Changes in Mass, Speed, Time Rate and Body Size

From SRT it is known that moving bodies experience not only changes in mass and speed, but also a reduction in their length l in the direction of movement, as well as the rate of time $\tau = \Delta t$. If these effects are taken into account, then equality (9) should be written:

$$F = F_v + F_m + F_l + F_\tau , \quad (17)$$

Equality (17) is consistent with the conclusion of E.L. Feinberg, which allows us to consider relativistic changes in the length and time scales as a result of the action of certain forces [3]. Naturally, these forces are part of the force F F that acts on the body. Considering that in SRT

$$l = l_0\sqrt{1 - v^2/c^2} , \quad (18)$$

We can write the equation:

$$F_l = -v\frac{m_0}{l_0} \cdot \frac{dl}{dt} = -v\frac{m_0}{l_0} \cdot \frac{dl_0\sqrt{1-v^2/c^2}}{dt} = -\frac{m_0 d\sqrt{1-v^2/c^2}}{dv} \cdot \frac{dv}{dt} = -m_0 a \cdot \frac{v^2}{c^2\sqrt{1-v^2/c^2}} . \quad (19)$$

The coefficient $-v\frac{m_0}{l_0}$ is selected so that the entire expression for F_l has the dimension of force, and the minus sign indicates that the length of the body decreases rather than increases with an increase in its speed.

Arguing similarly, given that the rate of time $\tau = \Delta t$ in SRT slows down in a moving body

$$\tau = \frac{\tau_0}{\sqrt{1-v^2/c^2}} \;, \qquad (20)$$

Find

$$F_\tau = -\frac{m_0 v}{\tau_0}\cdot\frac{d\tau}{dt} = -m_0 a\cdot\frac{v^2/c^2}{(1-v^2/c^2)^{1.5}} \;. \qquad (21)$$

The minus sign in formula (21) is explained by the fact that the rate of time of a moving body slows down rather than accelerates, i.e. the amount of time, as it were, "accumulated by the body" during movement becomes smaller, in contrast to the mass, the amount of which increases with increasing body speed.

Substituting (7, 10 - 14, 19, 21) into equality (17), we obtain the formula for Newton's second law in the form

$$F = m_0 a_0\sqrt{1-v^2/c^2} = \frac{m_0}{\sqrt{1-v^2/c^2}}a_0\left(1-\frac{v^2}{c^2}\right) = m_v a_v \;, \qquad (22)$$

Where

$$a_v = a_0\left(1-\frac{v^2}{c^2}\right) \qquad (23)$$

Formula (23) says that the greater the speed of the body, the smaller the acceleration that the body receives from the force *F*, becomes, and when the body reaches a speed equal to the speed of light, the acceleration of the body generally becomes equal to zero, i.e. the influence of the force *F* does not allow increasing the speed of the body above the speed of light. This solution explains the impossibility of moving at a speed greater than the speed of light by the fact that regardless of the magnitude of the applied force to a body moving at the speed of light, the acceleration of the body turns out to be zero, i.e. the speed of a body cannot exceed the speed of light.

Newton's Second Law Taking into Account SRT and Delay of Relativistic Effects

According to the work of S.I. Syrovatsky, relativistic effects are delayed, i.e. first there is some change in the speed of the object, and then the corresponding consequences appear: changes in the mass, length and rate of time of the moving object [4]. In accordance with A. Einstein's principle of relativity, among inertial reference frames there is no privileged frame of reference and it is impossible to detect the state of absolute motion. Therefore, anybody can be chosen as resting, so that at the initial moment of interaction of the body with an object that exerts a force on the body, its speed v=0, mass m_v=m_0 and acceleration a =a_0.Substituting these values of the parameters of the body at rest into (10), we obtain

$$F_v = m_v\cdot\frac{dv}{dt} = m_v a = m_0 a_0 = F_0 \;. \qquad (24)$$

Then equation (17), taking into account (24) and the conclusion of S.I. Syrovatsky can be written:

$$F = F_0 + F_m + F_l + F_\tau \;. \qquad (25)$$

Indeed, the object, as a source of force, begins to influence the body even when it is at rest, and the change in the mass, length and rate of time of the body occurs somewhat later, namely when the body has already acquired a certain minimum speed and began to move. Therefore, the forces expended to change the mass, length and rate of time of the body already correlate to this minimum speed, and therefore formulas (11), (19) and (21) must remain the same.

