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Motion and Radiation for Charged Particles Revolving Round Centre of Force of Attraction

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ABSTRACT

Nuclear and non-nuclear models of hydrogen atom are proposed, each with *N* equal to *42* or *43* orbits of revolution. Aberration of electric field is invoked to describe radiation from nuclear model with particle of electronic charge *-e* and mass *nm* of electronic mass *m*, revolving in *nth* orbit, with angular momentum *nL*, at radius *nr1* from a nucleus of charge *+Ne* and mass (*Nm*/*2*)(1 + *N*), where *n* = 1, 2, 3...*N*, and *r1* is radius of first orbit. The revolving particles have total charge *-Ne* and same total mass as the nucleus. The non-nuclear model has two particles of same mass *nm* and charges *+e* and *-e*, revolving diametrically in *nth* orbit, at radius *nr1* from a centre, with particles of total mass *Nm*(1 + *N*). Distinct masses *nm* gives discrete coplanar circular orbits. A particle dislodged from a circular orbit, revolves in an elliptic path with emission of radiation, at the frequency of revolution, in fine structure, before reverting to a stable circular orbit. The nuclear atom is identified with solid or liquid state of hydrogen and non-nuclear atom with gaseous state.

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Introduction

Revolution of a body, round a centre of force of attraction, is the most common motion in the universe [1]. This comes with revolution of binary stars round their centre of mass, the planets round the Sun, or an electron round the nucleus of an atom [2, 3]. Two bodies under mutual attraction, revolve round their centre of mass, with the heavier body nearer the centre. While the orbital path of a satellite is a closed ellipse, the orbit of an electrically charged particle, round a central force of attraction, is an unclosed elliptic path or a closed circle. A charged particle revolves in unclosed paths with emission of radiation, at the frequency of revolution, in a spread with *fine structure* [4]. The energy radiated is equal to the difference between change in kinetic energy and change in potential energy, in contrast to classical mechanics, relativistic mechanics and quantum mechanics [5-7].

Aberration of electric field, a phenomenon similar to aberration of light, discovered by Astronomer James Bradley, in 1728, is invoked to derive equations of orbit of motion and radiation of a charged particle revolving round a centre of force of attraction [8]. The discrete masses, nm, of revolving particles, give constant angular momentum *nL* in the nth orbit. Excited particles revolve in unclosed elliptic paths, with emission of radiation, with fine structure, at frequency of revolution, in accordance with Balmer-Rydberg formula, before reverting to a stable circular orbit [9]. The motions and energy radiations are treated outside the theory of special relativity or quantum mechanics [6, 7].

Revolution Round a Centre of Mass

In Figure 1, with origin at O, bodies M_1 and M_2, distance r apart, revolve, under mutual attraction, with radii r_1 and r_2, round their centre of mass at C. Definition of centre of mass, gives:

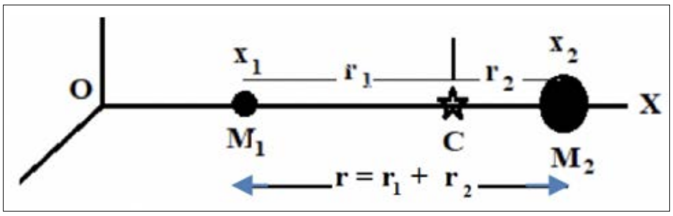

Figure 1: Two bodies M_1 at x_1 and M_2 at x_2, distance r apart, with centre of mass at C, relative to origin of coordinates O

$$\boxed{(M_1 + M_2)(x_1 + r_1) = M_1 x_1 + M_2(x_1 + r)} \quad (1)$$

$$r_1 = \frac{M_2 r}{M_1 + M_2} \quad (2)$$

$$\boxed{r_2 = \frac{M_1 r}{M_1 + M_2}} \quad (3)$$

If M_2 is much larger than M_1, $r_1 \approx r$, for revolution round a heavy M_2 at the centre of mass, as a nucleus. $M_1 = M_2$ gives non-nuclear revolution, with $r_1 = r_2$, round an empty centre of mass.

