

## Research Article

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## Effect of a Dc Oblique Magnetic Field on the Magnetic Susceptibility of the Super-Paramagnetics Nanoparticles in Very Low Damping

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**ABSTRACT**

The Magnetic Susceptibility of an individual Super-Paramagnetic nanoparticle in a presence of DC Oblique magnetic fields of arbitrary amplitude is investigated using Brown's continuous diffusion model. The susceptibility is calculated and compared when for extensive ranges of the anisotropy, the dc magnetic fields in the very low damping with Matrix continued Fraction. It is shown that the shape of the Spectrum of Super-Paramagnetic nanoparticles is substantially altered by applying a dc oblique field. There is also an inherent geometric dependence of the complex susceptibility on the damping parameter arising from coupling of longitudinal and transverse relaxation modes.

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**Introduction**

The Magnetics nanoparticles merit a big attention in view of their importance in the context of magnetic recording media [1, 2]. The pioneering theory of thermal fluctuations of the magnetization  $\mathbf{M}(t)$  of a single domain magnetic particle due to Néel was further developed by Brown using the classical theory of the Brownian motion [3, 4]. Brown proceeded by taking as Langevin equation, Gilbert's equation for the motion of the magnetization augmented by a random field [5, 6]. In the context of the Brown continuous diffusion model, the magnetization dynamics of magnetic nanoparticles is similar to the rotation of a Brownian particle in a liquid and is governed by a Fokker-Planck equation for the probability density function  $W$  of  $\mathbf{M}$  [7]. The Fokker-Planck equation is derived from Gilbert's equation with a random field, which takes into account the thermal fluctuations of  $\mathbf{M}$  in an individual Super-Paramagnetic nanoparticle [6]. Referring to magnetic relaxation in uniaxial particles, Brown estimated the reversal time of the magnetization for the case when  $\mathbf{H}_0$  is applied along the easy axis of the magnetization as (in our notation) [4, 5, 7].

$$\tau \approx \tau_N \pi^{1/2} \sigma^{-3/2} \left\{ (1-h^2) \left[ (1+h) e^{-\sigma(1+h)^2} + (1-h) e^{-\sigma(1-h)^2} \right] \right\}^{-1} \quad (1)$$

where  $\sigma = \beta K$  and  $h = \xi / 2 \sigma$  are the dimensionless barrier and field parameters, respectively;  $\xi = \beta M_S H_0$ ,  $\beta = v / (k T)$ ,  $v$  is the volume of the particle,  $T$  is the temperature,  $k$  is the Boltzmann constant,  $M_S$  is the saturation magnetization,

$$\tau_N = \beta M_S (1 + \alpha^2) / (2\gamma\alpha) \quad (2)$$

is the free diffusion time of the magnetization,  $\gamma$  is the gyromagnetic ratio, and  $\alpha$  is the dimensionless damping (dissipation) parameter.

Equation (1) in the low temperature (high barrier,  $\sigma \gg 1$ ) limit only. Cregg et al. have derived an approximate equation for  $\tau$  valid for all values of  $\sigma$  [8]. Aharoni and recently Coffey et al. and Klik and Yao. have reconsidered this problem [9,10 & 11]. They calculated  $\tau$  numerically and demonstrated a good agreement of their results with the Brown Eq.(1).

By applying an uniform magnetic field  $\mathbf{H}_0$  at an oblique angle  $\psi$  with respect to the easy axis, one can break the symmetry of the potential  $V$ , which will also depend on the azimuthal angle  $\varphi$ . In axially symmetric anisotropy Eq. (1) (with  $\mathbf{H}_0$  parallel to the easy axis) the energyscape is a uniform equatorial ridge (zone) separating two polar minima and has no saddle points, on the other hand, the external field  $\mathbf{H}_0$  generates azimuthally nonuniform energy distributions with a saddle point. Such a nonaxially symmetric energyscape leads to a new effect and strong intrinsic dependence of magnetics characteristics (such as the complex magnetic susceptibility and relaxation times) on the value of the damping parameter  $\alpha$  arising from coupling of the longitudinal and the transverse relaxation modes. Ouari and Kalmykov estimated the longitudinal reversal time and the magnetic Susceptibility in Biaxial Super-Magnetic nanoparticles in the presence of longitudinal magnetic field  $\mathbf{H}_0$  [12].

The biaxial (orthorhombic) anisotropy in the absence of DC magnetic field is [7]

$$\beta V_{biax}(\vartheta, \varphi) = \sigma \sin^2 \vartheta + \Delta \sin^2 \vartheta \cos^2 \varphi, \quad (3)$$

Here,  $\Delta$  and  $\sigma$  are the biaxiality and barrier parameters, respectively ( $\Delta=0$  corresponds to uniaxial anisotropy),  $\vartheta$  and  $\varphi$  are polar and the azimuthal angles of the spherical coordinate system.

