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A Rotor Looseness–Rub-Impact–Resonance Fault Mechanism Modeling

HU Jun[1, 2]

[1]School of mechanical engineering, University of Science and Technology Beijing, 100083

[2]Angang Steel Company Limited equipment engineering department, 114021

ABSTRACT

In view of the limitation of the numerical method for coupling faults of rotating machinery, a modeling method for coupling faults of rotor, Deformation-Motion Vector Analysis Method, is purposed in order to explain the fault mechanism more thoroughly and universally. Firstly, the "Deformation-Motion" is defined by fault and the relationship between fault deformation and periodic signal is established by Fourier series. Secondly, the "Vector Analysis" analyzes the deformation-motion in the dynamic coordinate system, and establishes the relationship between the fault deformation-motion and the periodic excitation force by using the dynamic and static coordinate systems. Thirdly, the analytical solution is obtained by harmonic balance method, and the coriolis acceleration term is first found in the analytical solution of rotating machinery failure. By further analysis of the analytical solution, we find that the general trend of the natural frequency of the rotor system will gradually approach the excitation source frequencies and eventually lead to the resonance: that is the law of Natural Frequency Migration Resonance Theory. Finally, the validity of the modeling method and the correctness of the analytical solution are verified by engineering practice.

***Corresponding author**

HU Jun, (School of mechanical engineering, University of Science and Technology Beijing, 100083; Angang Steel Company Limited equipment engineering department, 114021), Tel:13998055187; E-Mail: hujun_ag@qq.com

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Introduction

When we start the machinery malfunction diagnosis by the vibration signal analysis of rotating machinery, we must find that the condition deterioration of the equipment mainly displays the characteristic frequencies of the rotor system and the gradual emergence of various levels of harmonics and also the amplitude increase. These characteristics are often the common form of the looseness, in a sense, we can also be said that the looseness is the beginning of the fault. From the long-term, a large number of periodic monitoring data shows that the looseness of the rotor will lead to the development of other phenomena such as imbalance, misalignment, irregular bearings and so on, it is assumed that these failures do not dealt with in a timely manner, and then the increasing magnitude over the limit of design gap would induced the rotor rub-impact, cause the natural frequency migration resonance, at last lead to rapid damage to the equipment.. Therefore, the analysis of the mechanism of interaction of "looseness coupling rub-impact" is helpful to forecast the development trend of fault and provide theoretical guidance for the maintenance and repair of the equipment.

There are two kinds of researches on looseness and rub-impact of rotation machinery, which are divided into two categories: "model-free experimental test" and " dynamic modeling and numerical solution ".

The basic idea of the model-free experimental method simulates the rotor looseness fault, and extracts the fault features by the targeted method [1-3]. Although this kind of method used advanced wavelet and information fusion neural network, but its limitation is that the conclusion is only the statistical analysis of experimental data of the specific object, the general applicability of its conclusion needs further proof.

Another approach is to gain a deeper understanding of the failure mechanism. Most scholars adopt the method of fault mechanism modeling and numerical solution. However, the key content about the stiffness (damping) assumptions the excitation force assumptions the choice of the degree of freedom the numerical solution of the model is worthy to be analyzed and discussed.

Based on the assumption of piecewise stiffness, the stiffness and damping of the rotor system in the different vibration displacement stages are assumed to be non-linear jumps when loosening and rubbing occur [4-7]. So most of the scholars usually treat the stiffness and damping of the system with piecewise stiffness and constant value, but in the actual loosening process the rotor stiffness is the nonlinear continuously varying stiffness with the loose state, so the constant value of the piecewise linear stiffness and damping assumptions are limitations in the characterization of dynamic law of fault development. On the assumption of continuous stiffness, some scholars have established a simple continuous loosening model [8, 9]. The results show that the variation of the total stiffness of the loosening system is dependent on its own slowly changing characteristics. Although the model is better than the piecewise linear stiffness model, the research findings also cannot explain the actual looseness phenomenon well.

Based on the assumption of the nonlinear excitation of bearing, At present the non-linear excitation source of rotating machinery generally include bearing oil film force, rubbing friction, rotor crack, airflow excitation four kinds of excitation forces. Domestic and foreign scholars use various oil film dynamic characteristic expressions as the nonlinear support force of the rotor to establish the dynamic model of the rotor-bearing system. Gardner M. et al. used the multi-scale method to analyze the weak nonlinear motion of long and short bearing [10]. Adams M. and Abu. Mahfouz respectively studied the nonlinear dynamic behavior of rotor systems supported by cylindrical bearings and tilting-pad bearings by numerical integration method [11]. Zhang and Zhu discussed the stability margin of the sliding bearing with Lyapunov function [12]. Zhang and Songhua presented the Hopf bifurcation analysis of the limit cycle characteristics of eddy-film instability [13]. Huang Wen-Zhen study on the stability of the multi-span sliding bearing rotor system [14]. Chen Yu-Shu used short bearing assumption, included in the turbulence effect [15]. Zhao J. Y. studied the rotor system unbalance response supported by the oil film damper [16]. A dynamic model of a rotor-bearing system is established by Liu Xian Dong and C. Villa, J.-J. Sinou *, F. Thouverez. based on the Hertz nonlinear contact for a rolling bearing [17,18]. Chen Guo fully considered the bearing clearance, bearing ball and the raceway nonlinear Hertz contact force and bearing support stiffness change caused by the VC (varying compliance) vibration to establish the rolling bearing model [19].) Although the bearing force of the journal bearing and the Hertzian contact force of the rolling bearing are a kind of non-linear factor of the rotor-bearing system. But this nonlinearity is just a specific nonlinear characteristic of the looseness and rub-impact fault. It is only the response of the bearing non-linear force of rotor. On the other hand, if emphasizing only the nonlinearity of the bearing may confuse the fault type between the looseness and the bearing failure or rub-impact and the bearing failure, the bearing fault information was misinterpreted as a looseness or rub-impact fault signal.

About the choice of single-degree-of-freedom and multi-degree-of-freedom models, the initial loosening models such as Muszynska and Goldman are single disk Jeffcott rotor models with one-end loosening, where the stiffness is piecewise linear [20, 21]. Looseness multi-degree-of-freedom model is an improved model by analyzing the physical meaning of m, c, k, F and other parameters of different system. Zhang et al. established a nonlinear differential equation of motion for a rotor-bearing system with two-end bearing looseness [22]. Ma Hui et al. studied the dynamic characteristics of single-bearing and double-bearing loosening of two types of fault conditions respectively [23]. Luo Yuegang et al. established a nonlinear dynamic model of a three-bearing, two-span, elastic rotor-bearing system with a bearing-looseness failure, and analyzed the difference between the coupling fault characteristic and the single fault characteristic [24-26]. Lu Yanjun et al through the establishment of the mechanical model and finite element model of loosening - rubbing coupling fault of double-cantilever vertical rotor-bearing system. Liu Yang et al. developed a three-support double-disc rotor bearing system model with a loose-rub-impact coupled fault using a finite element method [27, 28]. However, the improvement of the multi-degree-of-freedom model only makes a simple linear addition and subtraction between the differential equations for the looseness displacement. For a rotor system with multiple coupling faults, the non-linearity of the fault state factors is not taken into account. So those multi-degree-of-freedom modes are essentially a kind of linear superposition. Of course, there is a big difference with the actual results ,because the superposition principle does not apply to nonlinear vibration systems.

Based on numerical integration method and analytic approximate method, such as Chu Fulei et al., Li Zhenping et al. [29], Li Hongkun et al. [30], Yang Yongfeng et al. [31], Liu Changli et al. [32],Chen Guo [33] MA H, ZHAO X, TENG Y, et al. [34], Xiong et al. [35] used Runge-Kutta method, Newmark-β method, shooting method, incremental method and incremental harmonic balance method [6, 29-35]. Numerical solution method generally needs the condition to be simplified, so objectively produces the disparity with the actual state; The numerical solution method needs to assign a value to the model, for this reason that this general model becomes the specific sampling, the solution obtained is equivalent to the experiment result. Actually it Is a specific conclusion for a particular type of rotor, Although it made a dimensionless processing, but there is not a high precision conclusion; because it is not yet an analytical solution, so the conclusion is only an explain of the typical failure phenomenon, but not an analysis of the failure mechanism, at last they all ignored the effect of resonance.