Aberration of Electric Field

Figure 2 depicts aberration of electric field, for an electron of charge *-e* and mass *m* at a point P, moving at time *t* with velocity **v**, at angle θ to force **F** of electric field **E** due to charge $+Q$ stationary at a point O. With respect to the moving electron, the source charge Q appears shifted to N, and the force transmitted with velocity of light c along PN, through aberration angle α, to give equations (4) and (5). Speeds v and c are magnitudes of the velocities. Equation (4) is Bradley's formula, a universal formula, independent of distance of separation PO. It is applicable at astronomical and atomic levels.

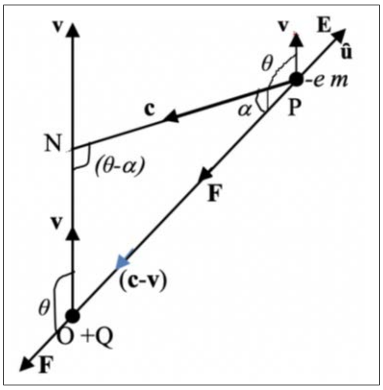

Figure 2: Aberration angle a due to electron of charge -e and mass m at P, moving with velocity **v** at an angle θ to the accelerating force **F** due to an electric charge +Q at O.

$$\sin\alpha = \frac{v}{c}\sin\theta \qquad (4)$$

In Figure 2, apparent displacement of charge Q, from PO to PN, through angle α, is aberration of electric field, such that relative velocity (**c** – **v**) between the electrical force, transmitted with velocity of light c, and the electron moving with velocity **v**, is along PO, line of action of the force. The electric field intensity is $\mathbf{E} = E\hat{\mathbf{u}}$ of magnitude E.

Equation (5), due to aberration of electric field, in terms of angles θ and α and unit vector $\hat{\mathbf{u}}$, in Figure 2, gives the accelerating force **F** on an electron moving with velocity **v** at time t, with constant mass m as the rest mass m_o, in accordance with Newton's second law of motion.

$$\mathbf{F} = \frac{eE}{c}(\mathbf{c}-\mathbf{v}) = -\frac{eE}{c}\sqrt{c^2+v^2-2cv\cos(\theta-\alpha)}\,\hat{\mathbf{u}} = m\frac{d\mathbf{v}}{dt} \qquad (5)$$

The speed of light being a limiting value, at **v** = **c**, is implicit in equation (5), the flagship of this paper.

Radiation of Energy

In equation (5), accelerating force **F** reduces to zero at velocity **v** = **c**. Force on a moving electron is less than the force -e**E** on a stationary one. The difference is radiation reaction force $\mathbf{R}_f$. Radiation power, scalar product $P = -\mathbf{v}.\mathbf{R}_f$, as expressed in equations (6) and (7).

$$\mathbf{R}_f = \frac{eE}{c}(\mathbf{c}-\mathbf{v}) - e\mathbf{E} \qquad (6)$$

$$P = -\mathbf{v}.\mathbf{R}_f = -\mathbf{v}.\left\{\frac{eE}{c}(\mathbf{c}-\mathbf{v}) - e\mathbf{E}\right\} = eEv\left\{\cos\theta - \cos(\theta-\alpha) + \frac{v}{c}\right\} \qquad (7)$$

In equation (7), radiation power is eEv^2/c for rectilinear motion with $\theta = 0$ or $\theta = \pi$ radians. No radiation if $\theta = \pi/2$ radians, in circular revolution.

Speed versus Time in Rectilinear and Circular Motions

Equations (4) and (5) give differential equation (8) for the speed v versus time t, in rectilinear motion for $\theta = 0$, with acceleration, and equation (9) for $\theta = \pi$ radians, with deceleration. Equation (10) is for circular revolution of constant radius r, at constant speed v and constant centripetal acceleration v^2/r round a centre of force of attraction, with $\theta = \pi/2$ and $\cos(\theta - \alpha) = \sin\alpha = v/c$.

$$eE\left(1-\frac{v}{c}\right) = m\frac{dv}{dt} \qquad (8)$$

$$eE\left(1+\frac{v}{c}\right) = -m\frac{dv}{dt} \qquad (9)$$

With $\theta = \pi/2$ radians and $\sin\alpha = v/c$, equation (5), with mass m equal to rest mass m_o, gives:

$$-eE\sqrt{1-(v/c)^2} = -m_o\frac{dv}{dt} = -m_o\frac{v^2}{r} \qquad (10)$$

Equation (10) is the result of motion of a charged particle perpendicular to an electric field.

