ISSN: 2754-4966

Journal of Nanosciences Research & Reports

SCIENTIFIC Research and Community

Research Article

Open Access

Analysis of Dynamics of Infected Active and Uninfected Active Populations Leading to Pandemics using a Discrete Model of Two Interacting Pacemakers Taking into Account the Time of Refractoriness

Sergey Belyakin[1*], Sergey Shuteev[2]

[1]Department of General Physics, Physics Faculty, Lomonosov Moscow State University, Moscow, Russia

[2]Laboratory of dynamic systems, Physics

ABSTRACT

In this publication, we generalize the proposed model of two interacting oscillators in the case of a strong difference in their periods (when the pacemaker pulses do not alternate) and propose a General model describing a network of oscillators coupled globally. Our goal is to make the model as simple as possible and enter the minimum number of parameters. Therefore, we will fully characterize the pacemaker of their internal lengths of the cycle and re-present them as pulse oscillators. Interaction of pacemakers is described by PRC.

***Corresponding author**

Sergey Belyakin, Faculty, Lomonosov Moscow State University, Moscow, Russia. E- Mail:belyakin1962@mail.ru

Received: September 26, 2020; **Accepted:** October 03, 2020; **Published:** October 10, 2020

Keywords: The Function Phase Response Curve, Pacemaker, Discrete System,The Neural Network Convolution

Introduction

The circle maps curve the phase response and the Arnold tongues of Consider some physical quantity ξ, which reflects the internal state of the biological oscillator. Let the eigenfrequency of the oscillator be equal T0. Let's call a marker any event that can be clearly seen in the experiment, which is reached by the value ξ only once per period. Such a marker may be, for example, the beginning of the action potential in the cardiac preparation. Let's define the oscillator phase as follows. The phase of an arbitrarily selected marking event (for example, the maximum value of ξ) is assumed to be zero. At any next time t, $0 < t <$ T0, the phase is defined as $\varphi = t/T_0$ (mod1). Since the rhythm is restored after the perturbation of the system, the introduced phase completely determines the state of the system.

Suppose that an external periodic perturbation acts on a nonlinear oscillator. Then each external influence shifts the state of the system to a new state (1):

$$\varphi_{n+1} = \varphi_n + f(\varphi n) \text{ (mod1)}. \quad (1)$$

The function f (φn) is called the phase response curve (PRC) [1-3] and determines the phase change after the stimulus. It is convenient to represent the points $f(\varphi n)$ of the system state lying on the circle of the unit radius. Then, by iterating the mapping (1), one point of the circle is converted to another point of the same circle. If the circle map is continuous, then it can be characterized by a number called the topological degree and equal to the number of passes through φ_{n+1} the unit circle during $f(\varphi n)$ the time it passes once. In periodic perturbations of self-oscillations with a stable limit cycle, the dynamics is often described by maps of a circle with a topological degree 0 (when the over-threshold response gives rise to a new cycle) or 1 (which expresses a sub-threshold response to stimulation). The different types of circle maps are shown in Fig.1.

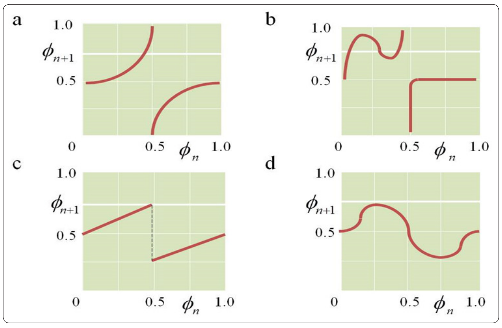


Figure 1: Different types of circle maps [1]: (a) reversible, topological degree 1; (b) irreversible, topological degree 1; (c) piecewise continuous; (d) topological degree 0.