This paper reviews the domestic and foreign scholars exploration on the theory of rotating machinery fault and combines with the engineering practice, the author believes that the key to solve the rotor fault is the more accurate differential equation and the analytical solution that can explain engineering problems in a more general sense. The differential equation of vibration of rotating machinery is to study the relationship between rotor deformation rotating motion of rotor deformation and force. In this paper, the logic starting point of fault analysis is the unification of fault mechanism and fault motion form, based on this point, **Deformation-Motion Vector Analysis Method** is proposed. This method interprets that a particular fault is the cause of a particular deformation, and the specific deformation motion is the differential manifestation on the fault rotor.

The fault deformation node hypothesis is introduced into the Deformation-Motion Vector Analysis Method. The deformation node hypothesis of the fault is shown in the Figure 1.

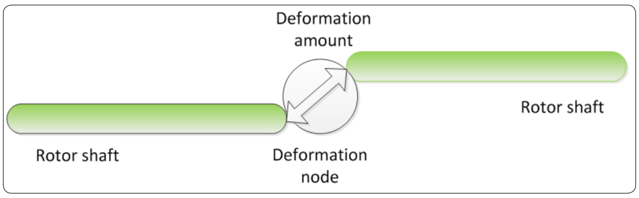

Figure1: Fault deformation node hypothesis

If the rotor before deformation is regarded as a rigid shaft, when a failure occurs, the rotor deforms at the focal point of stress where is the deformation node. At this time, the node deformed can be assumed as an additional hinge point at the fault point of the rigid body, and the deformation amount is set as the relative displacement of the rigid body to this hinge point. In this way, the direction of the force can be determined by the deformation shape of a specific fault, and the magnitude of the force can be determined by the deformation amplitude, which is the node hypothesis of fault deformation.

The complex problem of elastic deformation of rotor is transformed into the linkages motion of rigid body through the cause of the deformation node. The motion vector analysis of the linkages mechanism can be reasonably conducted by assuming the deformation node. Based on this assumption, the Deformation-Motion Vector Analysis Method is proposed. This method is used to make the modeling of coupling fault more concise and more consistent with the failure deformation mechanism. As shown in the Figure 2, this method can construct the rotor fault deformation model→mechanical model of deformation motion→analytical solution.

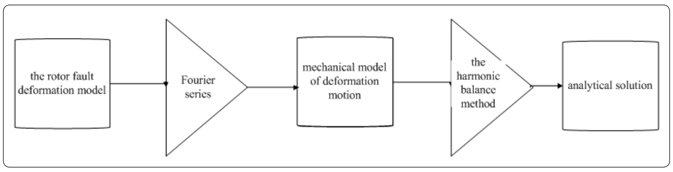

Figure 2: Deformation-Motion Vector Analysis Method graphical representation

Deformation-Motion Vector Analysis Method

First, analyzing the deformation characteristics of the rotor in the fault state, defined the rotor deformation by the fault type, established the relationship between the fault and its deformation, and got the deformation model of rotor in a specific fault state, then the fault deformation model is expressed by Fourier series, and the relationship between fault deformation and periodic signal is established; Secondly, the Fourier series is transformed into motion vector in the moving coordinate system, and the relationship between fault deformation and force is established; Finally, the harmonic balance method is used to modeling and solving. This method can be used to establish a differential equation and obtain an analytical solution for a single or coupled fault such as looseness and rub-impact, etc., which can be defined by any harmonic motion and can be expressed by a simple harmonic convergence series.

This method is expected to open up a new path for further study of fault mechanism besides numerical solution.

The Jeffcott rotor is an ideal model in this paper. Application of Deformation-Motion Vector Analysis Method resolves the typical looseness and rub-impact fault, by further analyzing the analytical solutions of the two kinds of equations, and solve the engineering problems with this conclusion.

Deformation-Motion Model

Rotor Deformation-Motion Model of Initialization Condition

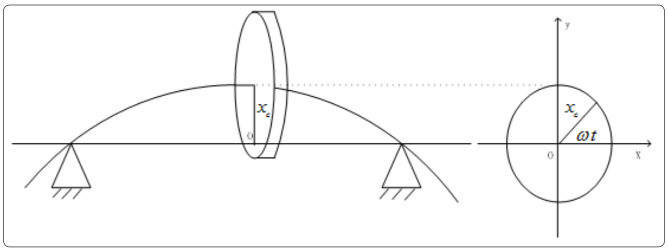

Figure 3: Rotor deformation-motion model of initialization condition

The initialization condition of rotor of rotating machinery refers to the constant speed fixed axis rotation of the single-DOF rotor under the condition of limited deformation caused by manufacturing and assembly deviation.

At this time, the deformation motion of the rotor is shown in Figure3,which is basically a forced motion with constant amplitude, and the axis trajectory is a perfect circle. This motion is in the initial state of the rotor , which is a steady state vibration. Its solution is:

$$m\ddot{x}_c + c\dot{x}_c + k_c x_c = \mathrm{F}_c \sin \omega t \tag{1}$$

$$x_c = \frac{\mathrm{F}_c}{\kappa_c \sqrt{(1-\lambda^2)^2 + (2\xi\lambda)^2}} \sin(\omega t - \varphi) \tag{2}$$

$$\text{where } \lambda = \frac{\omega}{\omega_n}, \qquad \xi = \frac{c}{c_c}$$

Rotor Deformation-Motion Model of Looseness Condition

The looseness condition of the rotor of rotating machinery refers to that the deformation of the rotor increased due to the decrease of stiffness. At this time, the deformation is the movement of rotor axis, and the direction of movement is along the direction where the stiffness decreases the most. So this kind of motion can be considered as the superposition of the rotary motion of the rotor and the rectilinear motion of the rotor along the inertia tangent in the direction of decreasing stiffness. The rotor should be returned by the elastic restoring force in a symmetrical way When the inertia force and the elastic force are balanced at the maximum deformation point, so the loose fault will cause the deformation fluctuation of the rotor similar the triangular wave.

The deformation movement of the rotor in the loose state is shown in Figure4, which can be decomposed into sine wave of initial deformation and triangular wave of moving deformation in the loose state. At this time, the rotor's movement changes from the fixed axis rotation to plane motion.

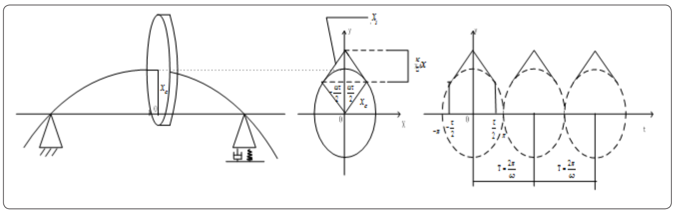

Figure 4: Rotor deformation -motion model of looseness condition

The rotational motion in the plane motion is the initial simple harmonic vibration and the moving motion is the periodic triangular wave vibration of loose deformation. Triangular wave vibration and simple harmonic vibration have the same 2π period. Suppose the pulse width and pulse amplitude of the triangle wave are and τ and ${}^{\kappa}_{\Delta}\mathrm{X}$, the expression of the periodic plane motion of the wave is shown in Formula 3:

$$\begin{cases} X_c \sin \omega t & \left[-\pi \quad -\frac{\tau}{2}\right) \\ X_c \cos\frac{\omega\tau}{2} + {}^{\kappa}_{\Delta}\mathrm{X} \begin{cases} \left[t - \frac{\tau}{2} \middle/ -\frac{\tau}{2}\right] & \left[\frac{-\tau}{2} \quad 0\right) \\ \left[t + \frac{\tau}{2} \middle/ \frac{\tau}{2}\right] & \left(0 \quad \frac{\tau}{2}\right] \end{cases} \\ X_c \sin \omega t & \left(\frac{\tau}{2} \quad \pi\right] \end{cases} \tag{3}$$

The Fourier series transformation of periodic triangular wave as:

$$x_s = \frac{{}^{\kappa}_{\Delta}\mathrm{X}}{2} + \frac{4\,{}^{\kappa}_{\Delta}\mathrm{X}}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin^2\left(\frac{n\pi}{2}\right)\cos(nwt) \tag{4}$$

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Rotor Deformation-Motion Model of Rub-Impact Condition

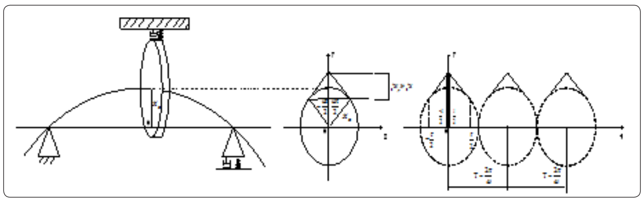

Figure 5: Rotor model of rub-impact condition and periodic rectangular excitation transformation

Deformation motion in the rub-impact state of rotating machinery rotor refers to that when the loose deformation exceeds the design gap between the rotor and the stator, and then friction occurs, so the loosening is the initial state of rubbing.

