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About Some Applications of Mathematics

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ABSTRACT

In this paper, a new mass-velocity relation $m = m_0 \text{ Exp}\left(v^2/c^2\right)$ is derived. The orbit of planetary motion is deduced based on this new relation and the Newtonian laws of motion along with the concept of proper time. The derivation does not use the tensor concept, but the relativistic orbit $\frac{d^2u}{d\phi^2}+u=\frac{m}{h^2}+mu^2$ is obtained as a first approximation of the modified Newtonian theory. Next, we discuss the possibility of introducing a field dependent metric for the EM field by considering the observation that $E^2 - c^2B^2$ or $E^2 - B^2$ with $c = 1$ is approximately an invariant quantity for EM Field [1-3]. It is possible to extend the metric for the gravitational case also [4]. The discussion of anomalous characteristic of Lorentz Transformation (LT) and the introduction of a non-linear transformation connecting Local Space-time coordinates (ct,x) with proper system $(c\tau,\ x_\tau)$ is continued.

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Keywords: We use the following keywords and notations: Local time interval (dt), Proper time interval ($d\tau$), gravitational field intensity $(\mathbf{E}_1^-)$ gravitational permittivity $\in_1$, gravitational constant $(G=\frac{1}{4\pi\in_1})$, mass of a source mass/Sun (M), scaled mass of Sun $(^-)$, reduced mass (μ) gravitational permeability (μ_1) such that $\mu_1 \in_1 c^2 = 1$ where is the maximum value of signal speed, gravitational potential (ϕ_1) and proper time interval

$$c^2d\tau^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 = c^2dt^2 - d\mathrm{R}^2 - \mathrm{R}^2d\theta^2 - \mathrm{R}^2\sin^2\theta d\varphi^2.$$

Similarly for the EM field we use $\mathbf{E}_2/\mathbf{H}_2$, ϵ_2, μ_2, such that $\epsilon_2\mu_2c^2 = 1$

Introduction

The Newtonian system needs a basis for concepts such as velocity, acceleration etc. relative to which these concepts are well-defined. It is a matter of difficulty to discover and distinguish true motions of particular bodies from the apparent, because the parts of the immovable space in which those motions are performed, do by no means come under the observation of our senses. If every motion is relative and everything in the world is in motion, a question arises according to Newton: how can one ever set up a determinate theory of motion? So, he introduced the concepts of absolute time and absolute space and deduced the law of gravitation: the inverse square law. The theory was not well received even in Newton's own university of Cambridge; the inverse square law without modifications lead to an elliptic orbit [5]. It is observed that the orbit of the planet Mercury is not exactly elliptic [2,3,5] but a rotating ellipse such that the major axis advances by 42 seconds of arc per century. The special theory of relativity was introduced, in order to remove some of the observational difficulties in electro-magnetism. Thus, the concepts of local time and proper time emerged; we shall use these according to Lorentz' interpretation [6-9,10-13]. Still there are some weaknesses; we come across relations such as $c^2dt^2 - dx^2 = c^2dt'^2 - dx'^2$ (a) and $cdtdx = cdt'dx'$ (b) [2,3,9,14,15]. These two equations imply $(cdt+idx)^2 = (cdt'+idx')^2$ so that $cdt = cdt'$ and $dx = dx'$ or $t = t'$ and $x = x'$, except for an additive constant which can hold according to Newtonian concepts only [9, 12]. This means equation (b) is invalid. The inconsistency between the primed and unprimed coordinates can be removed by replacing the primed quantities in favour of the proper coordinates $(cd\tau, dx_\tau)$. Thus, we have $c^2dt^2 - dx^2 = c^2d\tau^2$ (a') and $cdtdx = cd\tau\ dx_\tau$ (b').

These two equations can be rewritten as $cd\tau = cdt\sqrt{1-\left(\frac{dx}{cdt}\right)^2}$ (c) and $dx = dx_\tau\sqrt{1-\left(\frac{dx}{cdt}\right)^2}$ (d).

Thus, the inconsistency between equations (a) and (b) have been removed by dispensing with the primed coordinates. The former defines the proper time interval and the latter implies contraction hypothesis [6, 7, 9, 10, 12]. These clearly show that the equation $cdtdx = cdt'dx'$ with $dt' \neq dt$ is impossible. Thus, by replacing the primed coordinates by the proper values$(cd\tau, dx_\tau)$ where $d\tau$, dx_τ are the proper values estimated from(cdt, dx), the use of the primed coordinates is invalid in the relativistic sense. Hence the assertion, that the primed coordinates(cdt', dx') and the unprimed coordinates (cdt,dx) are equally valid inertial space time coordinates, is invalid due to the pseudo-Euclidean nature of space-time. On the other hand, the Lorentz Transformation can be interpreted as a general form of Dopplershift and Aberration formulae and hence the proper

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frame $(cd\tau, dx_\tau)$ is the unique preferred frame.