If E is constant, equation (11) is the solutions of equations (8), with constant $a = eE/m$, for acceleration from 0 initial speed. Equation 12 is the solution of equation (9) for an electron decelerated from the speed of light c. Equation (13) is a transposition of equation (10). Equation 14 is the relativistic mass-velocity formula, where m_o is the rest mass and γ is Lorentz factor.

$$\frac{v}{c} = 1 - \exp\left(-\frac{at}{c}\right) \qquad (11)$$

$$\frac{v}{c} = 2\exp\left(-\frac{at}{c}\right) - 1 \qquad (12)$$

$$eE = \frac{m_o}{\sqrt{1-(v/c)^2}}\frac{v^2}{r} \qquad (13)$$

$$m = \frac{m_o}{\sqrt{1-(v/c)^2}} = \gamma m_o \qquad (14)$$

In equation (11), the electron is accelerated to the speed of light c, as maximum, at constant mass and with emission of radiation. In equation (12) the electron is decelerated to stop in time $t = 0.693c/a$ and then accelerated in the opposite direction, to reach maximum speed -c.

Equation (14) is mathematically correct for circular motion, but physically wrong. Using equation (14), in rectilinear motion, is an expensive mistake. Lorentz factor here, has nothing to do with mass, but a consequence of motion of a charged particle perpendicular to an electric field.

Nuclear and Non-Nuclear Revolutions

Figure 3 depicts a particle of charge -e and mass nm, a multiple of the electronic mass m, at a point P, executing nuclear revolution, in angular displacement y, under a force of attraction of stationary charge +Q at O. In figure 4, two particles of charges -e and +e and same mass nm revolve diametrically round their (empty) centre of mass at O. Figure 3 gives a nuclear model of the hydrogen atom.

Figure 4 leads to a non-nuclear model.

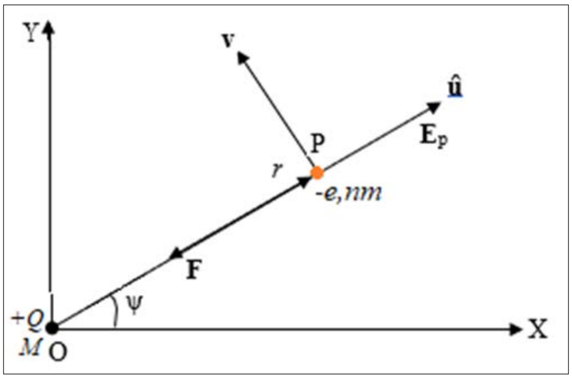

Figure 3: A particle of charge -*e* and mass *nm* at point P revolving with angular displacement ψ, in an orbit at velocity **v** under the attraction of charge +*Q*, the nucleus of mass *M* at a centre of attraction and revolution O.

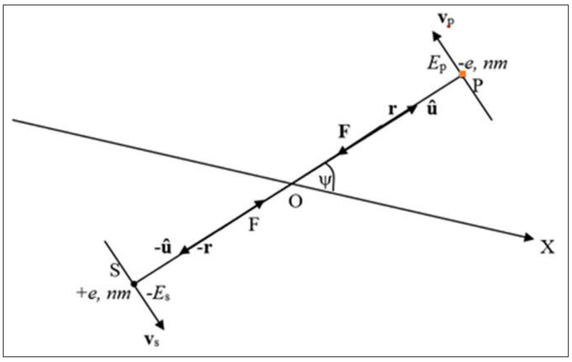

Figure 4: Two equal and oppositely charged particles at P and S having charges -*e* and +*e* and same mass *nm* of the electronic mass *m*, revolving in angle ψ, under mutual attraction, in an orbit of radius *r*, round a vacant centre of mass at O.

