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Short Communication

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Conservation of Energy and Momentum of Drifting Electrons in Materials

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ABSTRACT

In this research article, conservation of energy and momentum of drifting electrons in materials is expressed in a differential equation form. The equation is derived starting with Einstein's mass-energy equivalence relation of 1905 for relativistic masses. The equation is valid for both small and large objects. Conservation of momentum is added to the equation and it is shown that Noether's theorem is asserted. The drift velocity in metals is drastically reduced due to the much higher electron effective mass in metals.

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Introduction

I show that the conservation of energy and momentum of drifting electrons in materials is expressed in the differential form as:

$$\frac{dE}{E} = \frac{dp}{p} = \frac{dm}{m} \qquad (1)$$

Here, E,p,and m are energy, momentum, and mass of the free electrons in materials, and dE, dp, and dm are differential E, p, and m of the electrons when drifting in materials [1-3]. We start with Einstein's mass-energy equivalence relation of 1905 from his special theory of relativity.

$$E = mc^2 \qquad (2)$$

for a relativistic moving object of mass m where,

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (3)$$

Here, m_0 is the rest mass of the object, v is its velocity, and c is the speed of light. For a given mass m, having a constant velocity v, and at a given temperature, the energy $E=mc^2$ is constant and so the conservation of energy applies, meaning that the energy does not change with time. Two examples are: Pitcher throws a ball at constant velocity of 90 miles/hr in the game of Baseball, and a bowler in Cricket throws a full-toss ball at constant velocity of 140 Km/hr towards the batsman.

For large objects, like a ball, falling from the top of a building under constant acceleration due to gravity g, and ignoring air friction, the distance s covered by the ball in time t is given by the equation:

$$s = \frac{1}{2} g t^2 \qquad (4)$$

$$\frac{s}{t} = v = \frac{1}{2} g t \qquad (5)$$

As t increases, the velocity v increases linearly with time, and the mass m increases continuously with velocity from equation (3), however small increase it may be. The energy $E=mc^2$ therefore also increases continuously. The continuous change in E and m makes the energy equation differentiable. After differentiating once and rearranging, we get:

$$\frac{dE}{E} = \frac{dm}{m} \qquad (6)$$

The above example only shows the differentiability of the energy equation. The total mechanical energy which is the sum of potential and kinetic energies of the falling ball is constant as mgh or (1/2) mv_f^2, with v_f as the final velocity when the ball reaches the ground. So, the conservation of energy applies, meaning that the energy does not change with time. For constant energy, dE=dm=0 in the above equation.

For small objects, like electrons and holes in Si Semiconductor, the drift velocity at high electric fields in 100 KV/cm is constant, as shown in the figure in reference [4]. The kinetic energy E of the electron in materials is given as:

$$E = \frac{1}{2} m v^2 \qquad (7)$$

The constant energy for a given mass can be changed. It can be increased by heating Silicon, so as to supply thermal energy. Therefore, the equation can be differentiated, giving:

$$dE = (\frac{1}{2}v^2)dm + md(\frac{1}{2}v^2) \quad (8)$$

It is experimentally observed that the drift velocity of the electron at higher temperature decreases. Since the drift velocity decreases, and the total energy has increased, therefore the effective mass of the electron has to increase. It cannot remain constant, as shown in the second term of the differential equation. Thus, the second term is invalid, and we have only the first term. Dividing the first term by E on both sides gives:

$$\frac{dE}{E} = \frac{dm}{m} \quad (9)$$

Thus, for both large and small objects, the above equation forms the universal mass-energy equivalence relation. It is Universal because the energy E could be Potential, Kinetic, Electrical, Mechanical, Chemical, Nuclear, Thermal, etc. Adding conservation of momentum involves semiconductor physics in which the kinetic energy is expressed as:

$$E = \frac{p^2}{2m_{eff}} \quad (10)$$

Here p is the momentum and m_{eff} is the effective mass of the electron.