In this paper we present the results of the magnetic susceptibility of Super-Paramagnetic-nanoparticles subjected to an oblique dc magnetic field  $\mathbf{H}_0$  for wide ranges of the field strengths and

anisotropy energy parameters in the very low damping. We calculate the complex magnetic susceptibility  $\chi(\omega)$  using a matrix continued fraction method. The details of the calculation can be found in Ref [7-10]

### Basic Equations

In the presence of an oblique DC magnetic field, the free energy density take the following form

$$\beta V(\vartheta, \varphi, t) = \beta V_{biax}(\vartheta, \varphi) - \xi(\gamma_1 \sin \vartheta \cos \varphi + \gamma_2 \sin \vartheta \sin \varphi + \gamma_3 \cos \vartheta) \quad (4)$$

where  $\xi_0 = \beta H_0 M_S$ ,  $M_S$  is the saturation magnetization and  $\gamma_1 = \sin \psi \cos \phi$ ,  $\gamma_2 = \sin \psi \sin \phi$ ,  $\gamma_3 = \cos \psi$  are the direction cosines of the vector  $\mathbf{H}_0$ . In spite of the practical importance of biax anisotropy, which may yield an essential contribution to the free energy density of magnetic nanoparticles, the orthorhombic case in the presence of an oblique external field has not yet been solved due to the mathematical difficulties encountered.

In the context of Brown's model, the dynamics of the magnetization  $\mathbf{M}$  of a single domain Super-Paramagnetic nanoparticle may be described by Gilbert's equation augmented by a random field  $\mathbf{h}(t)$  with white noise properties accounting for the thermal fluctuations of the magnetization, viz [4, 5 & 6],

$$\dot{\mathbf{M}}(t) = \gamma [\mathbf{M}(t) \times [\mathbf{h}(t) - \partial V / \partial \mathbf{M} - (\alpha / \gamma M_S) \dot{\mathbf{M}}(t)]] \quad (5)$$

Brown derived from the Gilbert-Langevin Eq. (5), the Fokker-Planck equation for the distribution function  $W(\mathbf{M}, t)$  of the orientations of the magnetization vector  $\mathbf{M}$

$$\frac{\partial}{\partial t} W = L_{FP} W = \frac{1}{2\tau_N} \left\{ \beta \left[ \alpha^{-1} \mathbf{u} \cdot (\nabla V \times \nabla W) + \nabla \cdot (W \nabla V) \right] + \Delta W \right\}, \quad (6)$$

where  $L_{FP}$  is the Fokker-Planck operator,  $\nabla$  and  $\Delta$  are the gradient and Laplacian operators on the surface of unit sphere,  $\mathbf{u}$  is the unit vector directed along  $\mathbf{M}$ . A detailed discussion of the assumptions made in the derivation of the Fokker-Planck and Gilbert equations is given in (Refs. 7-8-13).

A concise theoretical description of the magnetization dynamics in a super-paramagnetic nanoparticle can be given by linear response theory (Refs. 7-8-13). Here it is supposed that a Super-Paramagnetic-nanoparticles in the presence of an Oblique magnetic field  $\mathbf{H}_0$  is subjected in addition to a small probe field  $\mathbf{H}_1$  [ $\beta(\mathbf{M}\mathbf{H}_1) \ll 1$ ]. Then the decay of the averaged magnetization  $\langle M \rangle(t)$  of the particle, when the field  $\mathbf{H}_1$  has been switched off at time  $t = 0$ , is [7-13].

$$\langle M \rangle(t) - \langle M \rangle_0 = \chi H_1 C(t), \quad (7)$$

where  $C(t)$  is the normalized relaxation (correlation) function of the longitudinal component of the magnetization defined as [13].

$$C(t) = \frac{\langle M(0)M(t) \rangle_0 - \langle M(0) \rangle_0^2}{\langle M^2(0) \rangle_0 - \langle M(0) \rangle_0^2} = \sum_k c_k e^{-\lambda_k t}, \quad (8)$$

$\lambda_k$  are the eigenvalues of the Fokker-Planck operator  $L_{FP}$  in Eq.(6),  $\sum_k c_k = 1$ ,  $\chi = \beta [\langle M^2(0) \rangle_0 - \langle M(0) \rangle_0^2]$  is the static susceptibility of the particle, and the brackets  $\langle \rangle$  and  $\langle \rangle_0$  designate the nonequilibrium and equilibrium ensemble averages, respectively. The equilibrium ensemble averages are defined as  $\langle A \rangle_0 = Z^{-1} \int_0^{2\pi} \int_0^\pi A(\vartheta, \varphi) e^{-\beta V(\vartheta, \varphi)} \sin \vartheta d\vartheta d\varphi$  ( $Z$  is the partition function). Having determined  $C(t)$ , one can calculate the oblique magnetic susceptibility of the particle