Velocity and Acceleration under a Central Force

In Figures 3 and 4, position vector of P, distance r from O, in direction of unit vector $\hat{\mathbf{u}}$, is expressed in equation (15) and the angular velocity $d\hat{\mathbf{u}}/dt$ is in equation (16). The velocity of the particle, $d\mathbf{r}/dt$ is in equation (17) and acceleration $d^2\mathbf{r}/dt^2$ in equation (18), with **k** as a constant unit vector normal to the plane of the orbit.

$$\mathbf{r} = r\hat{\mathbf{u}} \quad (15)$$

$$\frac{d\hat{\mathbf{u}}}{dt} = \frac{d\psi}{dt}\mathbf{k}\times\hat{\mathbf{u}} \quad (16)$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{dr}{dt}\hat{\mathbf{u}} + r\frac{d\psi}{dt}\mathbf{k}\times\hat{\mathbf{u}} \quad (17)$$

$$\frac{d\mathbf{v}}{dt} = \left\{\frac{d^2r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2\right\}\hat{\mathbf{u}} + \left(2\frac{dr}{dt}\frac{d\psi}{dt} + r\frac{d^2\psi}{dt^2}\right)\mathbf{k}\times\hat{\mathbf{u}} \quad (18)$$

The acceleration, in equation (18), is only in the radial direction of the force, which makes the second acceleration term zero.

Angular Momentum

Acceleration in the radial direction only, gives equation 19 (from equation 18) as zero. Equation (19) is expressed in terms of angular momentum L, as presented in equations (20) and (21).

$$nm\left(2\frac{dr}{dt}\frac{d\psi}{dt} + r\frac{d^2\psi}{dt^2}\right)\mathbf{k}\times\hat{\mathbf{u}} = 0 \quad (19)$$

$$\frac{nm}{r}\frac{d}{dr}\left(r^2\frac{d\psi}{dt}\right)\mathbf{k}\times\hat{\mathbf{u}} = \frac{n}{r}\frac{d}{dt}\mathbf{L}\times\hat{\mathbf{u}} = 0 \quad (20)$$

$$n\mathbf{L} = nmr^2\frac{d\psi}{dt}\mathbf{k} \quad (21)$$

Equation of the Nuclear Orbit of Revolution

Taking scalar product of equation (5) with the unit vector $\hat{\mathbf{u}}$, in reference to Figure 2, and a being a very small angle, so that the scalar product $\mathbf{c}.\hat{\mathbf{u}} = c\cos a \approx c$, with $\mathbf{v}.\hat{\mathbf{u}} = dr/dt$, and $d\mathbf{v}/dt$ from equation (18) for a particle of mass *nm*, gives equation (22) and (23).

$$\mathbf{F}.\hat{\mathbf{u}} = \frac{eE}{c}(\mathbf{c}.\hat{\mathbf{u}} - \mathbf{v}.\hat{\mathbf{u}}) = nm\frac{d\mathbf{v}}{dt}.\hat{\mathbf{u}} = nm\left\{\frac{d^2r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2\right\} \quad (22)$$

$$-eE\left(1 + \frac{1}{c}\frac{dr}{dt}\right) = nm\left\{\frac{d^2r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2\right\} \quad (23)$$

Putting $E = Q/4\varepsilon_o r^2$, in equation (23) gives the magnitude of the accelerating force, as in equations (24) and 25. Equation (26) gives the constant χ in terms of the charge Q.

$$-\frac{eQ}{4\pi\varepsilon_o r^2}\left(1 + \frac{1}{c}\frac{dr}{dt}\right) = nm\left\{\frac{d^2r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2\right\} \quad (24)$$

$$-\frac{\chi}{nmr^2}\left(1 + \frac{1}{c}\frac{dr}{dt}\right) = \frac{d^2r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2 \quad (25)$$

$$\chi = \frac{eQ}{4\pi\varepsilon_o} \quad (26)$$

Equation (25) is a mixed second order differential equation of motion. We need to reduce it to an equation of *r* as a function of angular displacement ψ only. Substituting $r = 1/u$ gives $dr/du = -1/u^2$ and with $d\psi/dt = L/mr^2$ (equation 21), gives equations (27) and (28).