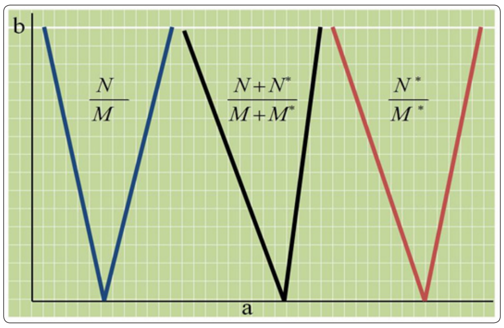


Figure 2: Schematic diagram of Arnold tongues. In shaded areas there is a steady phase capture. There are always other zones between any two capture zones.

The analysis of bifurcations of reversible circle maps was undertaken in the last century by A. Poincare and still attracts much -by V. I. Arnold (see also and the references given there). For fig.2 the bifurcation diagram of the circle diffeomorphism on the parameter plane (b, a) is shown. This diagram is divided into areas called language (or horns) of Arnold, which correspond to the sustainable capture phase ratio N/M (i.e., N cycles of the stimulator has M cycles of a nonlinear oscillator). Arnold languages exist for all rational relations N/M, where N and M are mutually Prime numbers. This means that there are an infinite number of Arnold languages that correspond to all possible ratios of frequencies of the stimulator and the perturbed oscillator. Between any two languages corresponding to N/M and N*/M* phase captures, there is another capture region corresponding to the capture of multiplicity phases (N+N*)/(M+M*). The structure shown in Fig.2, is the usual behavior for low stimulus amplitudes in simple theoretical models discussed below. However, as the amplitude of the periodic effect increases, this structure collapses [4,5].

To analyze the pandemic state within an infected society, we can use the simplest case (and the roughest) approximations of this PRC, which is the sinusoidal function. Unencrypted phase regions can be used as a quarantine state for an infected society. Where: δ - corresponds to the distance inside the infected society, a - corresponds to the given time spent inside the infected society, γ - corresponds to active behavior within the infected society.

Phase of Seizure with One-Sided Interaction of the Pacemaker Taking Into Account the Refractoriness

The simplest case of a period of refractoriness [6-12]:

$$x_{n+1} = \begin{cases} x_n + a, & 0 \le x_n \le \delta \quad (\mathrm{mod}\,1) \\ x_n + a - \gamma \sin(2\pi x_n), & \delta < x_n \le 1 \quad (\mathrm{mod}\,1). \end{cases} \qquad (3)$$

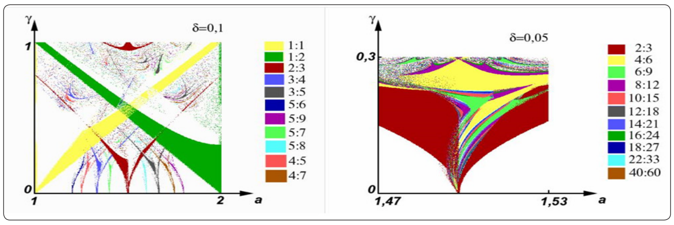


Figure 3: Phase diagram for the sine circle mapping with consideration for the refractoriness period (3) and stable phase captures of the sine map (3) inside the 2:3 capture splitting region.

Let's say, for example, δ=0.1 . The General structure of the phase capture regions obtained as a result of a numerical study of the system (3) is shown in (Fig.3.left) it is Clearly seen that taking into account the period of refractoriness splits the Central languages. In these areas, a detailed study of the phase pattern was conducted. It turned out that the presence of refractoriness time leads to the appearance of phase captures that are multiples of the main one. For example, regions with a multiplicity of 2k:3k for a whole k were found inside the 2:3 capture (Fig.3. right). A similar pattern can be observed inside the splitting of other captures. To analyze the transmission of infection from an infected member of society to an uninfected member of society, we use a system of two interacting nonlinear pulse oscillators.

Model of Two Interacting Pacemakers Taking Into Account the Refractoriness Time

In this section, we consider two interacting leading centers (pulse oscillators) that can be pacemakers in cardiac tissue, construct a model of such interaction, and investigate its behavior [13-19].