Relation Between (ct, x) and $(c\tau, x_\tau)$
The simplest transformation between local and proper space time coordinates may be taken as

$$c\tau = \sqrt{\left|c^2t^2 - x^2\right|} \qquad \text{(i)}$$

$$x_\tau = \frac{xt}{\sqrt{\left|c^2t^2 - x^2\right|}} \qquad \text{(ii)}$$

The inverse transformation is $c^2t^2 = \frac{1}{2}c^2\tau^2\left[\sqrt{1+\frac{4x_\tau^2}{c^2\tau^2}}+1\right]$ (iii)

$$x^2 = \frac{1}{2}c^2\tau^2\left[\sqrt{1+\frac{4x_\tau^2}{c^2\tau^2}}-1\right] \text{ (iv)}$$

These equations represent the transformation between the proper system $(c\tau, x_\tau)$ and the local system (ct, x)This can be generalised to 3+1-dimensional case by including $y_\tau = y$, $z \quad z$ The geometrical representation of $(c\tau, x_\tau)$ is as follows. We can represent the rectangular hyperbola of equation (i), $c\tau = \sqrt{\left|c^2t^2 - x^2\right|}$ in the *ct–x* plane, for a constant value of τ. Let it's vertex be $V(c\tau, 0)$; erect the tangent at vertex $V(c\tau, 0)$, cutting the conjugate/orthogonal hyperbola $2xct = 2x_\tau c\tau$ (ii) at the point $W(c\tau, x_\tau)$ for constant value of $(c\tau, x_\tau)$. Now VW represents x_τ and OV represents $c\tau$, where O is the origin of coordinates of the local frame.

Field Dependent Metric for EM Field/Gravitational Field
The matrix of LT for (ct', x', y') space to (ct, x, y) space is given by

$$L = \begin{bmatrix} \frac{1}{\sqrt{1-e^2}} & \frac{e}{\sqrt{1-e^2}} & 0 \\ \frac{e}{\sqrt{1-e^2}} & \frac{1}{\sqrt{1-e^2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where e stands for $\frac{v_0}{c}$ *and* v_0 is the speed of apparent relative motion and *c* is the maximum signal velocity of the gravitational field in the discussion. Similarly, *c* is signal velocity of the EM field when we consider EM Fields [1, 16]. In the general case of 3⊕1 dimensions, the Lorentz matrix can be given by

$$L = \begin{bmatrix} \sec\phi & \tan\phi\cos\alpha & \tan\phi\cos\beta & \tan\phi\cos\gamma \\ \tan\phi\cos\alpha & 1+(\sec\phi-1)\cos^2\alpha & (sec\phi-1)cos\alpha\, cos\beta & (\sec\phi-1)\cos\alpha\cos\gamma \\ \tan\phi\cos\beta & (\sec\phi-1)\cos\alpha\cos\beta & 1+(\sec\phi-1)cos^2\beta & (sec\phi-1)cos\beta\, cos\gamma \\ tan\phi\, cos\gamma & (sec\phi-1)cos\alpha\, cos\gamma & (sec\phi-1)cos\beta\, cos\gamma & 1+(sec\,\phi-1)\cos^2\gamma \end{bmatrix}$$

implies

$$c^2dt^2 - dx^2 - dy^2 - dz^2 \equiv c^2dt'^2 - dx'^2 - dy'^2 - dz'^2 \equiv c^2d\tau^2 \cdot$$

By letting

(i) $v_0 = v_0\left(i\, cos\alpha + j\, cos\beta + k\, cos\gamma\right)$

$$= v_0\left(\boldsymbol{i}\cos\omega\tau\sin\omega_0\tau + \boldsymbol{j}\sin\omega\tau\sin\omega_0\tau + \boldsymbol{k}\cos\omega_0\tau\right)$$

(ii) $v_0 = a + b\tau + \ldots = v_0(\tau)$ and

(iii)(a) $\tan\phi = v_0c^{-1}$ *or* $(iii)(b)\sin\phi = v_0c^{-1}$, (two distinct cases) we see that L can handle accelerated motion of O'/O relative to $O/O'$and the *metric* is pseudo-Euclidean but not non-Euclidean as in GTR.

The GTR excludes the possibility of a parallel theory for the motion of mass particles and charged particles by highlighting the observation that the energy momentum tensor of EM field has a vanishing trace. On the other hand, the vector fields can be generalized by means of contracted tensor fields.

We have the constitutive relations $\mathbf{D}_2 = \in_2 \mathbf{E}_2$ and $\mathbf{B}_2 = \mu_2\mathbf{H}_2$ for EM fields [1,2,1315,16]. We shall modify the vectors $\mathbf{E}_2$ and $\mathbf{H}_2$ by means of contracted tensor fields. We know that the metric in tensor calculus is given by $(ds)^2 = g_{ij}dx^idx^j = \left(g_{ij}dx^i\right)dx^j = dx_jdx^j$ where g_{ij} is a (0, 2) tensor and dx_j, dx^j are the covariant and contra-variant components of the same vector [17].

Keeping these ideas in view, let us introduce, for the gravitational field, two reciprocal/conjugate tensors $\in_{ij}$ and $\in^{ij}$ so that $\in^{i\alpha}\in_{\alpha j} = \in\delta^i_j$. we define $E_i = \in_{i\alpha} E^\alpha$ and $E^i = \in^{i\alpha} E_\alpha$ as the covariant and contra-variant components of **E**.

Now $\in_{ij} E^iE^j = \left(\in_{ij} E^i\right)E^j = E_jE^j = |\mathbf{E}|^2$ and

$\in^{ij} E_iE_i = \left(\in^{ij} E_i\right)E_j = E^jE_j = |\mathbf{E}|^2$ hence $|\mathbf{E}|^2 = \in_{ij} E^iE^j = \in^{ij} E_iE_i$

Similarly, we introduce two reciprocal/conjugate tensors μ_{ij} and μ^{ij} so that $\mu^{i\alpha}\mu_{\alpha j} = \mu\delta^i_j$ we define $H_i = \mu_{i\alpha}H^\alpha$ and $H^i = \mu^{ij}H_j$ as the covariant and contra-variant components of **H**

Also $\mu_{ij}H^iH^j = \left(\mu_{ij}H^i\right)H^j = H_jH^j = |\mathbf{H}|^2$ and

$\mu^{ij}H_iH_j = \left(\mu^{ij}H_i\right)H_j = H^jH_j = |\mathbf{H}|^2$. Therefore $|\mathbf{H}|^2 = \mu_{ij}H^iH^j = \mu^{ij}H_iH_j$.