In practice, the loose deformation cannot be increased indefinitely. When the loose deformation exceeds the design gap between the rotor and the stator, the original loose fault be developed into rub-impact fault, in this case, ωτ is both the loose limit angle and the rub-impact starting angle, and it is both the limit position of loosening and the position of where rub-impact occurs, and $^{\xi}_{\Delta}X$(the position of the loose axis varying with stiffness) ≈ $^{\xi}_{\Delta}X$(the position of the rub-impact axis varying with stiffness).

The deformation movement of the rotor in the rub-impact state is shown in Figure5. The rub-impact movement can be considered as a simplified periodic rectangular wave with the triangular wave of loose motion clipping the top, and it has a common 2^{π} period with the periodic triangular wave. Suppose the pulse width and pulse amplitude of the rectangular wave are and .Its periodic plane motion is expressed as:

$$\begin{cases} X_c\cos\frac{\omega\tau}{2}+{}^{\kappa}_{\square}X\begin{cases}\left[t-\frac{\tau}{2}\Big/-\frac{\tau}{2}\right] & [-\tau/2 \quad 0) \\ \left[t+\frac{\tau}{2}\Big/\frac{\tau}{2}\right] & (0 \quad -\iota]\end{cases} \\ {}^{k}_{\square}x_p(t+\iota)-{}^{k}_{\square}x_p(t-\iota)\ldots\ldots\ldots & (-\iota \quad \iota) \\ X_c\cos\frac{\omega\tau}{2}+{}^{\kappa}_{\square}X\begin{cases}\left[t-\frac{\tau}{2}\Big/-\frac{\tau}{2}\right] & [-\tau/2 \quad 0) \\ \left[t+\frac{\tau}{2}\Big/\frac{\tau}{2}\right] & (0 \quad \iota]\end{cases} \end{cases} \tag{5}$$

The Fourier series transformation of periodic rectangular wave as:

$$f(t)=\frac{{}^{\kappa}_{\Delta}X_{\rho}\iota}{T}+\frac{{}^{\kappa}_{\Delta}X_{\rho}\iota\omega}{\pi}\sum_{n=1}^{\infty}\mathrm{Sa}\left(\frac{n\omega\iota}{2}\right)\cos(n\omega t) \tag{6}$$

$${}^{k}_{\square}x_p \approx {}^{k}_{\square}x \tag{7}$$

Establishing the deflection motion model of the fault rotor is the first step of the Deformation-Motion Vector Analysis Method. The deformation motion model of ideal condition constitutes the initialization condition where the subsequent failure occurs, and the fault state is the superposition of fault deformation on the foundation deformation of the ideal condition. In this paper, periodic triangular wave and rectangular wave are used to simulate the looseness and rub-impact faults. Considering the different characteristics of different devices, other more accurate analog waveforms cannot be excluded. The relationship between fault deformation and periodic signal is established after the selected analog waveform is transformed by Fourier series.

Vector Analysis

The Vector Analysis of Displacement, Velocity and Acceleration of Rotor Deformation Movement in the Ideal State (Initial State)

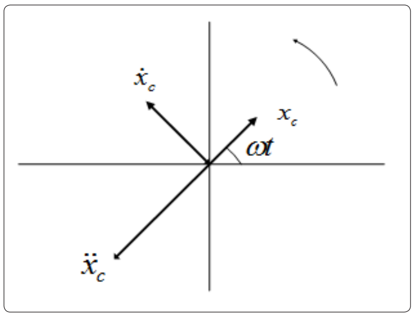

Figure 6: The rotor motion vector analysis in the initial condition

The vector analysis of displacement, velocity and acceleration the rotor in the initial state is shown in Figure 6

$$x_c=\frac{F_c}{\kappa_c\sqrt{(1-\lambda^2)^2+(2\xi\lambda)^2}}\sin(\omega t-\varphi) \tag{2}$$

In order to facilitate the calculation and not affect the result, the phase is now ignored.

$$x_c = X_c\sin\omega t \tag{7}$$

$$\dot{x}_c=\omega X_c\cos\omega t \tag{8}$$

$$\ddot{x}_c=-\omega^2 X_c\cos\omega t \tag{9}$$

The Vector Analysis of Displacement, Velocity and Acceleration of Rotor Deformation Movement in Loose State

The Fourier series Transformation of Looseness Deformation

$$x_s=\frac{{}^{\kappa}_{\Delta}X}{2}+\frac{4\,{}^{\kappa}_{\Delta}X}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin^2\left(\frac{n\pi}{2}\right)\cos(n\omega t) \tag{4}$$

In order to facilitate the calculation, order

$$\varepsilon_n=\frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin^2\left(\frac{n\pi}{2}\right) \tag{10}$$

$$x_s=\frac{{}^{\kappa}_{\Delta}X}{2}+\varepsilon_n\,{}^{\kappa}_{\Delta}X\cos(n\omega t) \tag{11}$$

$$\dot{x}_s=-n\omega\varepsilon_n\,{}^{\kappa}_{\Delta}X\sin(n\omega t) \tag{12}$$

$$\ddot{x}_s=-(n\omega)^2\varepsilon_n\,{}^{\kappa}_{\Delta}X\cos(n\omega t) \tag{13}$$

The Establishment of Moving Coordinate System
Since the deformation movement of the loose fault is superimposed on the basis of the initial deformation in the state of fixed axis rotation, so it is necessary to convert the displacement, velocity and acceleration of the loose fault deformation movement to the fixed reference coordinate system. Therefore, we need to set up two sets of coordinate systems, namely, dynamic coordinate system and static coordinate system:

An inertial coordinate system with respect to the earth takes the rotor axis O as the origin, and takes the X,Y and Z corresponding to the unit vectors I, J and K as the coordinate axes.

A moving coordinate system rotating with the rotor takes the rotor displacement vector in the initial state as origin o, and takes the x, y and z corresponding to the unit vectors i , j and k as the coordinate axes, as shown in Figure7.

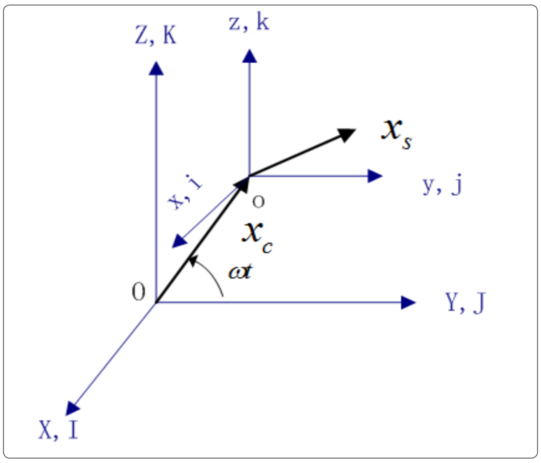

Figure 7: the moving coordinate system: System xyz moves within fixed system XYZ

Displacement Vector Analysis of Rotor Deformation Motion in Looseness condition

$$\overrightarrow{x_{c+s}} = \overrightarrow{x_c} + \overrightarrow{x_s} = X_c \sin\frac{\omega\tau}{2} + \cos\theta\left[\frac{{}^{\kappa}_{\Delta}X}{2} + \varepsilon_n\, {}^{\kappa}_{\Delta}X\cos(n\omega t)\right] \quad (14)$$

Velocity Vector Analysis of Rotor Deformation Motion in Looseness Condition

$$\overrightarrow{\dot{x}_{c+s}} = \overrightarrow{\dot{x}_c} + \overrightarrow{\dot{x}_s} = \left(\overrightarrow{\dot{x}_{cX}}\, I + \overrightarrow{\dot{x}_{cY}}\, J + \overrightarrow{\dot{x}_{cZ}}\, K\right) + \left(\overrightarrow{\dot{x}_{sx}}\, i + \overrightarrow{\dot{x}_{sy}}\, j + \overrightarrow{\dot{x}_{sz}}\, k\right) + \left(\overrightarrow{x_{sx}}\,\dot{i} + \overrightarrow{x_{sy}}\,\dot{j} + \overrightarrow{x_{sz}}\,\dot{k}\right) = \dot{x}_c + \dot{x}_s + \omega x_s \quad (15)$$