The Principle of Constructing a Model

Consider a system of two interacting nonlinear pulse oscillators fig.4. Let the momentum of the first oscillator with the period of undisturbed oscillations appear at the moment of time, and the momentum of the second oscillator with the period of undisturbed oscillations appear at the moment. Then the moments of time of occurrence of the following pulses are defined as:

$$t_{n+1} = t_n + T_1,\ \tau_{n+1} = \tau_n + T_2 .$$

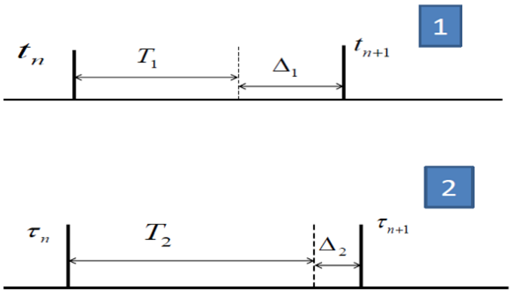


Figure 4: The scheme of construction of the model describing the system of two interacting nonlinear oscillators

Now, assuming that under the influence of the second pulse, the period of the first oscillator will change by some value $\Delta_1((\tau_n - t_n)/T_1)$ (where the expression in parentheses shows that this value depends only on the phase of the second pulse relative to the first), then the corresponding expression for t_{n+1} will look like: $t_{n+1} = t_n + T_1 + \Delta_1((\tau_n - t_n)/T_1)$. When you consider that $\tau_{n+1} > t_{n+1}$, that for τ_{n+1} get a similar expression:

$\tau_{n+1} = \tau_n + T_2 + \Delta_2((t_{n+1} - \tau_n)/T_2)$. Dividing both of these expressions by T_1, we find the corresponding expressions for the phases (4):

$$\begin{cases} \varphi_{n+1} = \varphi_n + \dfrac{1}{T_1}\Delta_1(\delta_n - \varphi_n), \\ \delta_{n+1} = \delta_n + \dfrac{T_2}{T_1} + \dfrac{1}{T_1}\Delta_2\left(\dfrac{t_n}{T_2} + \dfrac{T_1}{T_2} + \dfrac{1}{T_2}\Delta_1(\delta_n - \varphi_n) - \dfrac{\tau_n}{T_n}\right). \end{cases} \qquad (4)$$

Here $\varphi_n = t_n/T_1$ – phase of the first perturbed oscillator relative to the undisturbed (with the period T_1), $\delta_n = \tau_n/T_1$ – the second phase of the disturbed oscillator with respect to the same first oscillation with a period of T_1. Introducing the parameter $a = T_2/T_1$ (the ratio of the eigenfrequencies of both oscillators) and labeling $f_1 = \Delta_1/T_1$, $f_2 = \Delta_2/T_1$, after the transformations we obtain (5):

$$\begin{cases} \varphi_{n+1} = \varphi_n + f_1(\delta_n - \varphi_n), \\ \delta_{n+1} = \delta_n + a + f_2\left(\dfrac{1}{a}(\varphi_n + 1 + f_1(\delta_n - \varphi_n) - \delta_n)\right). \end{cases} \qquad (5)$$

Since we are interested in the phase difference of the described oscillators, the final expression, which will be used in the future, is as follows (6):

$$xn+1 = xn + a + f2[(1/2)(1 + f1(xn) - xn)] - f1(xn) \qquad (6)$$

where $x_{n+1} = \delta_n - \varphi n$.

Expression $g(x_n) = x_n + a + f_1(x_n)$, included in the right side of the equation is a circle map describing the effect of the constant perturbation on the nonlinear oscillator. Taking into account the mutual influence of oscillators leads to the appearance of an additional nonlinear term. Thus: $x_{n+1} = g(x_n) + f_2[a^{-1}(1 - g(x_n))]$ (mod1).