Hence our tensors have the properties that

(i $\in_{ij}$ and $\in^{ij}$ are reciprocal tensors
(ii) μ_{ij} and μ^{ij} are reciprocal tensors
(iii ∈'s and μ's are the dielectric parameter $\in_1$ and the susceptibility μ_1, for gravitational fields and $\in_2$, μ_2 for EM fields by extending the Maxwell-Lorentz theory to gravitational fields [17].

Thus for an infinitesimal region, it is possible to replace vector-fields by means of contracted tensor fields and the metric $d\mathbf{E}_2^2 - d\mathbf{H}_2^2$ (with $c = 1$) $= \in_{ij} dE^idE^j - \mu_{ij}dH^idH^j$ is a metric for the EM field. Similarly we can form the *metric* $d\mathbf{E}_1^2 - d\mathbf{H}_1^2$ with (c=1) $= \in_{ij} dE^idE^j - \mu_{ij}dH^idH^j$ for the gravitation field by changing $\in_2$ and μ_2 to $\in_1$ and μ_1 with corresponding definitions given above.

Characteristic of LT
The LT equations satisfy

$$c^2\tau^2 = c^2t'^2 - x'^2 \qquad (2.1)$$

$$c^2\tau^2 = c^2t^2 - x^2 \qquad (2.2)$$

Both (1) and (2) represent rectangular hyperbolas (RH).

By using the substitution

$$ct' = ct\cosh\varphi - x\sinh\varphi \qquad (2.3) \qquad (x' \text{ axis})$$

$$x' = x\cosh\varphi - ct\sinh\varphi \qquad (ct' \text{ axis})$$

the equation (2.1) becomes (2.2). A question arises at this stage: do P'(ct', x') and P (ct, x) represent points on two different RH in accordance with different time access as shown in Figure 1 or do they represent two points on the same RH and have the same time-axis, having two separate origins O, O' or coincident origins $O = O'$ or two nearby RH's with time axis distinct but inclined to each other at a small angle intersecting at the common world point (00).

We consider the following facts (see Figure 1). Equation (2.3) represents the x' - axis and (2.4) represents the ct'-axis. The slope of these lines are $\frac{1}{e}$ and e respectively where $e = \frac{v_0}{c} = \tanh\varphi$

If θ is inclination of ct'-axis with ct - axis, then $\tan\theta = e$ and slope of $x - axis = \frac{1}{e} = \cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$

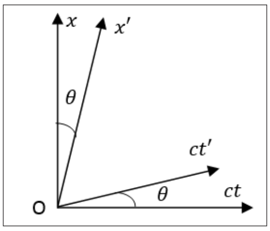


Figure 1

$\therefore x' - axis$ has inclination $\frac{\pi}{2} - \theta$ and hence $x' - axis$ and $ct' - axis$ are inclined at $\frac{\pi}{2} - 2\theta$ Therefore when (ct, x) axes are orthogonal, (ct', x') are not orthogonal and vice versa. Since $\sin\left(\frac{\pi}{2} - 2\theta\right) = \cos 2\theta = \cos^2\theta - \sin^2\theta = \frac{1-e^2}{1+e^2}$. We have the relation between the areal elements:

$$cdt\,dx = |J|\,cdt'dx' \sin\left(\frac{\pi}{2} - 2\theta\right) = \left(\frac{1-e^2}{1+e^2}\right) cdt'dx'$$ where we used

$|J| = 1$ and $\sin\left(\frac{\pi}{2} - 2\theta\right) = \left(\frac{1-e^2}{1+e^2}\right)$

i.e. $$dt\,dx = \left(\frac{1-e^2}{1+e^2}\right) dt'dx' \qquad (2.5)$$

The asymmetric nature of equation (2.5) reveals that there is no logic in stipulating that the LT represents the motion of O' relatively to O along a common x-axis. Figure 1 indicates that (ct,x) world point is a preferred representation to $(ct'x')$; the converse is true by re-drawing the figure and considering the inverse transformation. Thus, the LT implies that one or the other frame of reference is not a preferred one justifying the Lorentzian interpretation of a preferred frame. Hence the relativistic conclusion that there is no preferred frame of reference, is logically invalid; so is the assumption of a common $x / x' - axis$.

Mass - Velocity Relations

For a free particle, the Lagrangian L* and the Hamiltonian H represent the energy, one in the moving frame of the particle, and the other in the laboratory frame of the observer. Hence it is possible that both can be expressed in the form mc^2 where m is a function of the velocity of the particle. This is done by using the defining equations in classical dynamics, namely,

$$\left(\frac{\partial}{\partial \boldsymbol{v}}\right) L^* = momentum = m\boldsymbol{v} \qquad (2.1.1)$$

and

$$\left(\frac{\partial}{\partial \boldsymbol{p}}\right) H = \boldsymbol{v} \qquad (2.1.2)$$

Thus, by letting $L^* = f(\boldsymbol{v}) m_0 c^2 = mc^2$ in (2.1.1) and taking v along x-axis, we have

$f'(v) m_0 c^2 = c^2 \frac{dm}{dv} = mv \quad \therefore \frac{dm}{m} = \frac{vdv}{c^2}$ · Integrating

$$m = m_0 \operatorname{Exp}\left(\frac{v^2}{2c^2}\right) \qquad (2.1.3)$$

By defining $H = \boldsymbol{v} \cdot \left(\frac{\partial}{\partial \boldsymbol{v}}\right) L^* - L^*$ it is seen that (2.1.2) is satisfied. It can be similarly shown that the Lorentzian mass given by [1,15]

$$m = \frac{m_0}{\sqrt{\left|1 - \frac{v^2}{c^2}\right|}} \qquad (2.1.4)$$

can be obtained from (2.1.2) and the assumption. $mc^2 = g(v)\, m_0 c^2$. Thus, there are two possible mass-velocity relations, given by (2.1.3) and (2.1.4); we shall use the former in the following discussion.