$$\frac{dr}{dt} = \frac{dr}{du}\frac{d\psi}{dt}\frac{du}{d\psi} = \frac{-L}{m}\frac{du}{d\psi} \quad (27)$$

$$\frac{d^2r}{dt^2} = \frac{d}{dt}\frac{dr}{dt} = \frac{d\psi}{dt}\frac{d}{d\psi}\left(-\frac{L}{m}\frac{du}{d\psi}\right) = -\frac{L^2u^2}{m^2}\frac{d^2u}{d\psi^2} \quad (28)$$

Substituting equations (27) and (28) into equation (25) gives equation (29) and equation (30).

$$-\frac{L^2u^2}{m^2}\frac{d^2u}{d\psi^2} - \frac{L^2u^3}{m^2} = -\frac{\chi u^3}{nm}\left(1 - \frac{L}{m}\frac{du}{d\psi}\right) \quad (29)$$

$$\frac{d^2u}{d\psi^2} + \frac{\chi}{ncL}\frac{du}{d\psi} + u = \frac{m\chi}{nL^2} \quad (30)$$

Equation (30) is a second order differential equation with constant coefficients. The solutions are in equation (31) and (32).

$$u = \frac{1}{r} = A\exp(jb - q)\psi + \frac{m\chi}{nL^2} \qquad (31)$$

$$u = \frac{1}{r} = A\exp(-q\psi)\cos(b\psi + \beta) + \frac{m\chi}{nL^2} \qquad (32)$$

Equation of the Non-Nuclear Orbit of Revolution

The two particles in Figure 4 move with relative velocity **v.û** = 2(*dr*/*dt*) in the radial direction only. In this case, equation (22) remains as equation (33) but equation (23) becomes equation (34).

$$\mathbf{F}.\hat{\mathbf{u}} = \frac{eE}{c}(\mathbf{c}.\hat{\mathbf{u}} - \mathbf{v}.\hat{\mathbf{u}}) = nm\frac{d\mathbf{v}}{dt}.\hat{\mathbf{u}} = nm\left\{\frac{d^2r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2\right\} \qquad (33)$$

$$-eE\left(1 + \frac{2}{c}\frac{dr}{dt}\right) = nm\left\{\frac{d^2r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2\right\} \qquad (34)$$

Equation (24) becomes equation (35), equation (25) becomes equation (36) and equation (26) becomes equation (37) for the constant κ.

$$-\frac{e^2}{16\pi\varepsilon_o r^2}\left(1 + \frac{2}{c}\frac{dr}{dt}\right) = nm\left\{\frac{d^2r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2\right\} \qquad (35)$$

$$-\frac{\kappa}{nmr^2}\left(1 + \frac{1}{c}\frac{dr}{dt}\right) = \frac{d^2r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2 \qquad (36)$$

$$\kappa = \frac{e^2}{16\pi\varepsilon_o} \qquad (37)$$

Equation (36) is a mixed differential equation of motion. It is reduced to an equation of r as a function of y only by substituting $r = 1/u$ to give $dr/du = -1/u^2$ and with $dy/dt = L/mr^2$ (equation 21), to give equations (38) as a second order differential equation, with particular solution as equation (39).

$$\frac{d^2u}{d\psi^2} + \frac{2\kappa}{ncL}\frac{du}{d\psi} + u = \frac{m\kappa}{nL^2} \qquad (38)$$

$$u = \frac{1}{r} = A\exp(-b\psi)\cos(a\psi + \beta) + \frac{m\kappa}{nL^2} \qquad (39)$$

Orbits of the Nuclear and Non-nuclear Revolutions

In equations (32) and (39), the decay factors *q* (for nuclear revolution) and *b* (for non-nuclear revolution) are due to radiation. The particle revolves with speed v in an unclosed elliptic path and, after many cycles of ψ, with radiation, settles in a circle of radius nL^2/mc or nL^2/mk, the steady state or the stable circle. Energy radiated is equal to the difference between change in kinetic energy and change in potential energy. The free ellipse is the closed path traced if there were no radiation. The free ellipse WXYZ, with foci F_1 and F_2 and the stable circle CDEF, with centre at F_1 or O and radius as the latus rectum EF_1 or EO, for nuclear and non-nuclear revolutions, are shown in Figures 5 and 6.