Acceleration Vector Analysis of Rotor Deformation Motion in Looseness Condition

$$\overrightarrow{\ddot{x}_{c+s}} = \overline{\overrightarrow{\dot{x}_c} + \overrightarrow{\dot{x}_s} + \omega\overrightarrow{x_s}} \quad (16)$$

The first item: $\overrightarrow{\ddot{x}_c} = \ddot{x}_c$ (17)

The second item:

$$\overrightarrow{\ddot{x}_s} = \overrightarrow{\ddot{x}_{sx}}\, i + \overrightarrow{\ddot{x}_{sy}}\, j + \overrightarrow{\ddot{x}_{sz}}\, k + \overrightarrow{\dot{x}_{sx}}\,\dot{i} + \overrightarrow{\dot{x}_{sy}}\,\dot{j} + \overrightarrow{\dot{x}_{sz}}\,\dot{k} = \ddot{x}_s + \omega \times \dot{x}_s \quad (18)$$

The third item:

$$\overline{\omega \times x_s} = \dot{\omega} \times x_s + \omega \times \dot{x}_s + \omega \times (\omega \times x_s)$$

Because it is uniform motion, so $\dot{\omega} = 0$

$$\omega \times (\omega \times x_s) = \omega^2 x_s$$

$$\overrightarrow{\ddot{x}_{c+s}} = \ddot{x}_c + \ddot{x}_s + 2\omega\dot{x}_s + \omega^2 x_s \quad (19)$$

Vector analysis is the second step of the Deformation-Motion Vector Analysis Method. When comparing the analysis results of the two states, we can see that looseness is not only the geometric superposition of the initial state, but also the relative speed and Coriolis acceleration, which are the main causes of nonlinearity of the loose fault and instability of the system. (The stability of the system is discussed in another paper.)

5 Rotor Looseness Condition Differential Equations and The Solution

5.1 Rotor Stiffness Assumption for Looseness

When loosening occurs, the local support stiffness of the rotor system is reduced to $k_s = \eta_s k_c$ order $\eta_s \leq$. Looseness stiffness of a cycle of the rotor is a non-linear step-like stiffness, and the periodic stiffness is:

$$k_{c+s} = \begin{cases} k_c & {}^{\kappa}_{\Delta}x = 0 \\ k_s = \eta_s k_c \propto F(x_s) & 0 \leq x_s = {}^{\kappa}_{\Delta}x \leq x_P \end{cases} \quad (20)$$

Rotor Differential Equation in Looseness Condition
Although the rotor overlays the loose deformation on the initial fixed axis rotation, the rotor is still a closed system and satisfies the original equilibrium condition. Since the form of the external excited force is unchanged, the differential equation of the rotor in the looseness condition is:

$$m\overrightarrow{\ddot{x}_{c+s}} + c\overrightarrow{\dot{x}_{c+s}} + k_{c+s}\overrightarrow{x_{c+s}} = F_c \sin\omega t \quad (21)$$

Substitute equation (14), (15), (19), (20) into equation (21):

$$m(\ddot{x}_c + \ddot{x}_s + \omega^2 x_s + 2\omega\dot{x}_s) + c(\dot{x}_c + \dot{x}_s + \omega x_s) + k_c x_c + k_s x_s = F_c \sin\omega t \quad (22)$$

After decomposition and merge:

$$(m\ddot{x}_c + c\dot{x}_c + k_c x_c) + (m\ddot{x}_s + c\dot{x}_s + k_s x_s) + m(\omega^2 x_s + 2\omega\dot{x}_s) + c\omega x_s = F_c \sin\omega t \quad (23)$$

Where, $m\ddot{x}_c + c\dot{x}_c + k_c x_c = F_c \sin\omega t$ is the differential equation of the initial condition;

Where, $m\ddot{x}_s + c\dot{x}_s + k_s x_s = 0$ is the differential equation of the loose motion in the moving coordinate system;

Where, $m\ddot{x}_s + c\dot{x}_s + k_s x_s = -m\omega^2 x_s - 2m\omega\dot{x}_s - c\omega x_s$ is the differential equation of the loose motion in the static coordinate system.

It can be seen from the deconstruction of the above differential equation (23) that the initial state provides the initial condition for the loose motion in the static coordinate system.

In the differential equation, the $m\omega^2 x_s$ term is the inertial force caused by the amplitude growth of looseness, and the $2^2 x_s$ and $c\omega x_s$ are the interaction terms between the rotor's fixed axis rotation and the loose deformation, $2m\omega x_s$ is the cross product of the angular velocity in the static coordinate system and the loose deformation in the moving coordinate system ,and is the Coriolis acceleration. Now, $m\omega^2 x_s$ $2m\omega x_s$ and $c\omega x_s$ are the excited forces of the loose fault differential equation in the static coordinate system.

The Analytical Solution of Looseness Condition

Rotor loose fault is deconstructed into the following four sets of differential equation:

1、 $m\ddot{x}_c + c\dot{x}_c + k_c x_c = F_c \sin\omega t$

$$x_c = \frac{F_c}{k_c\sqrt{(1-\lambda^2)^2 + (2\xi\lambda)^2}}\sin(\omega t - \varphi) \quad \ldots \quad (2)$$

intial condition $X_{c0} = X_c \sin\omega\tau/2$, $\dot{x}_{c0} = 0$

2、 $m(\ddot{x}_s) + c(\dot{x}_s) + k_s x_s = -m\omega^2 x_s$

$$x_{s2} = \frac{-\lambda_s^2 \varepsilon_n {}_{\Delta}^{\kappa}x}{\sqrt{[1-(n\lambda_s)^2]^2 + (2\xi_s n\lambda_s)^2}}\cos(n\omega t - \varphi_s)\ldots \quad (24)$$

$$\lambda_s = \frac{\omega}{\omega_{ns}}, \tan\varphi_s = \frac{2\xi_s\lambda_s}{1-\lambda_s^2}, \quad \xi_s = \frac{c}{c_{cs}}, \quad c_{cs} = 2m\omega_{ns}$$

3、 $m(\ddot{x}_s) + c(\dot{x}_s) + k_s x_s = -2m\omega\dot{x}_s$

$$x_{s3} = \frac{2n\lambda_s^2 \varepsilon_n {}_{\Delta}^{\kappa}x}{\sqrt{[1-(n\lambda_s)^2]^2 + (2\xi_s n\lambda_s)^2}}\sin(n\omega t - \varphi_s)\ldots \quad (25)$$

4、 $m(\ddot{x}_s) + c(\dot{x}_s) + k_s x_s = -c\omega x_s$

$$x_{s4} = -\frac{c\omega \frac{{}_{\Delta}^{\kappa}x}{2}}{k_s} + \frac{-2\xi_s\lambda_s\varepsilon_n {}_{\Delta}^{\kappa}x}{\sqrt{[1-(n\lambda_s)^2]^2 + (2\xi_s n\lambda_s)^2}}\sin(n\omega t - \varphi_s)\ldots \quad (26)$$

the analytical soluton of loose fault:

$$x_{c+s} = \frac{F_c}{\sqrt{(1-\lambda^2)^2 + (2\xi\lambda)^2}}\sin\frac{\omega\tau}{2} - \frac{(m\omega^2 + c\omega){}_{\Delta}^{\kappa}x}{2k_s} - \sum_{n=1}^{\infty}\frac{4({}_{\Delta}^{\kappa}x)^2}{\pi^2 n^2\sqrt{[1-(n)^2]^2 + (2\xi_s n)^2}}\sin^2(\frac{n\pi}{2})\cos(n\omega t - \varphi_s) + \sum_{n=1}^{\infty}\frac{8({}_{\Delta}^{\kappa}x)^2(n\lambda_s^2 + \xi_s\lambda_s)}{\pi^2 n^2\sqrt{[1-(n\lambda_s)^2]^2 + (2\xi_s n\lambda_s)^2}}\sin^2(\frac{n\pi}{2})\sin(n\omega t - \varphi_s) \quad \ldots (27)$$

Analysis of analytical solution of loose fault:

The first term represents the initial condition when loosening occurs and is also the initial motion of the rotor; the second term represents the response of the system to the DC component; the third term represents the forced vibration caused by loosening inertia force, which is the steady-state vibration at the excitation frequency; the fourth term represents the forced vibration caused by the Coriolis acceleration, which is also the steady-state vibration at the excitation frequency, and Coriolis acceleration has not mentioned in previous studies of loose faults. Some general characteristics of loose fault can be obtained from the excitation source and frequency component of the analytical solution: The loose fault has the multiple harmonics of rotation frequency, and the harmonic amplitude converges with the law of $1/n^2$, and the multiple harmonics are the main theoretical basis to judge the rotor loosing fault.