Function $f_1(x)$, $f_2(x)$ called phase response curves, which generally do not coincide with each other. Both oscillators are sources of action potentials in the same tissue, have a similar nature, and can be considered functions $f_1(x)$ and $f_2(x)$ approximately the same. It is known that the response of the oscillator to an external stimulus depends only on the stimulus phase of its amplitude and the PRC changes its shape when the amplitude of the external influence changes. This means that the functions that define the type of phase response curves must depend on one parameter that determines the magnitude of the amplitude. In the case of this dependence can be considered multiplicative. Then the phase response curves will be written as: $f_1(x) = \gamma h(x)$, $f_2(x) = \varepsilon h(x)$.

Where $h(x)$ − periodic function, $h(x+1) = h(x)$. Under this assumption, the formula (6) will take the form:

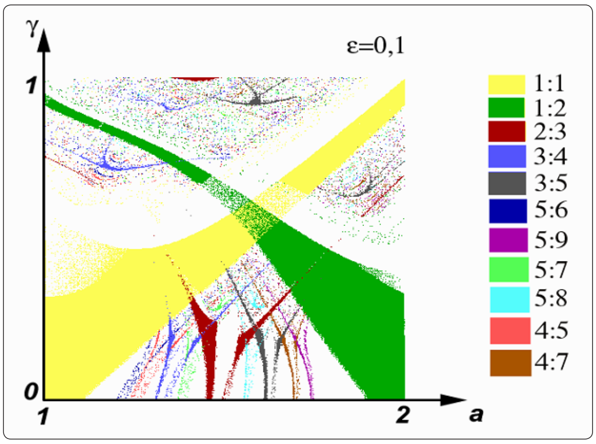
$$x_{n+1} = x_n + a + \varepsilon h[(1/a)(1 + \gamma h(xn) - xn)] - \gamma h(xn) \quad (\mathrm{mod}1), \ (7)$$

Let's focus on the study of the display (7) sinusoidal functions [20, 21].

General Case of Interaction between Two Pacemakers Phase Captures In Two-Way Interaction of Oscillators

As a model of two non-linearly interacting excitation sources, we consider two coupled oscillators, assuming the role of h(x) sinusoidal function without taking into account the refractoriness and assuming the value of the influence of the first oscillator on the second ε=0.1 . Then display (2) will take the form (8) [22,23]:

$$x_{n+1} = x_n + a + \frac{1}{10}\sin\left(\frac{2\pi}{a}(1+\gamma\sin(2\pi x_n) - x_n)\right) - \gamma\sin(2\pi x_n) \quad (8)$$