Derivation of Orbit of Planetary Motion from Newton-Lorentz Theory

We start with

$$\frac{d\boldsymbol{p}}{dt} = \mathbf{F}_1 \text{ (Newton's Law)} \qquad (3.1.1)$$

As an approximation $\frac{d}{dt}(m\boldsymbol{v}) = m\mathrm{E}_1^-$ where $\mathrm{E}_1^- = \frac{-\mathrm{MR}}{4\pi\varepsilon_1 \mathrm{R}^3} = \frac{-\mathrm{MI}}{4\pi\varepsilon_1 \mathrm{R}^2}$

$$i.e.\ m\frac{d\boldsymbol{v}}{dt} + \boldsymbol{v}\frac{dm}{dt} = m\mathrm{E}_1^- \qquad (3.1.2)$$

$$\therefore \boldsymbol{v} \times (d\boldsymbol{v}/dt) = \boldsymbol{v} \times \mathrm{E}_1^- \qquad (3.1.3)$$

But from $m = m_0 \operatorname{Exp}(v^2/2c^2)$ we have

$$\frac{dm}{dt} = m(\boldsymbol{v}/c^2)\cdot(d\boldsymbol{v}/dt) \therefore (3.1.2) \Rightarrow \frac{d\boldsymbol{v}}{dt} + \boldsymbol{v}\left[(\boldsymbol{v}/c^2)\cdot(d\boldsymbol{v}/dt)\right] = \mathrm{E}_1^-$$

$$\text{i.e. } \frac{d\boldsymbol{v}}{dt} + (\boldsymbol{v}/c^2)\times(\boldsymbol{v}\times d\boldsymbol{v}/dt) + (\boldsymbol{v}^2/c^2)(d\boldsymbol{v}/dt) = \mathrm{E}_1^-$$

$$i.e. (d\boldsymbol{v}/dt) + \left[\boldsymbol{v}\times(\boldsymbol{v}\times \mathrm{E}_1)\right]/c^2 \approx \mathrm{E}_1^- = \frac{-\mathrm{MI}}{4\pi\varepsilon_1 \mathrm{R}^2} \qquad (3.1.4)$$

discarding $\frac{\boldsymbol{v}^2}{c^2} \approx 0$ But $\boldsymbol{v} \times \mathrm{E}_1^- = \begin{vmatrix} \mathrm{I} & \mathrm{J} & \mathrm{K} \\ \dot{\mathrm{R}} & \mathrm{R}\dot{\theta} & 0 \\ \frac{-\mathrm{M}}{4\pi\varepsilon_1 \mathrm{R}^2} & 0 & 0 \end{vmatrix} = \frac{\mathrm{MR}\dot{\theta}\mathbf{K}}{4\pi\varepsilon_1 \mathrm{R}^2}$

and

$$\boldsymbol{v} \times \left(\boldsymbol{v} \times \mathrm{E}_1^-\right) = \begin{vmatrix} \mathrm{I} & \mathrm{J} & \mathrm{K} \\ \dot{\mathrm{R}} & \mathrm{R}\dot{\theta} & 0 \\ 0 & 0 & \frac{\mathrm{M}\dot{\theta}}{4\pi\varepsilon_1 \mathrm{R}} \end{vmatrix} = \frac{\mathrm{M}\dot{\theta}\left(\mathrm{R}\dot{\theta}\mathrm{I} - \dot{\mathrm{R}}\mathrm{J}\right)}{4\pi\varepsilon_1 \mathrm{R}} = \frac{\mathrm{M}\dot{\theta}^2 \mathbf{I}}{4\pi\varepsilon_1} - \frac{\mathrm{M}\dot{\mathrm{R}}\dot{\theta}}{4\pi\varepsilon_1 \mathrm{R}} \mathrm{J} \tag{3.15}$$

Since $\frac{d\boldsymbol{v}}{dt} = \left(\ddot{\mathrm{R}} - \mathrm{R}\dot{\theta}^2\right) I + \frac{1}{\mathrm{R}}\left[d\left(\mathrm{R}^2\dot{\theta}\right)/dt\right]\mathrm{J}$, (3.1.4) and(3.1.5) imply

$$\ddot{\mathrm{R}} - \mathrm{R}\dot{\theta}^2 = \frac{-\mathrm{M}}{4\pi\varepsilon_1 \mathrm{R}^2} - \frac{\mathrm{M}\dot{\theta}^2}{4\pi\varepsilon_1 c^2} \tag{3.16}$$

$$\text{and} \frac{d\left(\mathrm{R}^2\dot{\theta}\right)}{\mathrm{R}dt} = \frac{\mathrm{M}\dot{\mathrm{R}}\dot{\theta}}{4\pi\varepsilon_1 \mathrm{R}c^2} \tag{3.17}$$