$\chi(\omega) = \chi'(\omega) - i\chi''(\omega)$  given by [13].

$$\frac{\chi(\omega)}{\chi} = 1 - i\omega \int_0^\infty e^{-i\omega t} C(t) dt = \sum_k \frac{c_k}{1 + i\omega / \lambda_k}. \quad (9)$$

According to Eq.(9), the behavior of  $\chi(\omega)$  in the frequency domain is completely determined by the time behavior of  $C(t)$ . All the calculation numerical methods is detailed in reference [12-13]

### Results and Discussion

The magnetic susceptibility of single-domain Super-Paramagnetic Nanoparticles with biaxial anisotropy is given by the exact equation Eq.(9) formulated in terms of matrix continued fractions.

In Figs. 2-4 we have plotted the results of the calculation of the imaginary part of the normalized susceptibility  $\chi''(\omega)$  ( $\beta M_S^2 = 1$ ) from Eq(9), in the presence (line) and an absence (dashed line) of DC magnetic oblique field. Here is plotted for typical values of the model parameters  $\sigma$ ,  $\Delta$ ,  $\xi$ , in the very low damping  $\alpha=0.005$ . The results indicate that a marked dependence of  $\chi''(\omega)$  on  $\alpha$  and  $\xi$  exists and that three distinct dispersion bands appear in the spectrum of  $\chi''(\omega)$ . The characteristic frequency  $\omega_1$  and half-width  $\Delta\omega$  of the low-frequency band are completely determined by the smallest nonvanishing eigenvalue  $\lambda_1$  [7-13].

Thus the low frequency behavior of  $\chi(\omega)$  is dominated by the barrier crossing mode. In addition, a far weaker second relaxation peak appears at high frequencies. At the low fields, the amplitude of this band is far weaker than that of the first band. However, in magnetic oblique field  $\xi \ll 1$ , this band can dominate in the spectrum (see Fig.2-4). It is evident from all the figures, that the spectra depend strongly on the damping parameter, the magnetic field and on the potential barrier. The Methods for obtaining experimental and theoretical estimates of are discussed, for example, in [14],

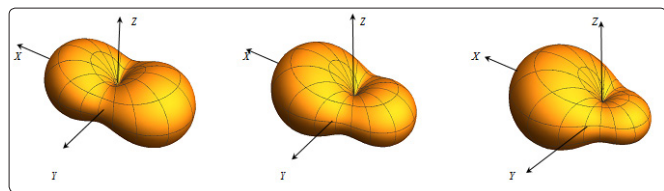
Just as in the absence of the bias field, there is an inherent geometric dependence of  $\chi(\omega)$  on the value of the damping parameter  $\alpha = 0.005$  arising from the *coupling of the longitudinal and transverse relaxation modes*. This coupling appears in the dynamic equation of motion, where the longitudinal component of the magnetization  $c_{||}(t)$  is coupled with the moments  $c_{\perp, \pm}(t)$  and results in the appearance of the third ferromagnetic resonance (FMR) peak in the spectrum of  $\chi''(\omega)$  due to excitation of transverse modes with characteristic frequencies close to the precession frequency of the magnetization, where  $\omega_{pr}^{H_0=0}$  is the precession frequency at  $\mathbf{H}_0 = \mathbf{0}$ .

The FMR peak appears only at low damping ( $\alpha \ll 1$ ) and strongly manifests itself at high frequencies. As  $\alpha$  decreases, the FMR peak shifts to higher frequencies and its half-width decreases (in our normalized units, see Fig. 1-3).