At present, the numerical solution is widely used to solve rotating machinery faults, however the numerical solution seriously limits the in-deep analysis of fault mechanism. the innovation of Deformation-Motion Vector Analysis Method can solve the differential equation in a briefly reasonable and effective way. In this paper, by analyzing the excitation source and the frequency components of the analysis solution of looseness fault, the clear physical meaning of the fault can be obtained, and the existence of Coriolis acceleration is found(The Coriolis acceleration is discussed in another paper).

Rotor Rub-Impact Condition Differential Equations and the Solution

Rotor Stiffness Assumption for Rub-Impact

With the aggravation of the looseness, the increase of the rotor amplitude will exceed the design gap between the stator and the rotor, resulting in friction, so rubbing is the result of increased loosening. In the local contact where friction occurs, it objectively plays the role of auxiliary support, so the system stiffness can be expressed as the parallel connection between the original stiffness and the auxiliary support stiffness of the rubbing site $k_p = \eta_p k_s$,order $\eta_p \geq 1$. The Rub-impact stiffness of a cycle of the rotor is a non-linear step-like stiffness, and the periodic stiffness is:

$$k_{c+s+p}\begin{cases} k_c & {}_{\Delta}^{\kappa}x = 0_c \\ k_s = \eta_s k_c & 0 < x_s = {}_{\Delta}^{\kappa}x \leq {}_{\square}^{k}x_p \\ k_p = \eta_p k_s \propto F(x_p) & {}_{\Delta}^{\kappa}x \geq {}_{\Delta}^{\kappa}x_p \end{cases} \quad (28)$$

Rotor Differential Equation in Rub-Impact Condition

When the rub-impact occurs, the time-domain signal has the obvious characteristic of cutting-top, indicating that somewhere in the stator has interfered with the rotor. The occurrence of rubbing fault will change the loosing characteristics of the rotor. In addition to the initial exciting force $F_c \sin\omega\tau$ the rotor also received a periodic rectangular wave of friction, and this incentive force always occurs at a fixed point in the static coordinate system. Since there is no relative motion, the response of the system to the rubbing excitation force can be transformed into the response of the system to the periodic rectangular wave excitation force under the initial condition of loose fault. the differential equation of the rotor in the rub-impact condition is:

$$m\ddot{x}_{c+s+p} + c\dot{x}_{c+s+p} + k_{c+s+p}x_{c+s+p} = F_c \sin\omega t + f_p(t)\ldots(29)$$

$$f_p(t) = k_p\left[\frac{\frac{k}{\square}x\iota}{T} + \frac{\frac{k}{\square}x\iota\omega}{\pi}\sum_{n=1}^{\infty} Sa(\frac{n\omega\iota}{2})\cos(n\omega t)\right]\ldots \quad (30)$$

So the system response to the periodic rectangular wave excitation force:

$$m\ddot{x}_p + c\dot{x}_p + k_p x_p = f_p(t) \quad (31)$$

$$\left.\begin{aligned} m\ddot{x}_p + c\dot{x}_p + k_p x_p &= k_p\frac{\frac{k}{\square}x\iota}{T} \\ m\ddot{x}_p + c\dot{x}_p + k_p x_p &= k_p\frac{\frac{k}{\square}x\iota\omega}{\pi}\sum_{n=1}^{\infty} Sa(\frac{n\omega\iota}{2})\cos(n\omega t)\end{aligned}\right\}\ldots(32)$$

$$x_p(t) = \frac{\frac{k}{\square}x\iota}{T} + \sum_{n=1}^{\infty}\frac{\frac{k}{\square}x\iota\omega Sa(\frac{n\omega\iota}{2})}{\pi\sqrt{[1-(n\lambda_p)^2]^2 + (2\xi_p n\lambda_p)^2}}\cos(n\omega t - \varphi_p)\ldots \quad (33)$$

$$\tan\varphi_p = \frac{2\xi_p n\lambda_p}{1-(n\lambda_p)^2},\quad \lambda_p = \frac{\omega}{\omega_p},\quad \xi_p = \frac{c}{c_{cp}},\quad c_{cp} = 2m\omega_{np}$$

The total responses of the system is:

$$\begin{aligned} x_{c+s+p} &= x_{c+s} + x_p \\ &= \frac{F_c}{\sqrt{(1-\lambda^2)^2 + (2\xi\lambda)^2}}\sin\frac{\omega\tau}{2} - \frac{(m\omega^2 + c\omega)\frac{\kappa}{\Delta}x}{2k_s} \\ &- \sum_{n=1}^{\infty}\frac{4(\frac{\kappa}{\Delta}x)^2}{\pi^2 n^2\sqrt{[1-(n)^2]^2 + (2\xi_s n)^2}}\sin^2(\frac{n\pi}{2})\cos(n\omega t - \varphi_s) \\ &+ \sum_{n=1}^{\infty}\frac{8(\frac{\kappa}{\Delta}x)^2(n\lambda_s^2 + \xi_s\lambda_s)}{\pi^2 n^2\sqrt{[1-(n\lambda_s)^2]^2 + (2\xi_s n\lambda_s)^2}}\sin^2(\frac{n\pi}{2})\sin(n\omega t - \varphi_s) \\ &+ \frac{\frac{k}{\square}x\iota}{T} + \sum_{n=1}^{\infty}\frac{\frac{k}{\square}x\iota\omega Sa(\frac{n\omega\iota}{2})}{\pi\sqrt{[1-(n\lambda_p)^2]^2 + (2\xi_s n\lambda_p)^2}}\cos(n\omega t - \varphi_p)\ldots(34)\end{aligned}$$

Analysis of analytical solution of rub-impact fault:

The analytical solution of the loose fault is retained in the analytical solution of the rub-impact fault as the initial condition of the rub-impact fault solution, which is of great significance, that is, it defines the existence condition of the rub-impact fault and the necessary signal component prior to the rub-impact signal; The second term represents the effect of the system on the DC component and does not cause vibration;

the third term represents the forced vibration caused by the rub-impact fault, which is the steady-state vibration at the excitation frequency.

By means of the excitation source and frequency component of the rub-impact analytical solution, Some universal characteristics of rub-impact fault can be analyzed: similar to loosing fault, rub-impact fault also has multiple harmonics of rotation frequency, but different from the features of decreasing amplitude of loose fault, the amplitude of harmonic converges with the law of 1/n . Therefore, the amplitude of rub-impact fault is higher than that of the loose fault at some special frequencies, and then combined with the characteristic of cutting-top, there are the theoretical basis to judge that the rotor system develops from the loose fault to the rub-impact fault and can be distinguished from the loose fault.

It is worth noting that the analytical solution of the rub-impact fault is developed from the initial state to the loosening phase, while the fault components before the rub-impact fault are hidden in the harmonics of the spectrum.

Natural Frequency Migration Resonance

From the analytic solution of the loose fault, we know that, when $1-(n\lambda_s)^2=0$, the denominator of the analytic solution tends to zero, the motion of rotor will have essential changes, so it represents the motion that can not be realized in practice. when $1-(n\lambda_s)^2=0$, it also means that a certain harmonic frequency of the interfering force is equal to the natural frequency of the loose state, that is $n\omega=\omega_{ns}$, If this piece is established, the system will resonate, called multi-harmonic resonance. Similarly, as in the loose state, when $1-(n\lambda_s)^2=0$, a certain harmonic frequency of the interfering force is equal to the natural frequency of the rubbing state, that is $n\omega=\omega_{np}$, the system will resonate, also known as multi-harmonic resonance.

