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Field Dependant Metric for Gravitational/EM Fields and a Non-Linear Transformation for Local to Proper Space-Time World

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ABSTRACT

In this paper, the introduction of a metric for Gravitational Field is examined based on [4]; this can be extended to EM Fields also by change of the parameters involved (ϵ_1, μ_1) to (ϵ_2, μ_2) We know that $E^2 - c^2B^2$ *or* $E^2 - B^2$ with is an invariant quantity for the EM Fields which can be extended to gravitational fields, as done in [1]. We discuss the metric for the gravitational case and extended to the EM fields as well. The discussion of anomalous characteristic of Lorentz Transformation (LT) and the introduction of a non-linear transformation connecting Local Space-time coordinates (*ct*,*x*) with proper system (cτ, x_τ) is continued.

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Field Dependent Metric for Gravitational Field

The matrix of LT for (ct', x', y') space to (ct, x, y) space is given by

$$L = \begin{bmatrix} \frac{1}{\sqrt{1-e^2}} & \frac{e}{\sqrt{1-e^2}} & 0 \\ \frac{e}{\sqrt{1-e^2}} & \frac{1}{\sqrt{1-e^2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where e stands for $\frac{v_0}{c}$ *and* v_0 is the speed of apparent relative motion of the frames S, S' and c is the maximum signal velocity of the gravitational field in the discussion [1]. Similarly, c is signal velocity of the EM field when we consider EM Fields [2-4] . In the general case of $3 \oplus 1$ dimensions, the Lorentz matrix can be given by

$$L = \begin{bmatrix} \sec\phi & \tan\phi\cos\alpha & \tan\phi\cos\beta & \tan\phi\cos\gamma \\ \tan\phi\cos\alpha & 1+(\sec\phi-1)\cos^2\alpha & (\sec\phi-1)\cos\alpha\cos\beta & (\sec\phi-1)\cos\alpha\cos\gamma \\ \tan\phi\cos\beta & (\sec\phi-1)\cos\alpha\cos\beta & 1+(\sec\phi-1)\cos^2\beta & (\sec\phi-1)\cos\beta\cos\gamma \\ \tan\phi\cos\gamma & (\sec\phi-1)\cos\alpha\cos\gamma & (\sec\phi-1)\cos\beta\cos\gamma & 1+(\sec\phi-1)\cos^2\gamma \end{bmatrix}$$

implies

$$c^2dt^2 - dx^2 - dy^2 - dz^2 \equiv c^2dt'^2 - dx'^2 - dy'^2 - dz'^2 \equiv c^2d\tau^2.$$

By letting

(i) $\boldsymbol{v}_0 = v_0\left(\boldsymbol{i}\cos\alpha + \boldsymbol{j}\cos\beta + \boldsymbol{k}\cos\gamma\right)$

$$= v_0\left(\boldsymbol{i}\cos\omega\tau\sin\omega_0\tau + \boldsymbol{j}\sin\omega\tau\sin\omega_0\tau + \boldsymbol{k}\cos\omega_0\tau\right)$$

(ii) $v_0 = a + b\tau + \ldots = v_0(\tau)$ and

(iii) $\tan\phi = v_0c^{-1}$ *or* $\sin\phi = v_0c^{-1}$, (two distinct cases) we see that L can handle accelerated motion of O′ /O relative to O′ /O and the *metric* is pseudo-Euclidean but not non-Euclidean as in GTR. Clearly τ is the best estimate of the absolute time.

The GTR excludes the possibility of a parallel theory for the motion of mass particles and charged particles by highlighting the observation that the energy momentum tensor of EM field has a vanishing trace [5, 6]. On the other hand, the vector fields can be generalized by means of contracted tensor fields.

We have the constitutive relations $\mathbf{D}_1 = \epsilon_1\mathbf{E}_1$ and $\mathbf{B}_1 = \mu\mathbf{H}_1$ for gravitational fields. We shall modify the vectors E1 and H1 by means of contracted tensor fields. We know that the metric in tensor calculus [7] is given by $(ds)^2 = g_{ij}dx^idx^j = (g_{ij}dx^i)dx^j = dx_jdx^j$ where g_{ij} is a (0, 2) tensor and dx_j, dx^j are the covariant and contra-variant components of the same vector.