Substituting equalities (11 - 14, 19, 21, 24) into (25), we get:

$$F = m_0 a_0\left(1-\frac{v^2}{+c^2\sqrt{1-v^2/c^2}}\right) = \frac{m_0}{+\sqrt{1-v^2/c^2}}\cdot a_0\left(+\sqrt{1-v^2/c^2}-v^2/c^2\right) = m_v a_v, \qquad (26)$$

where

$$a_v = a_0\left(+\sqrt{1 - v^2/c^2} - v^2/c^2\right). \qquad (27)$$

The "+" sign in formulas (26, 27) indicates that the mass of the body and its acceleration are positive values. From formula (27) it can be seen that the acceleration of the body will decrease with increasing body speed, and at some critical speed ω will be equal to zero. To find ω, we solve the equation

$$\sqrt{1 - v^2/c^2} - v^2/c^2 = 0 \qquad (28)$$

$$v^4 + c^2v^2 - c^4 = 0, \qquad (29)$$

Where do we get:

$$v = \omega = c\sqrt{\frac{\sqrt{5}-1}{2}} \approx 0{,}7862c \approx 235696.8871\ km/s\,. \qquad (30)$$

The square of the critical speed ω is the golden ratio of the squares of speeds that do not exceed the speed of light, i.e. he divides the range of squares of these speeds in the extreme and average ratio into two parts so that the boundary of this section is the geometric mean between these parts. However, in most literary sources, the reciprocal value of Φ=1.618034, is taken as the value of the golden section, which is usually denoted by the first letter of the name of the ancient Greek architect Phidias [5].

In Table. Figure 1 shows the characteristics of a body moving with acceleration, calculated by formal formulas (7, 27), as well as the values of the mass defect $\Delta m_r = m_r - m_0$, acceleration a_r, and mass m_r, which the body should have as a result of applying the reinterpretation principle, which is marked with the index r [6].

Table 1: Characteristics of a moving body under the action of force F (26)

v	m (7)	m_r	Δmr	av (27)	a_r
0	m_0	m_0	0	a_0	a_0
0.5c	1.155m0	1.155m_0	0,155a_0	0,616a_0	0,616a_0
0.75c	1.512m_0	1.512m_0	0,512a_0	0,099a_0	0,099a_0
ω	Φm_0	±Φm_0	+0.618m_0 -0.718m_0	0	0
0.8c	1.666m_0	–1.666m_0	– 2.666m_0	–0.040a_0	0.040a_0
0.9c	2.294m_0	– 2.294m_0	– 3.294m_0	– 0.374a_0	0.374a_0
$\sqrt{2(\sqrt{2}-1)}c$	2.385m_0	-2.385m_0	-3.385m_0	– (√2-1) a_0	(√2-1) a_0
c	∞	-∞	-∞	– a_0	a_0

When the object's speed is greater than the critical one, the acceleration of the body, defined by formula (27), formally becomes negative. Negative acceleration can be understood in three senses:

1. As a decrease in the absolute value of the acceleration of the movement of the body relative to the acceleration that was recorded at some point in time. If the acceleration of the body a_2 has become less than a_1, which was before this moment, then the difference between these accelerations a_2-a_1=-a_{21}, could be considered a negative acceleration.
2. As a deceleration, a decrease in the absolute value of the speed of the body, when the speed of the body begins to decrease relative to the speed that was recorded at some point in time.
3. Like a change in the sign of the direction of movement of the body, when the body from a certain moment begins to suddenly move in the opposite direction.

In our case, the absolute value of both the acceleration a_v, and the speed of the body at при $v > \omega$ (Table 1) increase rather than decrease. Therefore, the first two interpretations of negative acceleration are not acceptable. The third option remains, when the sign of the direction of motion of the body changes. However, in this case, formula (26) is violated, namely, the right side of the formula becomes negative, while the left side remains positive, since the direction of the acting force does not change. This is possible only if the mass of the body also becomes negative.

Discussion of the Results

In (7) J.P. Terletsky proposed to call particles with negative mass negatons. As shown in the work of the author, the negativity of acceleration for negatons means that they, in accordance with the principle of equivalence of the inertial and gravitational masses of Einstein [7]

$$m_i = m_a = m_p , \quad (31)$$

(*mi*, *ma*, *mp*- Inert, active and passive gravitational masses, respectively) must move in the opposite direction to the force. However, in the same paper, a new equivalence principle generalized for the case of particles with negative mass was substantiated [8]:

$$|m_i| = |m_a| = |m_p| . \quad (32)$$

Formula (3), taking into account (32), should be written as:

$$\mathbf{F} = |m_i|\mathbf{a} . \quad (33)$$

From formulas (32) and (33) it follows that the direction of particle acceleration, regardless of the sign of the gravitational charge of the mass of this particle, is always directed towards the action of the corresponding force. Therefore, the sign of particle acceleration is always positive, and the law of universal gravitation, taking into account (32), will look like:

$$F = |m_i|a = -G\frac{m_p m_a}{r^2} . \quad (34)$$

Particles with negative mass, obeying the equivalence principle (32), are proposed to be called negaons [8]. Thus, as soon as the speed of the particle becomes greater than the critical one, the mass of the particle becomes negative, although its speed is directed towards the acting force. Then, in order for the initial equality (3) to hold, it follows that the acceleration of the body must become positive. Such a change in the signs of mass and acceleration corresponds to the principle of reinterpretation, which allows changing the signs of the characteristics of an object, if this makes it possible to more rationally determine the state and essence of the object (for example, the Stückelberg-Feynman interpretation of positrons as electrons with negative energy moving backward in time) [6]. In the third and seventh columns of Table 1 shows the values of the mass m_r and the acceleration of the body a_r as a result of the reinterpretation.