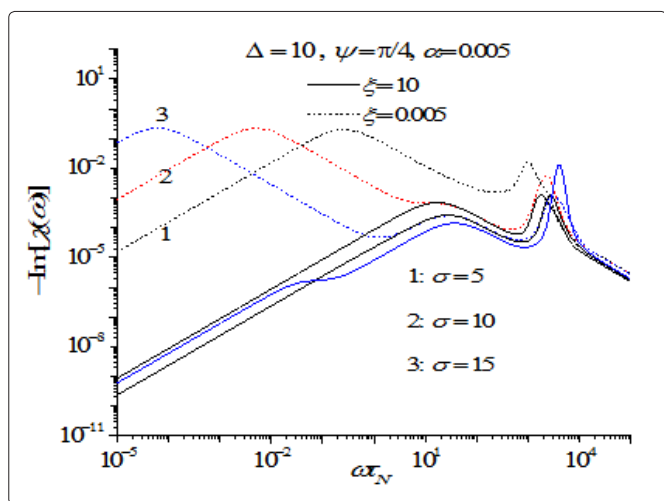
The imaginary and the real parts of the susceptibility as functions of the anisotropy parameter  $\sigma$  is shown in Fig. 5, for various various value of the biaxiality parameter  $\Delta$ . both in the absence ( $\xi \ll 1$ ) and presence ( $\xi \geq 1$ ) of a DC oblique magnetic field. The calculations indicate that a marked dependence of  $\chi(\omega)$  on the anisotropy  $\sigma$ , and the parameter of biaxiality  $\Delta$  exists.

To conclude, a rigorous numerical calculation of the susceptibility of an individual Super-Paramagnetic nanoparticle with biaxial anisotropy has been given using the matrix continued fraction method. The magnetic susceptibility as a function of the barrier  $\sigma$  and the frequency  $\omega$  was investigated in the very low damping.

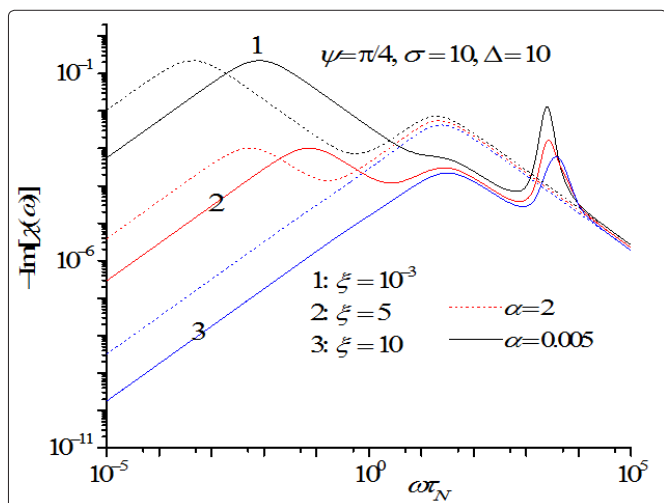
The magnetic process strongly depends on the DC oblique field, the damping, and the temperature. This dependence can be used in many applications like recording media. All the results have been given for the oblique field  $\mathbf{H}_0$ . The matrix continued fraction solution allows us to calculate all quantities of interest for an arbitrary orientation of  $\mathbf{H}_0$ .



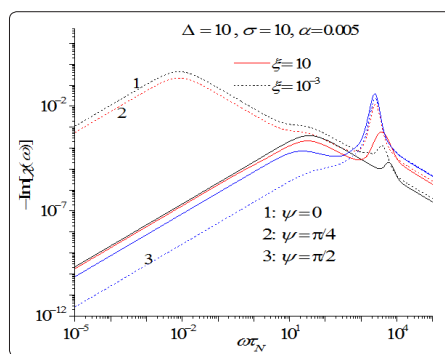
**Figure 1:** Effect of the oblique field on the Potential (Eq.5) ( $\Delta = \sigma \psi = \pi/4$   $\xi = 10^{-6}$  (left),  $\xi = 5$  (Middle),  $\xi = 1$  (Right)).



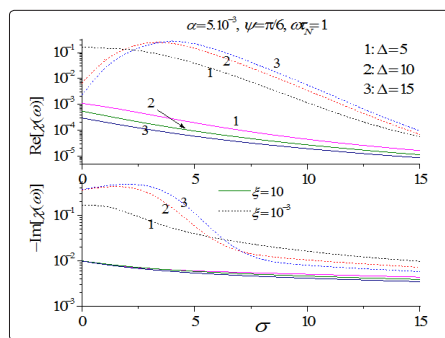
**Figure 2:** Imaginary part of the dynamic susceptibility -  $\text{Im}[\chi(\omega)]$  vs the dimensionless frequency  $\omega\tau_N$  for various dc field barrier parameters  $\sigma$ .



**Figure 3:** Imaginary part of the dynamic susceptibility -  $\text{Im}[\chi(\omega)]$  vs the dimensionless frequency for various dc field parameters  $\xi$ .



**Figure 4** Imaginary part of the dynamic susceptibility -  $\text{Im}[\chi(\omega)]$  vs. the dimensionless frequency  $\omega\tau_N$  for various value of the angle  $\psi$ .



**Figure 5** Imaginary part of the dynamic susceptibility -  $\text{Im}[\chi(\omega)]$  vs. the dimensionless frequency for various value of the biaxiality parameter  $\Delta$ .

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