Keeping these ideas in view, let us introduce, for the gravitational field, two reciprocal/conjugate tensors ϵ_{ij} and ϵ^{ij} so that $\epsilon^{i\alpha}\epsilon_{\alpha j} = \epsilon\delta^i_j$. We define $E_i = \epsilon_{i\alpha}E^\alpha$ and $E^i = \epsilon^{i\alpha}E_\alpha$ as the covariant and contra-variant components of E.

Now $\epsilon_{ij}E^iE^j = \left(\epsilon_{ij}E^i\right)E^j = E_iE^j = |\mathbf{E}|^2$ and $\epsilon^{ij}E_iE_j = \left(\epsilon^{ij}E_i\right)E_j = E^jE_j = |\mathbf{E}|^2$

Hence $|\mathbf{E}|^2 = \epsilon_{ij}E^iE^j = \epsilon^{ij}E_iE_j$.

Similarly we introduce two reciprocal/conjugate tensors μ_{ij} and μ^{ij} so that $\mu^{i\alpha}\mu_{\alpha j} = \mu\delta^i_j$ we define $H_i = \mu_{i\alpha}H^\alpha$ and $H^i = \mu^{ij}H_j$ as the covariant and contra-variant components of H where ϵ, μ stands for ϵ_1, μ_1 in the gravitational case and ϵ_2, μ_2 for the EM fields, such that $\epsilon\mu c^2 = 1$ where c is the maximum signal velocity of the gravitational/ EM fields.

Also $\mu_{ij}H^iH^j = (\mu_{ij}H^i)H^j = H_jH^j = |\mathbf{H}|^2$ and

$\mu^{ij}H_iH_j = (\mu^{ij}H_j)H_j = H^jH_j = |\mathbf{H}|^2$.

Therefore $|\mathbf{H}|^2 = \mu_{ij}H^iH^j = \mu^{ij}H_iH_j$. Hence our tensors have the properties that

(i) ϵ_{ij} and ϵ^{ij} are reciprocal tensors dependent on ϵ_1/ϵ_2
(ii) μ_{ij} and μ^{ij} are reciprocal tensors dependent on μ_1/μ_2
(iii) ϵ's and μ's are dependent on the dielectric parameter ϵ_1 and the gravitational susceptibility μ_1, for gravitational fields and ϵ_2, μ^2 for EM fields, such that $\epsilon\mu c^2 = 1$.

Thus, for an infinitesimal region, it is possible to replace vector-fields by means of contracted tensor fields and the metric $dE^2 - dH^2$ (with c = 1) is a metric for the gravitational field. Similarly, we can form the metric $dE^2 - dH^2$ with $(c = 1) = \epsilon_{ij}dE^idE^j - \mu_{ij}dH^idH^j$ for the EM field by changing ϵ_1 and μ_1 to ϵ_2 and μ_2 with corresponding definitions given above.

An Anomolous Characteristic of LT

The LT equations satisfy

$$c^2\tau^2 = c^2t'^2 - x'^2 \qquad (2.1)$$

$$c^2\tau^2 = c^2t^2 - x^2 \qquad (2.2)$$

Both (1) and (2) represent rectangular hyperbolas (RH). A question arises at this stage: do they represent (i) two different RH's or (ii) (ct', x') and (ct,x) are on the same RH. Relativistic arguments support (i). Let us examine the situation

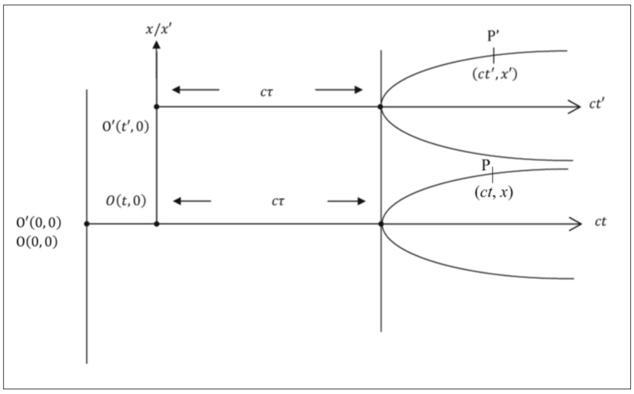

Figure 1

In the figure 1 the upper RH represents equation (2.1) and it is questionable to represent equation (2.2) by the lower one RH or any other RH with axis along ct-axis. By using the substitution

$$ct' = ct\cos h\varphi - x\sin h\varphi \qquad (2.3) \qquad (x' \text{ axis})$$

$$x' = x\cos h\varphi - ct\sin h\varphi \qquad (2.4) \qquad (ct' \text{ axis})$$

the equation (2.1) becomes (2.2). But the problem is: does P' and P represent points on two different RH as shown in the figure 1 or are they represent two points on the same RH and have the same time-axis as in figure 2.