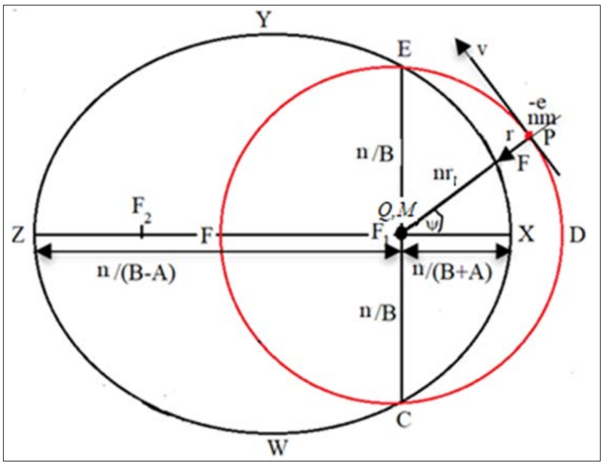

Figure 5: Free ellipse WXYZ and stable circular orbit CDEF of revolution of particle of charge -*e* and mass *nm* at P, of the nuclear model. The *nth* stable circle CDEF has the latus rectum EF_1 as radius with charge +*Q* of mass *M* as focus F_1.

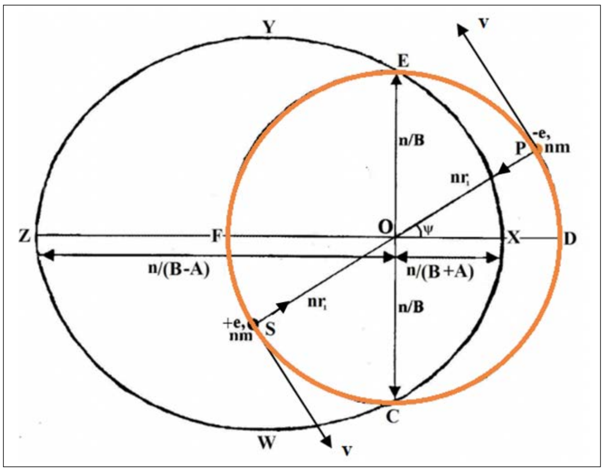

Figure 6: Free ellipse WXYZ and stable circular orbit CDEF of revolution of particles, of charges +*e* and -*e* in attraction, and mass *nm* at P or S, of non-nuclear model. The *nth* stable circle CDEF has latus rectum EO as radius with O as focus.

Number of Circular Stable Orbits

In the nuclear model, particles of charge -*e* and mass *nm* revolve in *N* orbits round a nucleus of charge +*Ne*, where *m* is the electronic mass and *n* is an integer *1*, *2*, *3*.....*N*. Total mass of negatively charged particle, revolving in *N* orbits, is the arithmetic sum of natural numbers *1* to *N*, which is $m(N/2)(N + 1)$, equal to the mass of the nucleus, making mass of the atom as $mN(N + 1)$. In the non-nuclear model, a particle of charge -*e* and mass *nm*, and another particle of charge +*e* and mass *nm* revolve diametrically round their common centre of mass, in *N* orbits. Total mass of orbiting particles, in *N* orbits, each with two particles, is also the arithmetic sum $mN(N + 1)$.