Resonance Formation and Inevitable Trend–Migration Resonance

If the long-term process of the failure and the development of the fault is studied, the $F(x_s)$ and $F(x_p)$ of steady-state analytical solutions of the looseness and rub-impact fault represent the deterioration degree of the fault and will continue to increase. According to the hypothesis of rotor stiffness in case of loose fault and rub-impact fault, although the stiffness of both loose and rub-impact can be approximately constant when observed in a short time, it is easy to understand from the perspective of the whole life cycle of the equipment that the peak value of loosening continues to increase because of the continuous decrease of system stiffness. Conversely, the reason for the continuous increase of the rub-impact peak is the continuous increase of the system stiffness. The change of system stiffness will inevitably lead to the change of the rotor vibration characteristics.

To understand the dynamic change of the rotor vibration characteristics theoretically and master its law as shown in Figure 8, the resonance characteristics of the rotor can be further analyzed in depth according to the research procedure of natural frequency migration resonance.

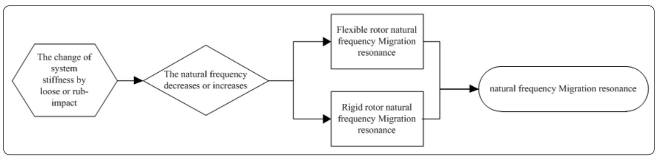

Figure 8: Flow of natural frequency migration resonance

From the basic conditions of normal rotor resonance, the stiffness of the rotor and the quality is not variable, so the natural frequency ω_n is a constant, but when the system respectively in loose or rub-impact fault and continuous deterioration, rotor system stiffness respectively to the smaller and larger into two directions, Causes a migration of the natural frequencies of the system, as shown in figure 9 The natural frequencies migration the coordinate system.

The natural frequency of the loose state: $\omega_{ns} = \sqrt{\frac{k_s}{m}} \prec \omega_n$

The natural frequency of the rub-impact state: $\omega_{np} = \sqrt{\frac{k_p}{m}} \succ \omega_n$

Taking the natural frequency of the initial state system as the origin of coordinates, and the frequency (Hz) as the unit. the forward coordinate is the direction of frequency increase, and rub-impact occurs, while the reverse coordinate is the direction of frequency decrease and loosens. So the migration coordinate system of the natural frequency is established.

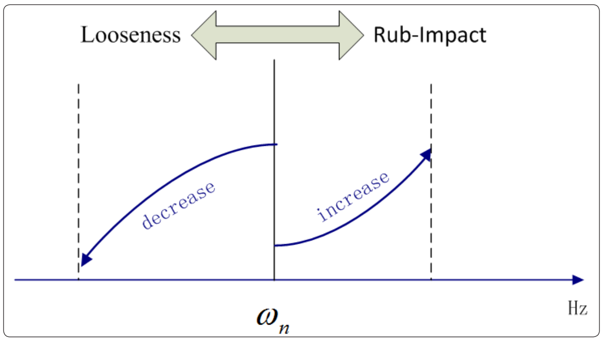

Figure 9: Natural frequency Migration coordinate system

Flexible Rotor Natural Frequency Migration Resonance
The natural frequency of the flexible rotor is less than the rotation frequency of the rotor, and the migration of its natural frequency and the coupling to the rotation frequency components are shown in FIG. 29. When the rotor has a loose fault, the natural frequency becomes smaller, and Sub-harmonic resonance occurs when it becomes small enough that $\omega_{ns}\left(=\sqrt{\frac{k_s}{m}} \prec \omega_n\right)$ and $\frac{1}{n}\omega\left(n \succ 1\right)$ are coupled.In contrast,when the rotor has a rubbing fault, the natural frequency increases, and the main resonance occurs when $\omega_{np}\left(=\sqrt{\frac{k_p}{m}} \succ \omega_n\right)$ is coupled with the rotational frequency ω.

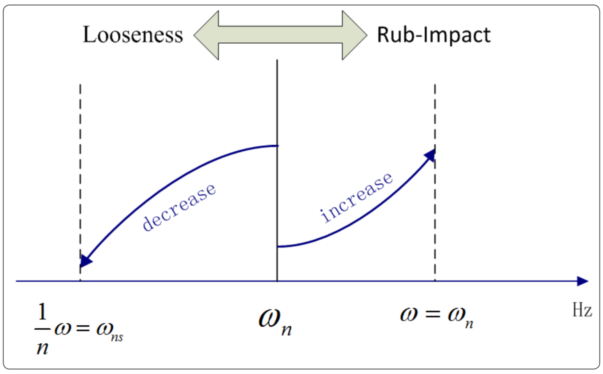

Figure 10: Variation of natural frequency of flexible rotor and coupling to rotation frequency components

Rigid Rotor Natural Frequency Migration Resonance
Same principle as flexible rotor, the natural frequency of the rigid rotor is greater than the rotor rotation frequency. The migration and change of the natural frequency of the rigid rotor and the coupling to the rotation frequency components are shown in FIG. 30. When the rotor has a loose fault, the natural frequency becomes smaller, and when it becomes smaller, High harmonic resonance occurs when $\omega_{ns}\left(=\sqrt{\frac{k_s}{m}} \prec \omega_n\right)$ decreases to the $\left(n-1\right)\omega\left(n \succ 1\right)$ coupled.When there is a rubbing fault of the rotor, the natural frequency increases, and when $\omega_{np}\left(=\sqrt{\frac{k_p}{m}} \succ \omega_n\right)$ becomes large enough to be coupled with the rotational frequency, high-order harmonic resonance will also occur.

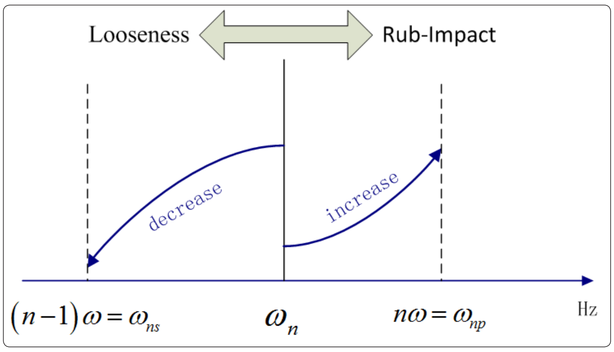

Figure 11: Variation of natural frequency of rigid rotor and coupling to rotation frequency components

The main conclusion can be drawn from the above analysis results: when the rotor is in a loose or rubbing fault, in addition to a large number of harmonics will cause various additional harmonic resonances, the migration of the natural frequency will also cause fault resonance. As the frequency and phenomenon of the resonance in the fault state are different from that of the conventional resonance, the migration of the natural frequency enriches the rotor resonance theory, as shown in Figure 12, which provides a theoretical basis for solving engineering problems that cannot be explained by the conventional resonance theory.

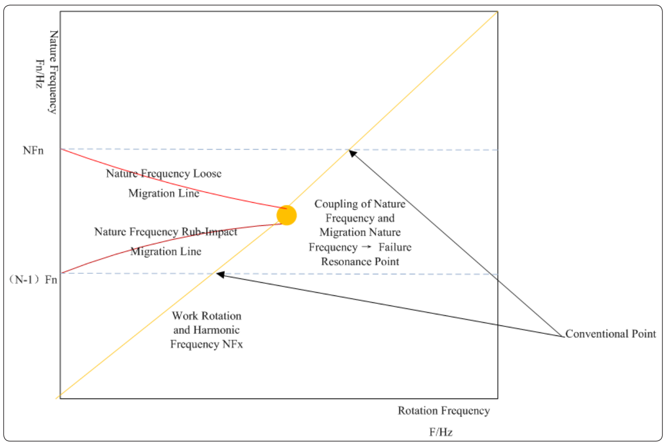

Figure 12: The natural frequency migration resonance Campbell diagram

Engineering Application
Rotor loose-rubbing fault is one of the most common faults in engineering. the analytical solution of looseness coupling rub-impact fault is used to diagnose a real fault.

Diagnosis and Treatment of Rotor "Loose-Resonance" Fault
In the process of signal analysis of fault diagnosis, frequency components that are lower than rotor rotation frequency are sometimes found to appear suddenly, which cannot be "justified" by existing theories. The following engineering problems of loose fan bearing are reasonably explained by natural frequency migration resonance theory.