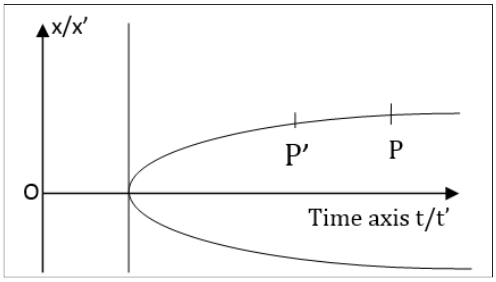

Figure 2

By substituting ct' by ct and x' by x in (2.1) this equation (2.1) turns to equation (2.2). But this substitution is not allowed if P (ct, x) and $P'(ct', x')$ lies on the same RH as in figure 2 or at least on two nearby RH's with their time axis distinct but inclined to each other at a 'small' angle and intersecting at the common world point (0, 0), since ct' -axis is not perpendicular to x' -axis (see Figure 3).

This conclusion follows from the following fact (see Figure 3). Equation (2.3) represents the x' - axis and (2.4) represents the ct' -axis. The slope of these lines is $\frac{1}{e}$ and e respectively where $e = \frac{v_0}{c} = \tan h\,\varphi$ If θ is inclination of ct' -axis with ct -axis, then $\tan\theta = e$ and slope of x-axis $= \frac{1}{e} = \cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$

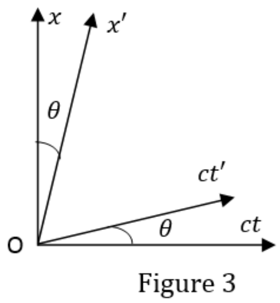

Figure 3

$\therefore x' - axis$ has inclination $\frac{\pi}{2} - \theta$ and hence x' -axis and ct' -axis are inclined at $\frac{\pi}{2} - 2\theta$. Since $\sin\left(\frac{\pi}{2} - 2\theta\right) = \cos 2\theta = \cos^2\theta - \sin^2\theta = \frac{1-e^2}{1+e^2}$. We have the relation between the areal elements: $cdt\,dx = |J|\,cdt'dx'\sin\left(\frac{\pi}{2} - 2\theta\right) = \left(\frac{1-e^2}{1+e^2}\right)cdt'dx'$

where we used $|J| = 1$ and $\sin\left(\frac{\pi}{2} - 2\theta\right) = \left(\frac{1-e^2}{1+e^2}\right)$ *i.e.*

$$dt\,dx = \left(\frac{1-e^2}{1+e^2}\right)dt'dx' \qquad (2.5)$$

This equation depicts the asymmetric nature of LT. As an approximation if we write $dt\,dx \cong dt'dx'$, then

$$2c\ dt\ dx \cong 2c\ dt'dx' \qquad (2.6)$$

Also, we have

$$c^2dt^2 - dx^2 = c^2dt'^2 - dx'^2 \qquad (2.7)$$

Equations (2.6) and (2.7) imply $(cdt + idx)^2 = (cdt' + idx')^2$

so that $cdt = cdt'$ and $dx = dx'$ or $t = t'$ and $x = x'$, except for an additive constant. The asymmetric nature of equation (2.5) reveals that there is no logic in stipulating that the LT represents the motion of O′ relatively to O along a common x-axis. Figure 3 indicates that (ct, x) world point is a preferred representation to (ct', x'); the converse is true by re-drawing the figure and considering the inverse transformation. Thus, the LT implies that one or the other frame of reference is a preferred one justifying the Lorentzian interpretation of a preferred frame. Hence the relativistic conclusion that there is no preferred frame of reference, is logically invalid; so is the assumption of a common x / x'-axis. Hence figure 2 and 4 give true representation of P and P' whereas figure 1 does not.