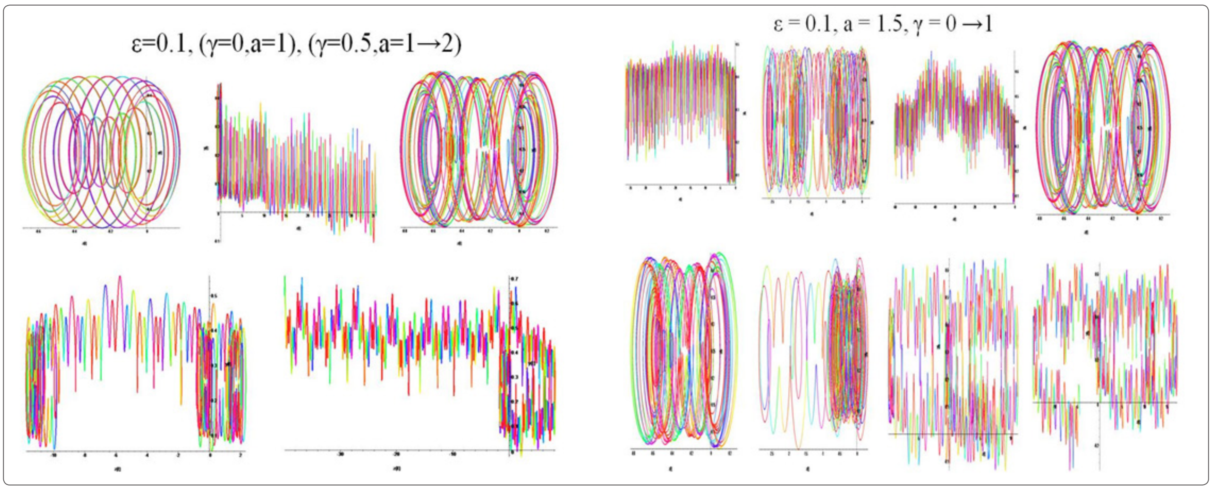


Figure 5: Phase diagram of the sine mapping of the circle (8) taking into account the mutual influence of oscillators

The location of the phase capture regions obtained as a result of numerical research (8) is shown in Fig.5. Similarly to the case of piecewise linear approximation of phase response curves, taking into account the mutual influence of two pulse systems leads to the curvature of the phase capture regions, their overlap at low γ and splitting of the main languages. Within the split regions, grabs that are multiples of the main one occur.

Analysis of the model of two interacting oscillators

To analyze the transmission of infection from an infected member of society to an uninfected member of society, we use a system of two interacting nonlinear pulse oscillators. To analyze the transmission of infection from an infected society to an uninfected society, we use a system of two interacting nonlinear pulse oscillators.

Unencrypted phase regions can be used as a quarantine state. Phase portraits with a stable periodic cycle correspond to infection, and areas of chaos can be used as a quarantine state.Phase portraits with a stable periodic cycle correspond to infection, areas of multiple bifurcations and chaos can be used as a quarantine state.

Approximation of the Active Medium as a Lattice of Pulse Oscillators

In this section, we will demonstrate a way to approximate discrete distributed environments based on the General model of coupled oscillators (8). Looking at the heart pacemaker at a microscopic level, it can be thought of as a large group of cells that generate heart rate and synchronize their action potentials to initiate heart contractions. Thus, instead of considering a single pacemaker, we can construct a lattice of coupled pulse oscillators. In this paper, we have limited ourselves to one-dimensional (chain) and two-dimensional (lattice) cases [24].

Assume that the Autonomous pacemakers are located at the nodes of a two-dimensional square lattice of size (N x M). We denote the lattice element with coordinates (i, j) as A_{ij}, where i=1,..., N and j=1,..., M. we restrict ourselves to considering a homogeneous medium and accept some restrictions on anisotropy. This means that the lattice pacemakers are identical, i.e. they have the same cycle length $T_{ij} \equiv T$, i=1,..., N; j=1,..., M, (however, in reality, the cells on the periphery of the sine pacemaker have the shortest cycle length, although its center acts as the leading pacemaker).

This restriction reduces the number of system parameters and therefore makes it easier to study the model. Now we will define the relationship between the elements. In works on lattices of concatenated maps, two main types of coupling are usually considered: nearest neighbor coupling and global coupling. Since in the previous sections we assumed that pacemakers all interact with each other, this time, as an example, we will consider lattices with a connection of the nearest neighbor type: first a two – dimensional lattice, and then a chain of coupled pulse oscillators [25].