Citation: HU Jun (2020) A Rotor Looseness – Rub-Impact – Resonance Fault Mechanism Modeling. Journal of Physics & Optics Sciences. SRC/JPSOS/133. DOI: doi.org/10.47363/JPSOS/2020(2)122

Introduction of fault equipment

The fault equipment is the fan of an iron making production line in a metallurgical enterprise, as shown in Figure. 13

Fan parameters:

Fan model 5-50 NO18D

Motor model Y450-6-315

Bearing model 22222CA/W33

Motor revolution 993CPM(16.56Hz)

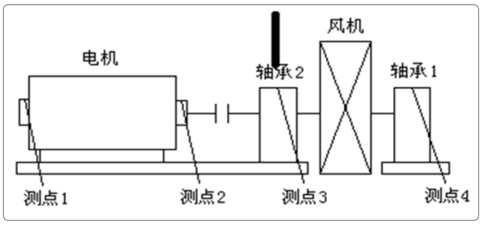

Figure 13: Structure diagram of failed fan

Introduction to the Faulty Equipment

The vibration of the bearing is too large during the operation of the fan. The fan bearing is replaced twice in a row, but the fault remains the same. Vibration monitoring of the fan reveals that the vibration value of the bearing at the load end of the fan is too large.

Table 1: effective vibration values of each measuring point of the fault fan (unit :RMS)

	Vertical	horizontal	axial	footing
Motor non-load side	2.13	2.93	3.54	0.65
Motor load side	2.59	3.12	2.87	0.98
Fan load side	6.52	7.45	4.73	3.93
Fan unloaded side	2.69	3.82	4.12	2.32

Fault Analysis and Treatment

Based on the analysis of the spectrum diagram of the fan load side bearing, it is found that there are a large number of high harmonics with rotating frequency. Calculated in the fan input side level time-domain waveform and spectrum figure 12 figure 13 can be found in the clear turn motor frequency of 16.56 Hz and a half times the frequency of 8.285 Hz, according to the analytical solution of resonant trend as a result, the discussion of this fault is the migration of flexible rotor natural frequency of the change and fraction frequency and phase composition of the coupling process, a half frequency doubling is the system as a result of loose sub-harmonic resonance happens, must check collapse.

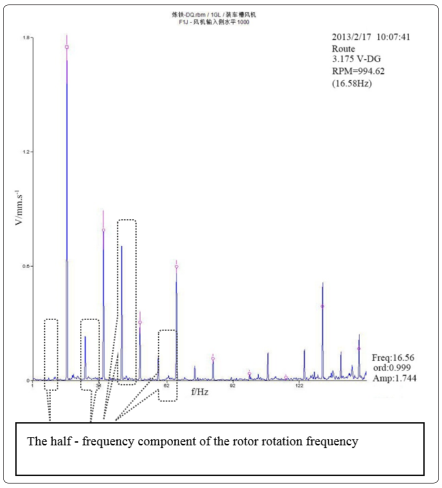

Figure 14: Frequency spectrum diagram of horizontal 1000HZ at the input side of fan

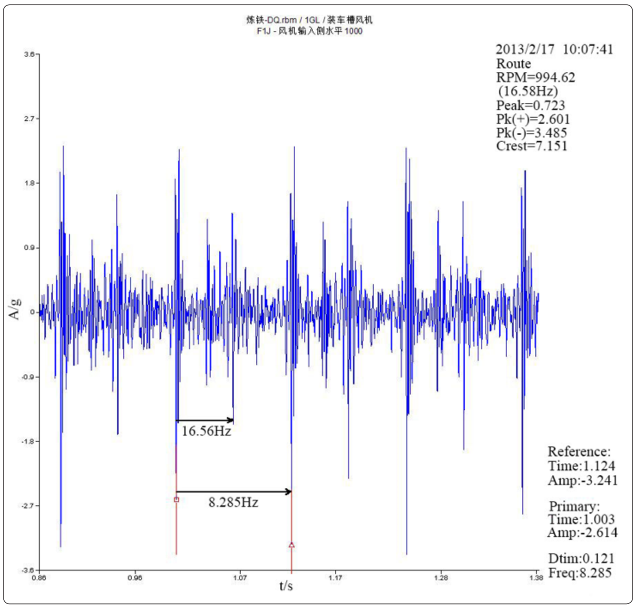

Figure 15: Time domain waveform of horizontal 1000Hz at the input side of the fan

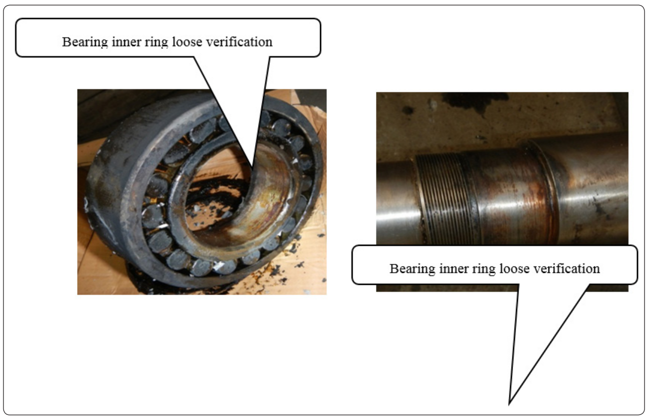

Figure 16: Verification of looseness of bearing inner ring

After the collapse of found loose bearing inner ring and the shaft has obvious phenomenon is shown in figure 14, after processing, such as contrast figure 15 shows low amplitude and a half times the frequency signal to eliminate.

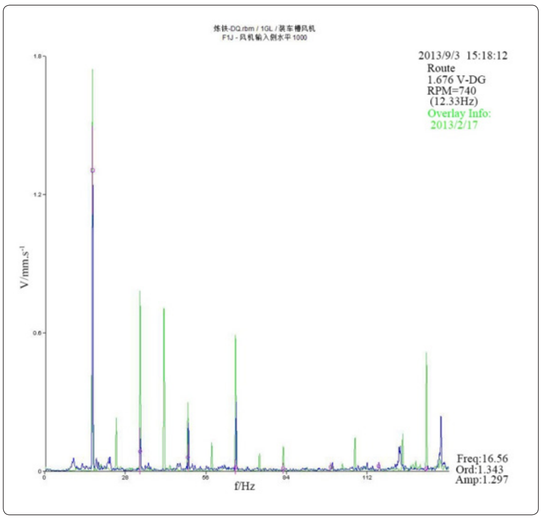

Figure 17: Elimination of loosing fault of bearing inner ring

Loose is just the beginning of the fault, if the fault after the start of looseness of bearing inner ring could not be the correct analysis and timely treatment, then, will continue to develop as the dynamic and static rotor rubbing, and resonance cause damage quickly. The following part will analyze a complete engineering case of rotor "loose-friction-resonance" fault.

Introduction of Fault Equipment

The fault equipment is the fan of an iron making production line in a metallurgical enterprise, as shown in Figure. 13

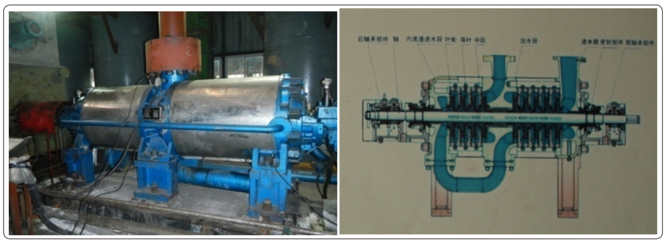

Figure 8: Structure and schematic diagram of the fault pump

The fluid enters through the inlet flange of the pump and into the guide vane after pressurization of the first stage impeller. The guide vane decelerates the fluid and guides it to the inlet of the next stage impeller. In this cycle, the fluid is pressured step by step through the impeller and then left to the low-pressure ring cavity of the outlet section. Thus, the pressurization of the fluid in the low-pressure section of the pump is completed. The low-pressure fluid flows into the high-pressure inlet section through the internal tube, and then passed through the high-pressure section. The fluid is pressurized step by step by the impeller and then slip into the outlet section ring chamber to complete the whole pressurization process. Mechanical structure of the pump: On the shaft forming the pump rotor, the impeller is arrange symmetrically at both ends of the shaft in the high and low pressure areas. the central part of the high and low pressure areas is sealed by a screw, and the gap between the seal and the shaft is 0.15mm. Process requirements when the pressure is insufficient to fill water, so frequent rate of rise and fall speed is the characteristics of this type of pump, motor rotation frequency between 46-49.