The presence of a preferred frame indicates that we can use the proper coordinates$(c\tau, x_\tau)$. Next, we attempt to find the relation between the proper coordinates and the local coordinates $(ct, x)/(ct', x')$ Since$OO' = v_0 t$ and $O'O = -v_0 t'$ their sum is not equal to zero; this is a strange situation [8, 9]. So, we seek to find the required transformation to satisfy $OO' = v_0\tau$ and $O'O = -v_0\tau$

Equation (2.2) can be re-written as

$$c\tau = ct\sqrt{1 - \frac{x^2}{c^2t^2}}$$

which can be re-written as

$$c\tau = ct\sqrt{1 - \frac{v^2}{c^2}}$$

where$v = \frac{x}{t}$. This is the formula time-dilation [10-12]. Also, the formula of Fitzgerald-Larmor-Lorenz contraction [2, 13-16] is

$$L_0 = \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}}$$

In this equation, replace L_0 by x_τ, L by x and $v = \frac{x}{t}$, Thus we get

$$x_\tau = \frac{x}{\sqrt{1 - \frac{x^2}{c^2t^2}}}$$

Hence the transformation we sought is

$$c\tau = \sqrt{|c^2t^2 - x^2|} \qquad (2.8)$$

$$x_\tau = \frac{xt}{\sqrt{|c^2t^2 - x^2|}} \qquad (2.9)$$

The inverse transformation is given by

$$c^2t^2 = \frac{1}{2}c^2\tau^2\left[\sqrt{1 + \frac{4x_\tau^2}{c^2\tau^2}} + 1\right] \qquad (2.10)$$

$$x^2 = \frac{1}{2}c^2\tau^2\left[\sqrt{1 + \frac{4x_\tau^2}{c^2\tau^2}} - 1\right] \qquad (2.11)$$

Equations (2.8) to (2.11) represent the transformation between the proper system $(c\tau, x_\tau)$ and the local system (ct, x). This can be generalised to four-dimensional case by including $y_\tau = y$, $z_\tau = z$.

The geometrical representation of $(c\tau, x_\tau)$ is as follows:

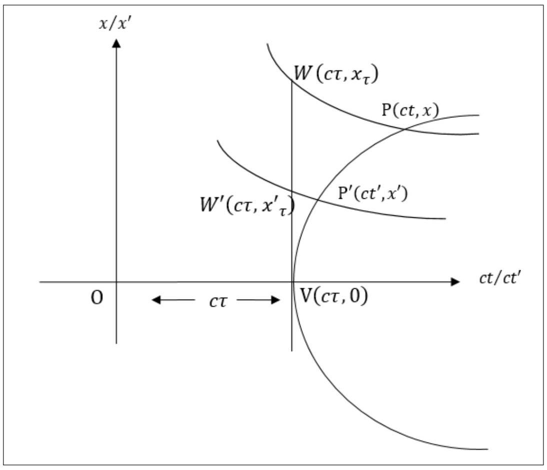

Figure 4

In figure 4 we represent the RH of equation (2.2) in the ct–x plane. Its vertex is $V(c\tau,0)$; erect the vertical at $V(c\tau,0)$, cutting the conjugate/orthogonal RH $2xct = 2x_\tau c\tau$ at the point $W(c\tau,x_\tau)$. Thus, VW represents x_τ and OV represents $c\tau$. If we consider another point $P'(ct', x')$ on the hyperbola of equation (2.2) we get another point $W'(c\tau, x'_\tau)$. Now VW' represents x'_τ and $W'W = x_\tau - x'_\tau = \frac{xt - x't'}{\tau}$. We define this quantity as $v_0\tau$ or $v_0 = \frac{xt - x't'}{\tau^2}$ is the relative speed of O wrt O′ or vice versa.

Conclusion

It is possible to introduce field dependent contracted tensors from ϵ_{ij}, μ^{ij} to define filed vectors **E|D**, **B|H** and the *metric* $d\mathbf{E}^2 - d\mathbf{H}^2$ for Gravitational/EM fields. The linear LT can be considered as a relationship between two local frames of references, whereas equations (2.8) to (2.11) give the relationship among true/proper values and their observed values in the local frames. These equations justify a general principle of fuzziness of measurements as well as the existence of a unique preferred frame viz, the proper frame of reference as suggested by H.A. Lorentz.

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