The latter can be re-written as

$$\frac{d\left(\mathrm{R}^2\dot{\theta}\right)}{\mathrm{R}^2\dot{\theta}} = \frac{\mathrm{M}d\mathrm{R}}{4\pi\varepsilon_1 c^2 \mathrm{R}^2} \text{Integrating we get}$$

$$\mathrm{R}^2 \frac{d\theta}{dt} = h_0 \ \mathrm{Exp}\left(-\mathrm{M}/4\pi\varepsilon_1 c^2 \mathrm{R}\right) = h_0 Exp\left(-\phi_1/c^2\right) \text{ or } h_0 \mathrm{Exp}\left(\frac{-\mathrm{MG}}{c^2\mathrm{R}}\right) \tag{3.18}$$

$$\text{As an approximation } \mathrm{MR}^2\dot{\theta} \approx \mathrm{H}_0 \tag{3.19}$$

$$\text{or } \mathrm{R}^2\dot{\theta} \approx h_0 \tag{3.1.10}$$

Letting

$u = \frac{1}{\mathrm{R}}$, we have $\dot{R} = \left(\frac{-1}{u^2}\right)\left(du/d\theta\right)\dot{\theta} = -h_0\left(du/d\theta\right)$,

$$\ddot{\mathrm{R}} = -h_0\left(d^2u/d\theta^2\right)\dot{\theta}$$

$$\ddot{\mathrm{R}} - \mathrm{R}\dot{\theta}^2 = -h_0\left(d^2u/d\theta^2\right)h_0u^2 - \left(1/u\right)\left(h_0u^2\right)^2$$

$$= -h_0^2u^2\left[\left(d^2u/d\theta^2\right) + u\right] \tag{3.1.11}$$

Now (3.1.6) can be re-written as

$$-h_0^2u^2\left(\frac{d^2u}{d\theta^2} + u\right) = \frac{-\mathrm{M}u^2}{4\pi\varepsilon_1} - \frac{\mathrm{M}\cdot\left(h_0u^2\right)^2}{4\pi\varepsilon_1 c^2}$$

$$\therefore \frac{d^2u}{d\theta^2} + u = \frac{\mathrm{M}}{h_0^2 \cdot 4\pi\varepsilon_1} + \frac{\mathrm{M}u^2}{4\pi\varepsilon_1 c^2}$$

$$\therefore \frac{d^2u}{d\theta^2} + u = \frac{\mathrm{MG}}{h_0^2} + \frac{\mathrm{M}u^2}{4\pi\varepsilon_1 c^2} = \frac{\mathrm{MG}}{h_0^2} + \frac{\mu_1 \mathrm{M}}{4\pi}u^2 \tag{3.1.12}$$

This is the equation for planetary motion [1,15] in which we have used the local time interval dt. Next, we shall use the proper time interval $d\tau$. The simplest way to obtain the orbit by using $d\tau$ is to proceed as follows:

The metric obtainable from proper time interval $d\tau$ is

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - d\mathrm{R}^2 - \mathrm{R}^2 d\theta^2 \tag{3.1.13}$$

by restricting to motion in a plane, using polar co-ordinates (R, θ).

From Equation (3.1.8) we have

$$\mathrm{R}^2 \frac{d\theta}{dt} = h_0 \mathrm{Exp}\left(\frac{-\mathrm{MG}}{c^2\mathrm{R}}\right)$$

$$\therefore \mathrm{R}^2 \frac{d\theta}{d\tau} = h_0 \frac{dt}{d\tau} \mathrm{Exp}\left(\frac{-\mathrm{MG}}{c^2\mathrm{R}}\right) = h_0 \tag{3.1.14}$$

implies

$$\frac{dt}{d\tau} = \mathrm{Exp}\left(\frac{\mathrm{MG}}{c^2\mathrm{R}}\right) \tag{3.1.15}$$

and

$$R^2 \frac{d\theta}{d\tau} = h_0 \tag{3.1.16}$$

By using the principle of time-dilation and contraction hypothesis we have $dtd\mathrm{R} = \left(dt \ \mathrm{Exp}\left(\frac{-\mathrm{MG}}{c^2\mathrm{R}}\right)\right)\left(d\mathrm{R} \ \mathrm{Exp}\left(\frac{\mathrm{MG}}{c^2\mathrm{R}}\right)\right)$ is approximately an invariant for the field of a spherically symmetric potential field of a gravitating mass M. This assertion is consistent with existing theory; we come across [2, 3] the metric

$ds^2 = \mathrm{A}\left(r\right)dt^2 - \mathrm{B}\left(r\right)dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2$.

Therefore when dt is changed to $dt \ \mathrm{Exp}\left(\frac{-\mathrm{MG}}{c^2\mathrm{R}}\right) = d\tau$,

dR will be changed to $d\mathrm{R} \ \mathrm{Exp}\left(\frac{\mathrm{MG}}{c^2\mathrm{R}}\right)$ i.e when

$\frac{dt}{d\tau} = \mathrm{Exp}\left(\frac{\mathrm{MG}}{c^2\mathrm{R}}\right)$, $\frac{d\mathrm{R}}{d\tau}$ will be replaced by $\frac{\mathrm{R}}{d}\mathrm{Exp}\left(\frac{\mathrm{MG}}{c}\right)$

(3.1.13) implies $1 = \mathrm{K}\left(\frac{dt}{ds}\right)^2 - \left(\frac{d\mathrm{R}}{d\tau}\right)^2 - \mathrm{R}^2\left(\frac{d\theta}{d\tau}\right)^2$

where K is a scaling factor and is taken to be unity i.e. $$1 = K\,\mathrm{Exp}\left(\frac{2MG}{R}\right) - \mathrm{Exp}\left(\frac{2MG}{R}\right)\left(\frac{dR}{d\tau}\right)^2 - R^2\left(\frac{d\theta}{d\tau}\right)^2 \qquad (3.1.17)$$

$$\left[1 + R^2\left(\frac{d\theta}{d\tau}\right)^2\right]\mathrm{Exp}\left(\frac{-2MG}{R}\right) = K - \left(\frac{dR}{d\tau}\right)^2$$

$$\therefore \left(1 + \frac{h_0^2}{R^2}\right)\left(1 - \frac{2MG}{R} + \cdots\right) + \left(\frac{dR}{d\tau}\right)^2 - K = 0 \qquad (3.1.18)$$