In the initial stage, the rotor of the pump is running at the rated speed. As shown in Figure10, the pump operates at a small amplitude during the stabilization phase, which corresponds to the rotor initial state in Figure3.

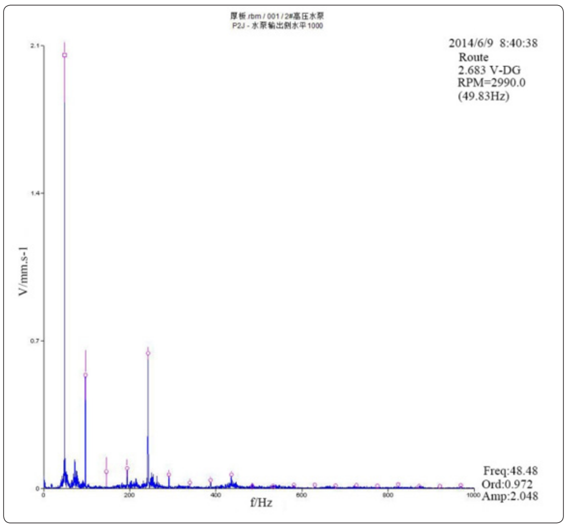

Figure 9: The pump operates with small amplitude

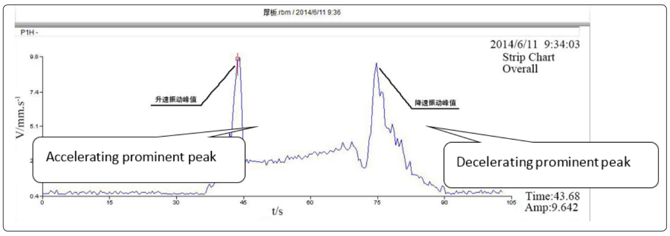

Figure 10: The pump increase and decrease RMS record

The record of the vibration velocity value of the pump measured on June 11, 2014 in the process of water replenishment is shown in Figure10. The pump has two prominent peak value in the process of rise and fall, indicating that the pump has passed the critical speed twice, respectively. The low amplitude phase between the two peaks represents the normal operation of the pump at rated speed.

The waterfall diagram of the pump in the initial stage is shown in Figure11. In the figure, it can be clearly observed that the natural frequency of the pump rotor in normal operation is 43.5Hz.

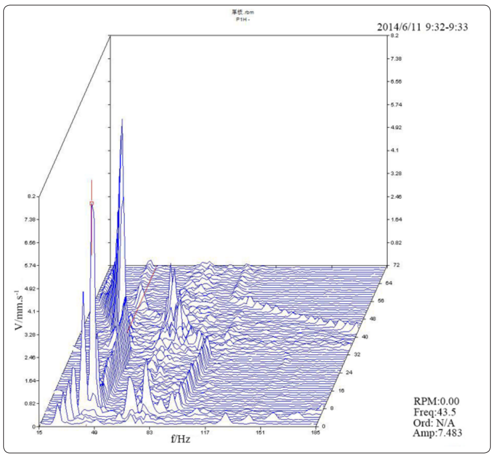

Figure 11: The pump increase speed and decrease speed waterfall plot

Vibration signal Analysis of Looseness Coupling Rub-Impact

According to the cycle test (1 times / week) as shown in Figure12, the vibration velocity value of the pump increased significantly from the second week.

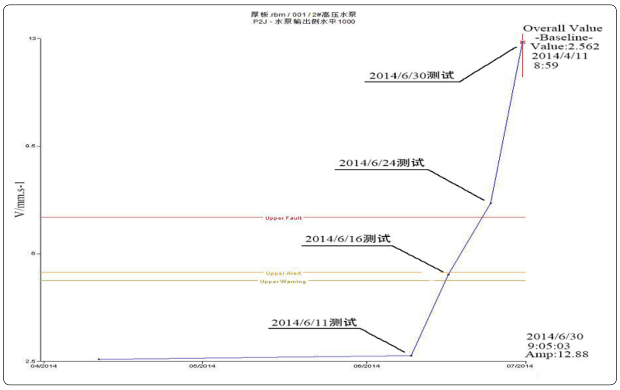

Figure 12: Vibration trends of the pump

The high amplitude state in the spectrum diagram corresponds to the high order harmonic component representing the loose fault, as shown in Figure 13.

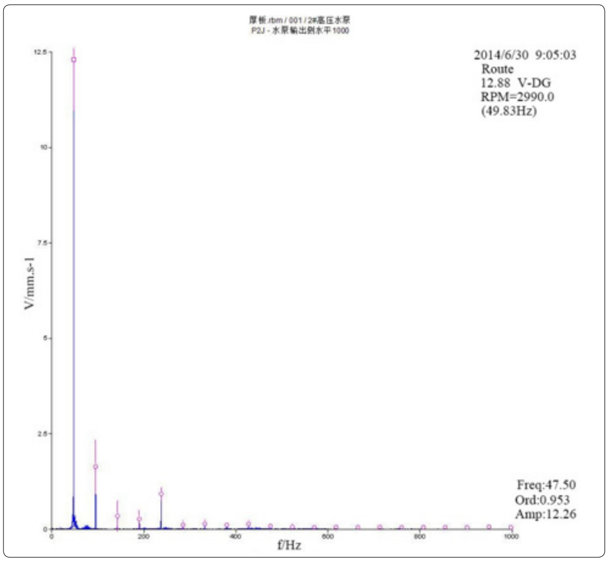

Figure 13: The high order harmonic of the loose fault

In the time domain waveform diagram, the chipping characteristic representing the rub-impact fault is shown in Figure14.

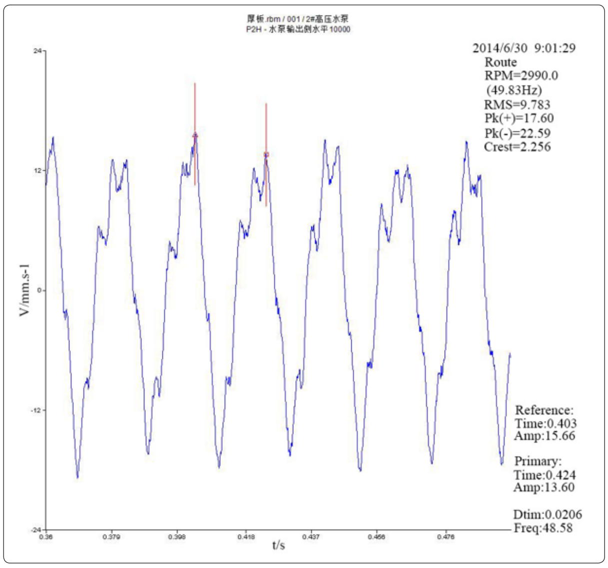

Figure 14: Clipping signal of rub-impact fault

Clipping frequency is 48.58Hz, within the frequency range of 46-49Hz. The reason for chipping is that the rotor shaft is too deformed due to the loosening fault of the rotor in the early stage, resulting in scraping and bumping between the rotor and the seal.

The Actual Verification

After the pump was disassembly on August 13, 2014, it was found that a large area of the middle seal of the pump was scratched, as shown in Figure15, which verified the correctness of the analytical solution of the rotor loose and rub-impact fault with the Deformation-Motion Vector Analysis Method.

Figure 15: Scraping touch on the pump sealing

Conclusions

1. A method of modeling and solving the coupling faults of rotating machinery is presented, which is called the Deformation-Motion Vector Analysis Method. Through this method , first, the relationship between the fault Deformation Motion and the periodical signal is established by the Fourier series. Secondly, the relationship between the fault Deformation Motion and the excitation forces is established by the plane motion vector analysis method. Finally, the harmonic balance method is used for modeling and solving.
2. The advantage of the Deformation-Motion Vector Analysis Method is that any periodic deformation motion can be defined and expressed by the Fourier series, such as looseness, rub-impact and other coupling faults can be expressed by the differential equation and its analytical solution can be obtained. At the same time, the conclusion of the analytical solution is also the theoretical basis for judge the looseness and rub-impact fault.
3. As far as the numerical solution method is concerned, the great difference of the Deformation-Motion Vector Analysis Method lies in that this method finds the coriolis acceleration which is ignored by numerical solution, so the Deformation-Motion Vector Analysis Method is more scientific and reasonable. The analytical solution of the new method also overcomes the specific limitation of the numerical solution, which is more conductive to the deeper fault mechanism analysis.

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