Radius, Speed and Frequency of Revolution in NTH Stable Circle

The radius r_n, speed v_n and frequency of revolution $f_n = v_n/2\pi r$ cycles per second, in the nth stable circle, are in equations (40), (41) and (42), respectively, for the nuclear model of the hydrogen atom, where S is a constant.

$$r_n = \frac{4\pi\varepsilon_o nL^2}{mNe^2} \quad (40)$$

$$v_n = \frac{Ne^2}{4\pi\varepsilon_o nL} \quad (41)$$

$$f_n = \frac{mN^2e^4}{2\pi(4\pi\varepsilon_o)^2L^3n^2} = \frac{cS}{n^2} \quad (42)$$

The radius r_n, speed v_n and frequency of revolution $f_n = v_n/2pr_n$ cycles per second, in the nth stable circle, are given in equations (43), (44) and (45), respectively for the non-nuclear model of the hydrogen atom, where R is Rydberg constant.

$$r_n = \frac{16\pi\varepsilon_o nL^2}{me^2} \quad (43)$$

$$v_n = \frac{e^2}{16\pi\varepsilon_o nL} \quad (44)$$

$$f_n = \frac{me^4}{2\pi(16\pi\varepsilon_O)^2L^3n^2} = \frac{cR}{n^2} \quad (45)$$

At some temperature and pressure, the nucleus breaks up, and the particles form the non-nuclear atom.

In equation (43), $r_1 = \frac{16\pi\varepsilon_o L^2}{me^2}$ is the radius of first orbit of the non-nuclear model. Also, $r_1 = \frac{\varepsilon_O h^2}{\pi me^2}$ = 5.292×10^{-7} m, the radius in the first orbit, is the same as the Bohr radius, obtained through quantum mechanics, for the hydrogen atom. This makes the angular momentum $L = h/4\pi$, where $h = 6.626x10^{-34}$ J-sec, Planck constant, a fundamental physical constant.

Radiation from the Hydrogen Atom

Figure 7 shows the first seven circular orbits of the nuclear model. In Figure 8 are similar circular orbits of the non-nuclear model. Interactions between revolving charged particles and electric fields of other particles, involving changes in potential and kinetic energies, give emission of radiation. Let us follow the motion of two particles at positions P and Q with radii nr_1 and qr_1 respectively, revolving at positions P and Q as shown at the initial time $t = 0$. The relative positions of the points O, P and Q are as shown, with OP and OQ in an angular displacement ψ_o at the initial stage, time $t = 0$. In time t, the line OP moves to OP_t through angle ψn in the nth orbit, and line OQ moves to OQ_t through angle ψq in the qth orbit. The difference in angular displacement ψ_t, at time t, angle P_tOQ_t, is expressed in equation (46), angular velocity in equation (47) and the frequency in equation (48).

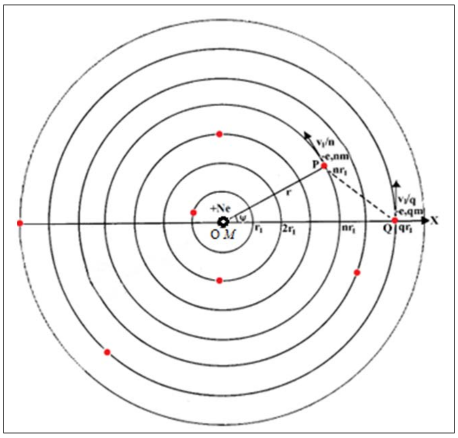

Figure 7: Seven of N circular orbits of nuclear model, with negatively charged particle, revolving in angle ψ, under attraction of a nucleus of charge $+Ne$ and mass M, at O. A particle in nth orbit has multiple nm of electronic mass m and charge $-e$.

In Figure 7, the nucleus of charge $+Ne$ and mass $(N/2)(N + 1)m$ also rotates as a ring, confined by magnetic fields of the revolving negative particles of total mass $(N/2)(N + 1)m$. At a certain temperature and pressure, the centre cannot hold. The nucleus falls apart for particles to form the non-nuclear model, with mass of positive particles as $(N/2)(N + 1)m$ equal to mass of negative particles.