Letting $u = \frac{1}{R}$ and omitting higher powers (3.1.18) yields

$\left(1 + h_0^2 u^2\right)\left(1 - 2\bar{m}u\right) + \left(\frac{dR}{d\tau}\right)^2 - K = 0$ where

$$\bar{m} = \frac{MG}{c^2} = MG \text{ when } c = 1$$

$$\frac{dR}{d\tau} = \frac{d}{d\tau}\left(\frac{1}{u}\right) = \frac{-1}{u^2}\frac{du}{d\theta}\cdot\frac{d\theta}{d\tau} = -h_0\frac{du}{d\theta}$$

$$\therefore \left(1 - 2\bar{m}u + h_0^2u^2 - 2\bar{m}h_0^2u^3\right) + h_0^2\left(\frac{du}{d\theta}\right)^2 - K = 0$$

i.e. $$\left(\frac{du}{d\theta}\right)^2 + u^2 = \frac{K-1}{h_0^2} + \frac{2\bar{m}u}{h_0^2} + 2\bar{m}u^3 \qquad (3.1.19)$$

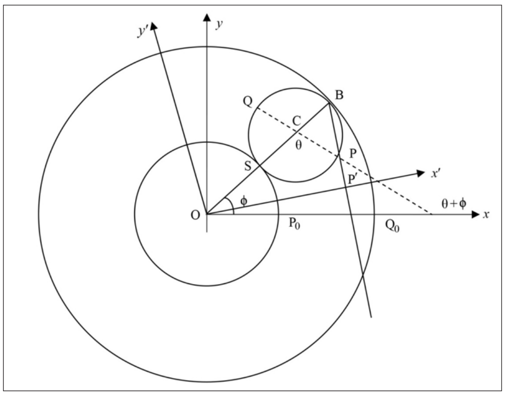
Differentiating (3.1.19) wrt θ and cancelling $2\frac{du}{d\theta}$ we get

$$\frac{d^2u}{d\theta^2} + u = \frac{\bar{m}}{h_0^2} + 3\bar{m}u^2 \qquad (3.1.20)$$

This is only an approximate result obtainable from (3.1.18); it is the equation of planetary motion given in books on general relativity but Einstein's GTR is refuted by M.W. Evans [14]. Replacing $\bar{m}$ by the reduced mass μ of the Sun-planet system, we get the orbit of the two-body problem; this replacement has the advantage that the masses of the two bodies are involved in the reduced mass. By omitting the term $3mu^2$ in (3.1.20) the orbit is an ellipse, if we do not omit it then the orbit is a precessing ellipse, the major axis of the ellipse rotating about a central point. Since circles and ellipses are special cases of trochoids, the epi-cycloid/hypo-cycloid is a possible orbit obtainable from (3.1.18). This possibility is examined in the next section.

Motion in a Epicycloid/Hypocycloid

Consider two concentric circles with centre O and constant radii R + aand R − a. In the figure OC = R, OS = R − a, OB = R + a and SC = CB = a



Consider a circle of diameter 2a resting initially with its horizontal diameter having endpoints P_0 and Q_0 touching the inner circle and the larger outer circle of radii R – a and R + a at P_0 and Q_0.

Assume that this circle rolls with constant angular

speed $= \omega = \frac{d\theta}{dt}$. At time $t P_0Q_0$ takes the position PQ. θ and ϕ are as shown in the figure. Arc SP = Arc SP_0 i.e. $a\,\theta = (R-a)\phi$ or $\theta = \left(\frac{R}{a} - 1\right)\phi$ or $\theta + \phi = \frac{R}{a}\phi$ since $\theta = \angle SCP$ and $\phi = \angle SOQ_0 = \angle SOx$. CP has inclination and $\angle CBP = \frac{1}{2}\angle SCP = \frac{1}{2}\theta$. Since OC = R and CP = a we have $\angle CSP = \frac{\pi}{2} - \frac{\theta}{2}$

$\therefore$ Inclination of SP $= \phi - \left(\frac{\pi}{2} - \frac{\theta}{2}\right) = \phi + \frac{\theta}{2} - \frac{\pi}{2}$. Also $SP = 2$ as in $\theta/2$.

Projection of SP on horizontal $= 2a\sin\frac{\theta}{2}\sin\left(\phi + \frac{\theta}{2}\right) = a\cos\phi - a\cos\frac{R}{a}\phi$.

Projection of SP on vertical line $= 2a\sin\frac{\theta}{2}\sin\left(\phi + \frac{\theta}{2} - \frac{\pi}{2}\right)$

$= 2a\sin\frac{\theta}{2}\cos\left(\phi + \frac{\theta}{2}\right)$

$= -a\sin(\phi + \theta) + a\sin\phi$

Let P = (x, y) relative to OP_0Q_0X as x-axis and OY perpendicular to Ox as y-axis.