References

1. Kaneko K (1989) spatiotemporal chaos in one-dimensional and two-dimensional coupled map lattices. // Physica D 37: 60-82.
2. Arnold VI, Gindikin SG, Guillemin VW, Kirillov AA, Kostant B, et al. (1991) Cardiac arrhythmias and circle mappings. // Izrail M. Gelfand: Collected Papers. / Eds.: Springer-Verlag. 1989. P.3 Reprinted in Chaos 1: 20-24.
3. Loskutov A, Rybalko SD, Zhuchkova EA. (2003) A model of cardiac tissue as a conductive system with interacting pacemakers and refractory time. // Int. J. Bifurcation and Chaos 14: 2457-2466.
4. Boyland P L (1986) Bifurcations of circle maps: Arnol'd tongues bistability and rotation intervals. // Commun. Math. Phys 106: 353-381.
5. Kaneko K (1984) Supercritical behavior of disordered orbits of a circle map. // Progr. Theor. Phys 72: 1089-1103.
6. Berklinblit MB, Gelfand IM, Garfinkel VS, Fomin SV, Tsetlin ML. Eds. (1971) Models of the Structural–Functional Organization of Certain Biological Systems. // Cambridge: MIT Press.
7. Glass L, Belair J (1986) Continuation of Arnold tongues in mathematical models of periodically forced biological oscillations. // Nonlinear Oscillations in Biology and Chemistry. Lect. Notes in Biomathematics. No.66. / Ed. Othmer H.G. Berlin: Springer–Verlag. 232-243.
8. Glass L, Mackey MC A (1979) simple model for phase locking of biological oscillations. // J. Math. Biol 7: 339-352.
9. Keener JP, Hoppensteadt FC, Rinzel J (1981) Integrate and fire models of nerve membranes response of oscillatory inputs. // SIAM J. Appl. Math 41: 503-517.
10. Glass L. (1991) Cardiac arrhythmias and circle maps – A classical problem. // Chaos 1:13-19.
11. Keener JP (1981) On cardiac arrhythmias: AV conduction block. // J. Math. Biol 12: 215-225.
12. Glass L, Guevara MR, Shrier A (1987) Universal bifurcations and the classification of cardiac arrhythmias. // Ann. (NY) Acad. Sci 504: 168-178.
13. Shrier A, Dubarsky H, Rosengarten M, Guevara MR, Nattel S, Glass L (1987) Prediction of atrioventricular conduction rhythms in humans using the atrioventricular nodal recovery curve. // Circulation 76: 1196-1205.
14. Hoppensteadt FC, Keener J (1982) Phase locking of biological clocks. // J. Math. Biol 15: 339-349.
15. Guevara MR, Glass L (1982) Phase locking, period doubling bifurcations and chaos in a mathematical model of periodically driven oscillator: A theory for the entrainment of biological oscillators and the generation of cardiac dysrhythmias. // J. Math. Biol 14: 1-23.
16. Keener J,Glass L (1984) Global bifurcations of a periodically forced oscillator. // J. Math. Biol 21: 175-190.
17. Zaslavsky GM (1978) The simplest case of a strange attractor. // Phys. Lett. A 69: 145-147.
18. Ding EJ. (1987) Analytic treatment of a driven oscillator with a limit cycle. // Phys. Rev. A 35: 2669-2683.
19. Guevara MR,Glass L, Shrier A (1981) Phase locking, period doubling bifurcations, and irregular dynamics in periodically stimulated cardiac cells. // Science 214: 1350-1353.
20. Moe GK, Jalife J, Mueller WJ, Moe BA (1977) mathematical model for parasistole and its application to clinical arrhythmias. // Circulation 56: 968-979.
21. Jalife J, Antzelevitch C, Moe G (1982) The case for modulated parasistole. // Pace 5: 911.
22. Ikeda N (1982) Model of bidirectional interaction between myocardial pacemakers based on the phase response curve. // Biol. Cybern 43: 157-167.
23. Ikeda N,Yoshizawa S, Sato T(1983) Difference equation model of ventricular parasystole as an interaction between cardiac pacemakers based on the phase response curve. // J. Theor. Biol 103: 439-465.
24. Courtemanche M, Glass L, Belair J, Scagliotti D, Gordon D. (1989) A circle map in a human heart. // Physica D 40: 299-310.
25. Willeboordse FH, Kaneko K (1994) Bifurcations and spatial chaos in an open flow model. // Phys. Rev. Lett 73: 533-536.