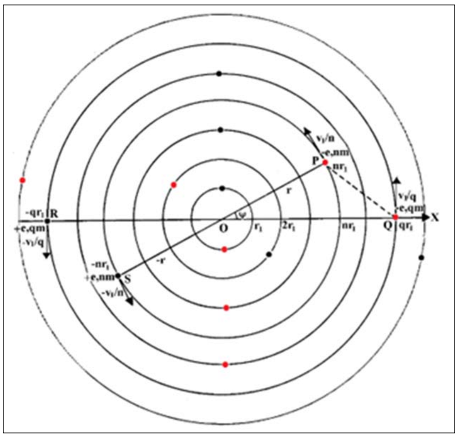

Figure 8: Seven of N circular orbits of non-nuclear model, with 2 oppositely charged particles, revolving, in angle ψ, round centre of mass O. Each particle in the nth orbit has mass nm, one carries a revolving charge $-e$ and the other $+e$.

$$\psi_t = \psi_o + \psi_n - \psi_q \quad (46)$$

$$\frac{d\psi_t}{dt} = \frac{d\psi_n}{dt} - \frac{d\psi_q}{dt} = \omega_n - \omega_q = 2\pi f_n - 2\pi f_q = 2\pi f_{nq} \quad (47)$$

$$f_n - f_q = f_{nq} \quad (48)$$

Combining equation (48) with equations (42) where $f_n = cS/n^2$ and equation (45) where and $f_n = cR/n^2$, gives the discrete frequencies of radiation, in equation (49) for the nuclear model and equation (50) for the non-nuclear model.

$$f_{nq} = cS\left(\frac{1}{n^2} - \frac{1}{q^2}\right) \quad (49)$$

$$f_{nq} = cR\left(\frac{1}{n^2} - \frac{1}{q^2}\right) \qquad (50)$$

Equation (50) is Balmer-Rydberg-Bohr formula for discrete frequencies of radiation from the atom of hydrogen. However, the frequencies have a small spread, called *fine structure*, as a disturbed particle takes time to execute many lively revolutions before settling in a stable circular orbit.

Results and Discussion

1. Rutherford-Bohr's nuclear model of the hydrogen atom (proton and electron) gives one orbit with quantized radius and does not differentiate between gaseous, liquid or solid states of hydrogen.
2. In equation (5), velocity of light c is a limit, since accelerating force F reduces to zero at velocity v = c, with motion at constant mass as the rest mass mo
3. Equation (7) shows that circular motion of an electron, with $\theta = \pi/2$ radians, under force of attraction, are without radiation and stable; no need of Bohr's quantum mechanics to fix it.
4. Electrical particles, such as positronium, protium, deuterium and tritium, have charge +e or -e and a multiple nm of the electronic mass m, in N orbits of revolution, where n = 1, 2, 3…..N.
5. Lorentz factor g in equation (14) has nothing to do with mass, but a result of motion of a charged particle perpendicular to an electric field.
6. Relativistic mass-velocity formula (14) is correct for circular motion only, with constant mass.
7. Ratio of frequency of radiation in the nth orbit of the nuclear model, to frequency in the non-nuclear model, is $16N^2 \approx 28{,}700$, putting radiation from the nuclear model in the x-rays regions.
8. Explanation of limiting speed of light, at constant mass of a moving particle, is achieved, without recourse to the theory of special relativity.
9. Radiation from charged particles, accelerated by an electric field, is explained as the difference between change in potential energy and change in kinetic energy, outside quantum mechanics.
10. The Balmer-Rydberg-Bohr radiation-frequency formula, equation (50), is for the non-nuclear model of the atom of hydrogen gas, equation (49) is for the nuclear model.
11. Two models of hydrogen atom are given, with 42 or 43 orbits, where revolving charged particles interact and radiate, outside the notion of probability of location in quantum electrodynamics.
12. The paper deals with hydrogen atom (protium), constituting over 90% of atoms in the universe.

Conclusion

The paper has succeeded in showing that a charged particle, dislodged from a circular orbit, round a centre of force of attraction, revolves in unclosed elliptic paths, emitting radiation, in many cycles at increasing frequency of revolution, with a small spread exhibiting *fine structure*, before reverting to a stable circular orbit, in lively actions indicative of rudimentary memory.

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