$\therefore$ x = OS cosφ + Projection of SP on horizontal.

$$\therefore x = (R - a)\cos\phi + a\cos\phi - a\cos\frac{R}{a}\phi$$

$$x = R\cos\phi - a\cos\left(\frac{R}{a}\right)\phi \quad (3.2.1)$$

Similarly

$$y = R\sin\phi - a\sin\left(\frac{R}{a}\right)\phi \quad (3.2.2)$$

$$\therefore r^2 = x^2 + y^2 = R^2 + a^2 - 2aR\cos\left(\frac{R}{a} - 1\right)\phi$$

$$= R^2 + a^2 - 2aR\cos\theta$$

$$= (R + a)^2\sin^2\frac{\theta}{2} + (R - a)^2\cos^2\frac{\theta}{2} \quad (3.2.3)$$

$\therefore x\dot{y} - y\dot{x} = (R^2 + aR)(1 - \cos\theta)$ is not a constant. Its areal velocity is not a constant relative to fixed axes Ox, Oy. Draw OP′ perpendicular to BP or parallel to SP and Oy' perpendicular to OP′ or parallel to BP. Relative to the axes OP′x', Oy' we have

$$P = (OP', P'P) = \left[(R + a)\sin\frac{\theta}{2}, (R - a)\cos\frac{\theta}{2}\right]$$

Since $OP' = OB\sin\frac{\theta}{2}$ and $P'P = BP' - BP = OB\cos\frac{\theta}{2} - BS\cos\frac{\theta}{2}$

$$\therefore x' = (R + a)\sin\frac{\theta}{2} \quad (3.2.4)$$

$$y' = (R - a)\cos\frac{\theta}{2} \quad (3.2.5)$$

$$\Rightarrow \frac{(x')^2}{(R+a)^2}+\frac{(y')^2}{(R-a)^2}=1 \quad (3.2.6)$$

Therefore, P lies on the ellipse (3.2.6)

$x'\frac{dy'}{dt'}-y'\frac{dx'}{dt}=-\frac{1}{2}\frac{d\theta}{dt}(R^2-a^2)=-\frac{1}{2}w(R^2-a^2)=h$ say (a constant)

$$\therefore r^2\frac{d\psi}{dt}=h \qquad (3.2.7)$$

where $\psi=tan^{-1}(\frac{y'}{x'})$ i.e areal velocity is a constant; hence P has a central acceleration towards O and there is no transverse acceleration. From (3.2.7), by letting $r=\frac{1}{u}$ we have $\frac{d\psi}{dt}=hu^2$ and

$$\frac{dr}{dt}=\frac{1}{u^2}\frac{du}{d\psi}\frac{d\psi}{dt}=-h\frac{du}{d\psi}$$

$$r\frac{d\psi}{dt}=rhu^2=hu$$

$$\therefore v^2=\left(\frac{dr}{dt}\right)^2+\left(r\frac{d\psi}{dt}\right)^2=h^2\left(\frac{du}{d\psi}\right)^2+h^2u^2 \qquad (3.2.8)$$

But $v^2=\left(\frac{dx'}{dt}\right)^2+\left(\frac{dy'}{dt}\right)^2=\frac{1}{4}\left[2\left(R^2+a^2\right)-r^2\right]\left(\frac{d\theta}{dt}\right)^2=A^2-B^2r^2$

where A and B are constants. (3.2.8) and (3.2.9) imply

$$h^2\left[\left(\frac{du}{d\psi}\right)^2+u^2\right]=A^2-B^2r^2 \text{ or } \left(\frac{du}{d\psi}\right)^2+u^2=a^2-b^2r^2 \qquad (3.2.10)$$

Differentiating (3.2.10) wrt ψ and dividing the result by $2\frac{d\psi}{d\theta}$ we get

$$h^2\left[\left(\frac{d^2u}{d\psi^2}\right)^2+u\right]=0-b^2(-1)u^{-3} \text{ or } \frac{d^2u}{d\psi^2}+u=\frac{b^2}{h^2}u^{-3} \qquad (3.2.11)$$

But $\frac{dr}{dt}=-h\frac{du}{d\psi}$ implies $\frac{d^2r}{dt^2}=-h\frac{d^2u}{d\psi^2}\cdot\frac{d\psi}{dt}=-h^2u^2\frac{d^2u}{d\psi^2}$

and $r\frac{au}{dt}=hu$

$$\therefore \frac{d^2r}{dt^2}-r\left(\frac{d\psi}{dt}\right)^2=-h^2u^2\left[\frac{d^2u}{d\psi^2}+u\right]=-h^2u^2\left(\frac{b^2u^{-3}}{h^2}\right)$$

i.e. radial acceleration $=-b^2r$ and transverse acceleration is zero in x′y′ co-ordinates but not zero in the xy co-ordinates.

Therefore, P has a trochoidal motion relative to fixed axes (Ox, Oy) and a harmonic oscillation as a precessing ellipse, relative to (Ox′, Oy′)

Deduction of Helmoltz' Equation $(\nabla^2+k^2)\varnothing=0$ from Newtonian Theory

The Poisson's equation is $\nabla^2\varnothing+\left(\frac{\rho}{\epsilon}\right)=0$ (3.3.1)

This equation will turn into Helmotz' equation provided we can show that $\rho=\epsilon k^2\varnothing$; we shall prove this.

The fast-revolving leaves of a fan cannot be distinguished, but has the appearance of a circular disc. Similarly, the vibrating spherical membrane has the appearance of a cloud within a spherical shell. Assuming that an electron has the appearance of a cloud within spherical shell of radius r and thickness Δr, the average electron mass/charge density ρ must satisfy the condition $4\pi r^2\rho\Delta r=m/q$ or $\rho=\frac{m/q}{4\pi r^2\Delta r}$. But the potential due to nucleus of mass M/charge Q at a distance r is given by $\varnothing=\frac{M/Q}{4\pi\epsilon r}$ with usual notations.