Differentiating twice, the above equation, gives the effective mass as:

$$\frac{d^2E}{dp^2} = \frac{1}{m_{eff}} \quad (11)$$

Integrating the above equation once, and separating the variables, gives:

$$dE = \frac{p}{m_{eff}}dp \quad (12)$$

Now, if the energy E is constant, then dE is equal to zero. This implies that dp is also zero. This implies further that the momentum p of an electron is constant. Momentum of an electron can be written as:

$$p = mv \quad (13)$$

If momentum p has to remain constant then, as the drift velocity v of the electron increases, the electron effective mass m would decrease and vice versa. The conservation of momentum in the differential form can be added as:

$$\frac{dE}{E} = \frac{dp}{p} = \frac{dm}{m} \quad (14)$$

Energy of the free electron in materials is given by the theorem of equipartition of energy as:

$$\frac{1}{2}m_{eff}{v_{th}}^2 = \frac{3}{2}kT \quad (15)$$

Here, v_{th} is the thermal velocity of electron which is about 10^5 meters/sec at 300K temperature, k is the Boltzmann constant and T is the temperature in Kelvin. The theorem states, that for a system in thermal equilibrium, the total energy of the system is evenly distributed as (½)kT for each degree of freedom. In materials, the average kinetic energy of a free electron or hole is equal to (3/2)kT.

In high resistivity semiconductors, v_{th} of an electron at high electric fields of KV/cm and at 300K, is nearly the same as the drift velocity at about 10^5 meters/s. The density of free electrons is of the order of $10^{14}/cm^3$. In metals, the constant drift velocity is about 10^{-4} meters/sec because it has a much higher density of free electrons at $10^{22}/cm^3$ causing more collisions with atoms and defects. Comparing the square of the drift velocities in semiconductors and metals, it can be observed that the square of the velocity has reduced 10^{18} times in metals, resulting in the electron effective mass to be 10^{18} times larger. It is about 10^{-12} kg, given that the mass of free electron is about 10^{-30} kg. There is a small heat loss in materials due to inelastic collisions of electrons with atoms and defects.

Noether's theorem is asserted according to which, if a conservation law governs a system, then there is presence of symmetry in the physical action of the system and vice versa. The electrons, drifting at constant velocity, have zero acceleration and so no force is applied to the objects. The position, velocity, and momentum of an object will remain the same with time implying conservation of energy and momentum. The laws of physics will accurately describe the behavior of the objects in a reference frame moving at constant velocity. Here, the inertial reference frame is that of the electrons moving at constant velocity. Thus, conservation of energy and momentum of drifting electrons in materials at constant velocity implies time symmetry of nature.

A new science in Electron Physics which is already published is highlighted again in this communication. After finding the universal mass-energy equivalence relation (dE/E) = (dm/m), a new science in Physics was discovered. The intrinsic Fermi energy level in a semiconductor given as dE below the conduction band of the semiconductor was found to be equal to relative longitudinal electron effective mass in materials as (dm/m), multiplied by the bandgap of the material as E. The electron at the intrinsic Fermi energy level E_i, possesses potential energy equal to dE relative to the conduction band and it possesses kinetic energy at the conduction band equal to dE relative to the intrinsic Fermi energy level E_i in the bandgap. From the conservation of energy, the change in potential energy equals the change in kinetic energy. For example: Si (100) has the relative longitudinal electron effective mass of one conduction valley as (dm/m) = 0.49, and the Si bandgap is E =1.12 eV, both at 300K temperature. The two multiplied together gives the intrinsic Fermi energy level dE below the Si conduction band as E_i = 0.49 x 1.12 = 0.5488. This value becomes 0.550 eV if a third decimal place value of 3 or 4 in the Si bandgap value of 1.12 eV is assumed. This value of 0.55 eV is well accepted but the physics was unclear till now. The E_i in all materials having a bandgap can be found. The electron and hole effective mass add to give the free electron mass with the electron mass always smaller than the hole mass. Only one of them needs to be determined. The implication of this new finding is that all the properties of a Metal-Oxide-Semiconductor (MOS) device on ANY parabolic semiconductor can be found without fabricating the MOS device.

Conclusion

Conservation of energy and momentum applies to all materials such as metals, insulators and semiconductors that can be expressed in a differential equation form. Inelastic collisions and effective masses of electrons in materials result in loss of energy. Overall, taking the lost energy into account, the conservation of energy and momentum is applicable.

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