The existence of substances with negative mass does not contradict any fundamental physical laws. It is easy to see that the law of conservation of mass, the law of conservation of momentum, and the law of conservation of energy are satisfied [9]. Therefore, there are no logical prohibitions on their existence. Moreover, by now, a mechanism has been developed for the appearance of mass in fundamental particles. It is based on their interaction with the space-filling homogeneous *Higgs field*, according to which fermions acquire masses [10]:

$$m_f = \frac{\xi \cdot \varphi_0}{c^2} , \quad (35)$$

where φ_0 is the strength of the Higgs field, ξ is the coupling constant of the corresponding fermion with this field, c is the speed of light. In the case of a real Higgs field, only two points of energy minimum $\varphi_0=+a$ and $\varphi_0= - a$ are possible [11]. Together with (35), these value of φ_0 give us grounds to assume the existence of matter with a negative mass.

From all of the above, it follows that the appearance of Newton's second law (formulas 15, 22, 26) is formally the same, although the dependences of the acceleration of the body on their speed at the current moment are different. The most realistic in the pragmatic sense is an alternative solution based on the conclusion of E.L. Feinberg, which requires taking into account changes in the rate of time and body size. The proposed alternative theory makes it possible to detect the hypothetical possibility of the existence of bodies in a state with a negative mass, or at least some anomaly in the behavior of the body when the critical speed is reached ω≈235696.8871 *km/s*. Unfortunately, at the Large Hadron Collider, no attempts were made to detect in the products of the acceleration of particles of matter to velocities exceeding this value, and there are no special signs of the transition of the matter of these particles to a state with a negative mass in the literature.

Conclusions

1. It is shown that Newton's second law, which takes into account only changes in the mass and velocity of a body, contradicts Einstein's postulates.
2. A formula of Newton's second law is proposed, taking into account the change in mass, speed, time rate and dimensions of the body, explaining the impossibility of moving at a speed greater than the speed of light, since, regardless of the magnitude of the applied force to a body moving at the speed of light, the acceleration of the body turns out to be zero.
3. A formula for the second law is proposed, taking into account SRT and the delay of relativistic effects, according to which the acceleration of the body will decrease with an increase in the speed of the body, and at some critical speed ω≈235696.8871 km/s it will be equal to zero.
4. When the speed of the body is greater than the critical mass of the body becomes negative.

References

1. Golovkin BG (2022) Proof of Einstein`s postulates. Ann Math Phis 5: 13-20.
2. Richard C Tolman (1911) XLIV. Non-Newtonian mechanics:—The direction of force and acceleration. The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science 22: 458-463.
3. E.L. Feinberg (1978) Is it possible to consider the relativistic change in the scales of length and time as a result of the action of some forces?. Einstein collection (1975-1976). M. Science 43-77.
4. S.I. Syrovatsky (1978) On the question of the "delay" of the relativistic contraction of moving bodies. Einstein collection (1975-1976). M. Science 78-81.
5. Fernando Corbalan (2010) Golden section. Mathematical language of beauty. M. Deagostini. ISBN 978-5-9774-0680-2. Fernando Corbalan (2013) Golden Ratio The mathematical language of beauty. M.: De Agostini 160.
6. Bilaniuk OM, ECG Sudarshan (1969) Particles beyond the Light Barrier. Physics Today 22: 43. Bilanyuk O, Sudarshan E (1974) Particles behind the light barrier. Einstein collection. M. Science 112-133.
7. Terletsky JP (1981) Negative Masses and the Universe. Experimental technique of physics. B 29: 331-332.
8. Golovkin BG (2019) New Particle Mechanics with Negative Mass. World Scientific News 126: 283-290.
9. Golovkin BG (2019) Negative mass is a component of the Universe. Noosphere. Society. Human 1. URL: noocivil.esrae.ru/256-1797.
10. Ishkhanov BS, Kapitonov IM, Yudin NP (2019) Particles and atomic nuclei. M.: LENAND 551: 672.
11. Bogush AA (1987) Introduction to gauge field theory of electroweak interactions. Minsk. Science and technology 185: 359.