Using the value of M/Q from the latter in the former we have

$$\frac{\rho}{\epsilon}=\left(\frac{m}{M}/\frac{q}{Q}\right)\frac{4\pi r\varnothing}{4\pi r^2\Delta r}=\frac{\left(\frac{m}{M}/\frac{q}{Q}\right)\varnothing}{r\Delta r} \text{ i.e. } \rho=\frac{\left(\frac{m}{M}/\frac{q}{Q}\right)\varnothing}{r\Delta r}=\epsilon\varnothing k^2 \quad (3.3.2)$$

where $k^2=\frac{\left(\frac{m}{M}/\frac{q}{Q}\right)\varnothing}{r\Delta r}=\frac{1}{\lambda^2}$. Using equation (3.3.2) in equation (3.3.1), this equation (3.3.1) becomes Helmoltz' Equation. To satisfy $\lambda^2=\left(\frac{m}{M}/\frac{q}{Q}\right)^{-1}r\Delta r$, we may use Bohr's choice:

$\left(\frac{m}{M}/\frac{q}{Q}\right)^{-1}r=n\lambda,\Delta r=\frac{\lambda}{n}=r_1$ so that $r=n^2r_0$ where $r_0=\left(\frac{m}{M}/\frac{q}{Q}\right)r_1$ and $\Delta r=\left(\frac{m}{M}/\frac{q}{Q}\right)r_0$ Thus, we get Helmoltz' equation

$$\left(\nabla^2+k^2\right)\varnothing=0 \qquad (3.3.3)$$

Next, we show that (3.3.3) is consistent with Newtonian Theory. The filiform [5] solutions i.e. solutions which are zero everywhere, except at points very close to a space curve of the Helmoltz' wave equation

$$\nabla^2\psi-\frac{v^2}{c^4}\frac{\partial^2\psi}{\partial t^2}=0 \qquad (3.3.4)$$

are the null geodesics of the metric for which line element is

$$\frac{c^4}{v^2}dt^2-dx^2-dy^2-dz^2=0 \quad (3.3.5)$$

It is readily shown that the null geodesics of this metric are curves which satisfy the Newtonian equation,

$$m\frac{d^2x}{dt^2}=-\frac{\partial V}{\partial x} \qquad (3.3.6)$$

From Debroglie's wave theory, we have $\lambda\omega=v_p$ and $v_pv=c^2$ so that (3.3.4) becomes $\nabla^2\psi-\frac{1}{v_p^2}\frac{\partial^2\psi}{\partial t^2}=0$ or $\nabla^2\psi-\frac{k^2}{\omega^2}\frac{\partial^2\psi}{\partial t^2}=0$ (3.3.7)

By taking $\psi = \phi e^{-i\omega t}$ in (3.3.7) we get Helmoltz' equation (3.3.3).

Hence (3.3.4) is the wave equation associated with the Newtonian equation (3.3.6) and $\psi = \phi(x, y, z)e^{-i\omega t}$ is the associated wave function.

The De Broglie's equation and Schrodinger's equation can be deduced from Helmoltz' equation. We have $(\nabla^2 + k^2)\varnothing = 0$(3.3.3).

This can be re-written as $\nabla^2\varnothing + \left(\frac{\omega^2}{c^2} - \frac{m_0^2c^2}{\hbar^2}\right)\varnothing = 0$ since

$h^2w^2 = h^2k^2 + m_0^2c^4$ in de Broglie theory, i.e $\nabla^2\varnothing + \frac{\omega^2}{c^2}\varnothing = \frac{m_0^2c^2}{\hbar^2}\varnothing$

(3.3.8). Multiply this by $e^{-i\omega t}$ we get $e^{-i\omega t}\nabla^2\varnothing + e^{-i\omega t}\frac{\omega^2}{c^2}\varnothing = e^{-i\omega t}\frac{m_0^2c^2}{\hbar^2}\varnothing$

i.e. $\nabla^2(\varnothing e^{-i\omega t}) - \frac{1}{c^2}\frac{\partial^2}{\partial u^2}(\varnothing e^{-i\omega t}) = \frac{m_0^2c^2}{\hbar^2}(\varnothing e^{-i\omega t})$ i.e.

$$\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2}{\partial u^2}\psi = \frac{m_0^2c^2}{\hbar^2}\psi \quad (3.3.8)$$

This is De Broglie's equation / Klein-Gordon equation.

Equation (3.3.3) can be re-written as $\nabla^2\varnothing + \frac{2m}{\hbar^2}(E - V)\varnothing = 0$ (3.3.3′),

by using the condition of Hamiltonian energy, $\frac{p^2}{2m} + V = E = \hbar\omega$

and $\boldsymbol{p} = \hbar k$. Multiplying (3.3.3′) by $e^{-i\omega t}$ and letting $\psi = \phi e^{-i\omega t}$ and we have $\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0$

i.e. $\frac{-\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$ (3.3.9)

Since $\psi = \phi e^{-i\omega t}$ imply $\frac{\partial\psi}{\partial t} = -\mathrm{i}\omega\psi$, we have $i\hbar\frac{\partial\psi}{\partial t} = i\hbar(-\mathrm{i}\omega\psi) = \hbar\omega\psi = E\psi$.

Therefore equation (3.3.9) becomes Schrodinger's equation:

$$\frac{-\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t} \quad (3.3.10)$$

Conclusions

Schrodinger's equation and Broglie's equation / Klein-Gordon equation can be deduced from Helmoltz' equation which can be deduced from Newtonian premises. An elliptic orbit, relative to set of axes Ox', Oy' need not be an elliptic orbit relative to another set of axes Ox, Oy. It is possible to introduce a field dependent metric for EM fields; this can be extended to gravitational fields also [4]. The primed and unprimed set of co-ordinates involved in the Lorentz' transformation are not equally inertial; one set is a preferred set to the other. The exact preferred frame is the one having $(x_\tau y_\tau z_\tau c\tau)$ as coordinates where $y_\tau = y$ and $z_\tau = z$.

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